Modeling of the Small Strain Shear Modulus on a Fiber Reinforced Sand

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Abstract. The mechanical behavior of fiber reinforced soils has been extensively studied in the last decades. Previous studies have shown that inclusion of fibers increases the shear strength of the reinforced soil. However in some cases the presence of fibers can reduce the stiffness of the composite material. In this paper, we study the change on the initial stiffness in an alluvial sand reinforced with polypropylene fibers. A model based on Hertz elastic contact theory is developed in order to explain the trends of the maximum shear modulus in the fiber reinforced sand as the fiber content is varied. The model assumes that the shear wave is transmitted through elastic distortions at the contacts, so the stiffness of the contacts governs the initial shear modulus, which in turn is affected due to fibers addition. Furthermore, the ratio between the amount of grain to fiber contacts and the total of contacts on the shear wave path influence the maximum shear modulus. An experimental testing program involving confined compression tests with shear wave velocity measurements of unreinforced and fiber-reinforced sand specimens was undertaken to validate the proposed model trends. The model predictions were found to agree well with the experimental results.

Keywords. Sand with fibers, shear modulus, shear wave velocity.

1. Introduction

Soil reinforcement by means of fiber addition has been reported in the last few decades by several investigators ([1], [2], [3], [4], [5], and [6]). In general, studies show that addition of fibers increase the shear strength at large strains of the reinforced soil. However, as far as we know, only a few studies dealing with the effect of fiber inclusion on the stiffness of reinforced soil at low strain levels have been published. Among these, Heineck et al. ([7]) observed that inclusion of fibers do not change the initial stiffness at low strain levels (10^{-5}) of the reinforced soil, when the fiber content is up to 0.5% by weight of dry soil. Furthermore, Diambra et al. ([8]) observed that shear modulus at medium strain levels (10^{-3}) is not affected by fibers incorporation. However, other investigators suggest that when fiber content is higher than 0.5% by weight, the stiffness of the reinforced soil at low strain levels is reduced ([9]).

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This work presents a model based on Hertz theory (elastic contact theory) developed with the aim to explain the trends of the maximum shear modulus (G_{max}) in a fiber reinforced sand as the fiber content is varied. The model predictions are validated by means of an experimental testing program.

2. Physical Model of Contacts

2.1. Introduction to the Model

At low strain levels (10^{-5} or less), it can be assumed that soils behave elastically, so there is a unique and direct relation between shear wave velocity (V_s) and initial shear modulus or maximum shear modulus (G_{max}) given by:

$$G_{max} = \rho \cdot V_s^{\,2} \quad (1)$$

The shear wave velocity is calculated by measuring the time that a mechanical shear wave needs to travel a certain distance along a soil specimen. This shear wave is transmitted through elastic distortions at the contacts of the soil grains and the contacts of fibers to soil grains (Figure 1).

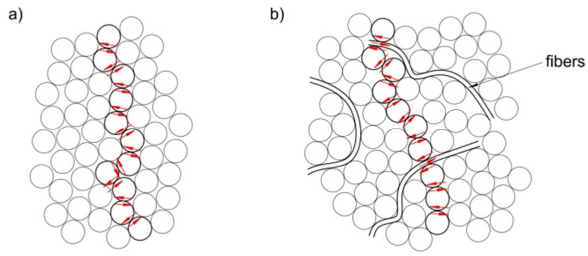

Figure 1. (a): The shear wave is transmitted through elastic distortions at the grain-to-grain contacts; (b): The shear wave is transmitted through elastic distortions at the grain-to-grain and fiber-to-grain contacts.

Thus, the initial shear modulus of the fiber reinforced soil (G'_{max}) is a function of the grain-to-grain contact stiffness (μ_{gg}), the fiber-to-grain contact stiffness (μ_{gf}), the number of grain-to-grain contacts (N_{gg}) and the number of fiber-to-grain contacts (N_{gf}). We define the parameters α as the ratio between the fiber-to-grain contact stiffness and the grain-to-grain contact stiffness (Eq. (2)), and β as the ratio of the number of fiber-to-grain contacts to the total of contacts through which the shear wave is transmitted (Eq. (3)).

$$\alpha = \frac{\mu_{gf}}{\mu_{gg}} \quad (2)$$

$$\beta = \frac{N_{gf}}{N_{gf} + N_{gg}} \quad (3)$$

When fiber content is zero, the initial shear modulus is the shear modulus of the soil without reinforcement (G_{max}) and β is equal to zero. Contrarily, if all contacts are of the fiber-to-grain type, the shear modulus will take a value G''_{max} directly related to the grain-to-fiber stiffness, and β will be equal to one. Finally, the shear modulus G'_{max} of the fiber reinforced soil is given as:

$$G'_{max} = \beta \cdot G''_{max} + (1 - \beta) \cdot G_{max} \tag{4}$$

For grains with a stiffness very high compared with the stiffness of the contacts, it is reasonable to assume the relationship showed in Eq. (5).

$$\frac{G''_{max}}{G_{max}} = \frac{\mu_{gf}}{\mu_{gg}} = \alpha \tag{5}$$

And then, the initial shear modulus of the fiber reinforced soil is obtained as:

$$G'_{max} = G_{max} \cdot [1 - \beta \cdot (1 - \alpha)] \tag{6}$$

2.2. Determination of α

The parameter α relates the stiffness between fiber-to-grain contacts and grain-to-grain contacts. In order to quantify this parameter, Hertz Theory of Contacts Mechanics is used. Particles of soils are idealized as spheres, while fibers are assumed to be cylinders (Figure 2). Both materials are considered elastic.

According to the Hertz Theory, the shear stiffness of a contact (μ) between two elastic bodies is:

$$\mu = 2 \cdot G^* \cdot r_c \tag{7}$$

Where r_c is the contact radius between the two bodies, and G* is the effective shear modulus given by a combination of the elastic properties of the two bodies under consideration (Eq. (8)).

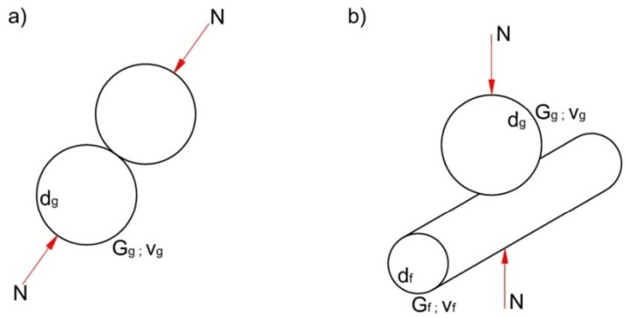

Figure 2. Hertz theory of contact mechanics, a) particles of soils are idealized as spheres and, b) fibers are considered cylinders.

$$G^* = \left(\frac{2 - \nu_1}{4 \cdot G_1} + \frac{2 - \nu_2}{4 \cdot G_2}\right)^{-1} \tag{8}$$

Introducing (8) into (7) we obtain:

$$\mu_{gg} = \frac{4 \cdot G_g}{1 - \nu_g} \cdot r_c \tag{9}$$

In Eq. (9), G_g is the shear modulus and ν_g is the Poisson ratio of the material of the grains of soil, and r_c is the contact radius between two grains of soils of diameter d_g, given by Eq. (10):

$$r_c = \sqrt[3]{\frac{3}{16} \cdot \frac{d_g \cdot (1 - \nu_g)}{G_g} \cdot N} \tag{10}$$

being N the contact force.

On the other hand, the fiber-to-grain stiffness is given by the following equation:

$$\mu_{gf} = 2 \cdot \frac{1}{\frac{2-\nu_g}{4 \cdot G_g} + \frac{2-\nu_f}{4 \cdot G_f}} \cdot r'_c \tag{11}$$

Where G_f is the shear modulus and ν_f is the Poisson ratio of the material of the fiber, respectively, and r'_c is the contact radius between the fiber and the soil grain, which can be approximated by:

$$r'_c = \sqrt[3]{\frac{3}{8} \cdot \left(\frac{1 - \nu_g}{2G_g} + \frac{1 - \nu_f}{2G_f} \right) \cdot \left[\frac{1}{d_g} \cdot \left(\frac{1}{d_g} + \frac{1}{d_f} \right) \right]^{-1/2} \cdot N} \tag{12}$$

Combining Eqs. (9), (10), (11) and (12), we obtain the parameter α as follow:

$$\alpha = \frac{\mu_{gf}}{\mu_{gg}} = \frac{2}{1 + \frac{2-\nu_f}{2-\nu_g} \cdot \frac{G_g}{G_f}} \cdot \left[\left(1 + \frac{1 - \nu_f}{1 - \nu_g} \cdot \frac{G_g}{G_f} \right) \cdot \left(\frac{1}{1 + \frac{d_g}{d_f}} \right)^{1/2} \right]^{1/3} \tag{13}$$

2.3. Determination of β

In order to evaluate β, it is assumed that fibers are uniformly distributed and randomly oriented in the soil mass. To quantify the number of contacts inside a soil cube, only contacts between particles of soil and contacts of soil particles and fibers are considered, but not contacts at the boundaries of the soil mass. In addition, in order to minimize the influence of the boundaries, a volume of soil large enough is considered to calculate β.

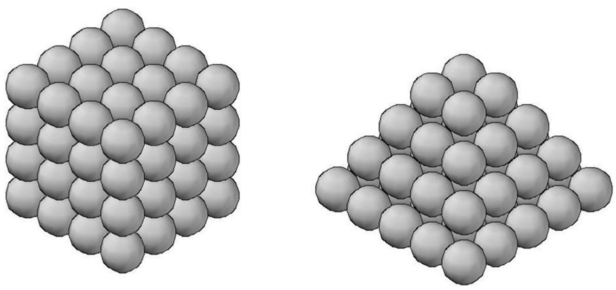

Figure 3. Simple cubic packing (left) and face-centered cubic packing (right).

Now, we analyze the influence of the packing on β parameter. For this, we studied two cases: a simple cubic packing for a soil in a loose state, and a face-centered cubic packing for a soil in a dense state (Figure 3).

2.3.1. Simple Cubic Packing

In the simple cubic packing, particles of soils are idealized as spheres of equal diameter, each of one in contact with other 6 spheres, so the coordination number (CN) is 6. Taking a cubic region of n^3 particles, the number of contacts will be:

$$N_c = 3 \cdot (n^3 - n^2) \tag{14}$$

Figure 4a shows the variation of the ratio "number of contacts" (N_c) to the "number of spheres" (N_s) with the sample weight, considering particles with specific gravity (γ_s) equal to 2,67. It can be seen from Figure 4a that for weights of soil over 250 gr the relation N_c/N_s is practically constant and approximately equal to 3.

Thus, considering that the number of contacts (N_c) is equal to three times the number of spheres (N_s), and the number of spheres is equal to the weight of soil specimen divided the weight of one sphere, the number of contacts can be described as a function of the soil sample weight (W_s), particle diameter (d_g) and specific gravity of soil (γ_s):

$$N_c = \frac{18}{\pi} \cdot \frac{W_s}{d_g^3 \cdot \gamma_s} \tag{15}$$

In order to evaluate the number of fiber-to-grain contacts, the number of fibers for certain fiber content (CF) is calculated (Eq. (16)). Then, the number of contacts between one single fiber and grains of soil is approximated by Eq. (17).

$$n^{\underline{o}} fibers = \frac{4 \cdot W_s \cdot CF(\%)}{\pi \cdot d_f^2 \cdot L_f \cdot \gamma_f \cdot 100} \tag{16}$$

$$n_{cf} = 2 \cdot \frac{L_f}{d_g} \tag{17}$$

In Eq. (16) γ_f is the specific gravity of the fiber, and in Eq. (17) L_f is the fiber length. The number of fiber-to-grain contacts is given by the combination of Eqs. (16) and (17):

$$N_{cf} = \frac{2}{25\pi} \cdot \frac{W_s \cdot CF(\%)}{\gamma_f \cdot d_g \cdot d_f^2} \tag{18}$$

Therefore, the parameter β will be:

$$\beta = \frac{N_{cf}}{N_c + N_{cf}} = \left(1 + \frac{225}{CF(\%)} \cdot \frac{\gamma_f}{\gamma_s} \cdot \frac{d_f^2}{d_g^2} \cdot \right)^{-1} \tag{19}$$

2.3.2. Face-centered Cubic Packing

The coordination number for the face-centered cubic packing is 12. Taking a cubic region of n spheres by side, the total number of spheres is in this case:

$$N_s = n^3 - n^2 + \frac{n}{2} \tag{20}$$

and the number of contacts is given by Eq. (21):

$$N_c = 6 \cdot n^3 - 16 \cdot n^2 + 14 \cdot n - 4 \tag{21}$$

Figure 4b shows the variation of the number of contacts to the number of spheres ratio with the weight of the soil mass. As it was highlighted for the simple cubic packing, for weights of soil over 250 gr. the relation N_c/N_s is nearly constant and, in this case, approximately equal to 6.

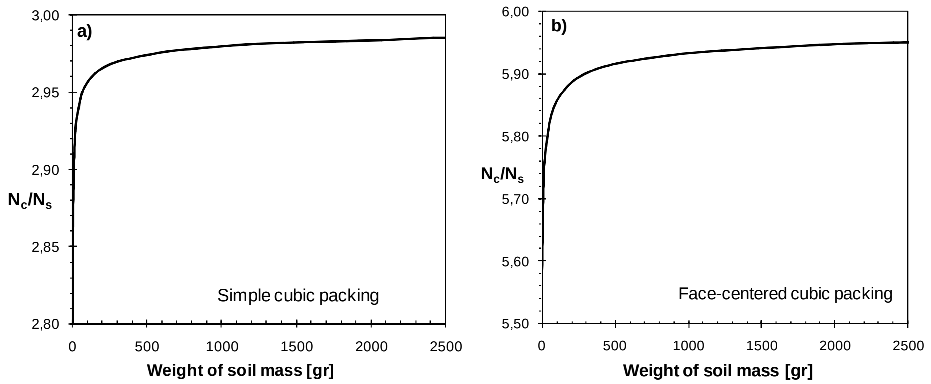

Figure 4. Number of contacts to number of spheres ratio versus weight of soil specimen: a) simple cubic packing, b) face-centered cubic packing.

Now we calculate the number of contacts as a function of soil weight:

$$N_c = \frac{36}{\pi} \cdot \frac{W_s}{d_g^3 \cdot \gamma_s} \tag{22}$$

The number of fiber-to grain contacts is calculated in the same way than for the simple cubic packing, but considering that each fiber has two times the contacts that the same fiber in a simple cubic packing, obtaining the following result:

$$N_{cf} = \frac{4}{25\pi} \cdot \frac{W_s \cdot CF(\%)}{\gamma_f \cdot d_g \cdot d_f^2} \tag{23}$$

The parameter β is obtained from combination of Eqs. (22) and (23):

$$\beta = \frac{N_{cf}}{N_c + N_{cf}} = \left(1 + \frac{225}{CF(\%)} \cdot \frac{\gamma_f}{\gamma_s} \cdot \frac{d_f^2}{d_g^2} \cdot \right)^{-1} \tag{24}$$

It can be seen that β does not depend on the packing, or the void ratio of the soil mass, but it is a function of fiber content, the specific gravity of each material, and the diameter of fiber to diameter of grain ratio.

3. Validation of the Model

3.1. Experimental Program

The soil used in the present study was an alluvial siliceous well-graded sand. Main geotechnical properties of the sand are listed in Table 1. Polypropylene fibers of 10 mm length and 0.16 mm in diameter were used throughout this work.

Table 1. Geotechnical properties of the soil used in this work.

U.S.C.S.	**C_U**	**C_g**	**%PT#200**	**γ_s**	**d_{50}**
SW	7.9	1.4	4.9	2.67	0.6 mm

U.S.C.S.: Unified Soil Classification System; C_U: coefficient of uniformity; C_g: coefficient of gradation; %PT#200: percentage of passing weight through the sieve IRAM Nº 200 (75um); γ_s: specific gravity; d_{50}: diameter of 50% passing weight.

To quantify the maximum shear modulus of the reinforced soil at low strain levels ($\varepsilon \approx 10^{-5}$), confined compression with shear wave velocity measurement tests were performed. These tests were conducted in a modified oedometer with the incorporation of bender elements in its upper and bottom caps. A detailed description of this equipment is given in [10].

3.2. Quantification of the Model Parameters

Table 2 shows the data used to calculate parameters α and β. Properties of fibers were obtained from the manufacturer. The grains diameter was assumed to be the d_{50}. Finally, elastic parameters of the soil grains were assumed to be equal to the granite properties, because of the nature of the sand.

Table 2. Soil and fibers data used in the calculation of the model's parameters.

Gg	**νg**	**dg**	**γg**	**Gf**	**νf**	**df**	**γf**
20000 MPa	0.25	0.6 mm	2.67	400 MPa	0.50	0.16 mm	0.90

3.3. Results and Discussion

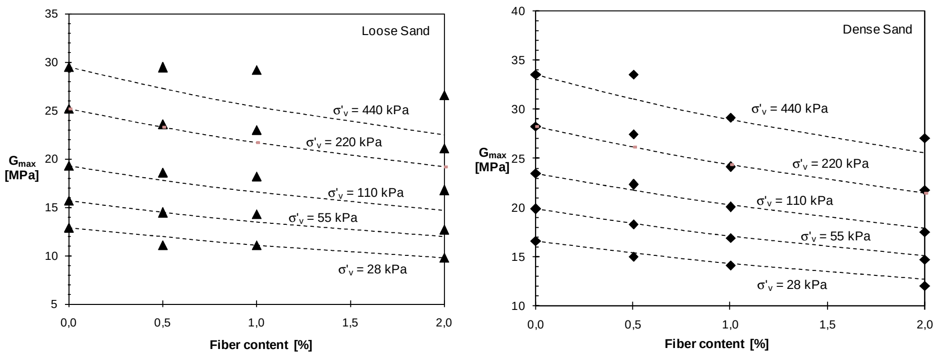

Figure 5. Maximum shear modulus versus fiber content, for different vertical pressures. Comparison between test results and predictive model; a) sand in a loose state; b) sand in a dense state.

Figure 5 shows the effect of fiber content on the maximum shear modulus, for vertical stresses ranging from 28 kPa to 440 kPa, corresponding to the sand in a loose state (5a) and the sand in a dense state (5b).

From these figures it can be seen that inclusion of fibers tends to reduce the initial stiffness of the reinforced soil at low strain levels. Also, a good agreement is observed between the trends of the experimental data and the model predictions.

4. Conclusions

A physical model based on Hertz theory is presented in order to explain and justify the maximum shear modulus drop as synthetic fibers are added to a fiber reinforced sand.

The mathematical model assumes that shear wave velocity and maximum shear modulus of the reinforced sand depends mainly on the stiffness of the grain to grain and fiber to grain contacts.

The model predictions are compared to experimental results obtained by means of bender element measurements in an alluvial clean silica sand reinforced with polypropylene fibers tested in confined compression state. The model predictions fit very well with the laboratory measurements.

The proposed model allow concluding that the maximum shear modulus of the reinforced sand decreases as the fiber content increases because of the drop of stiffness at particle contact level when fibers are added to the soil mass.

The ratio between the amount of grain to fiber contacts and the total of contacts on the shear wave path controls the maximum shear modulus value.

References

[1] D. H. Gray, H. Ohashi, Mechanics of Fiber Reinforcement in Sand, *Journal of Geotechnical Engineering* **109**, No. 3 (1983), 335-353.

[2] M. H. Maher, D. H. Gray, Static Response of Sands Reinforced with Randomly Distributed Fibers, *Journal of Geotechnical Engineering* **116**, No. 11 (1990), 1661-1677.

[3] R. L. Michalowski, A. Zhao, Failure of Fiber-Reinforced Granular Soils, *Journal of Geotechnical Engineering* **122**, No. 3 (1996), 226-234.

[4] J. G. Zornberg, Discrete framework for limit equilibrium analysis of fibre-reinforced soil, *Géotechnique* **52**, No. 8 (2002), 593-604.

[5] N. C. Consoli, K. S. Heineck, M. D. T. Casagrande, M. R. Coop, Shear Strength Behavior of Fiber-Reinforced Sand Considering Triaxial Tests under Distinct Stress Paths, *Journal of Geotechnical and Geoenvironmental Enginnering* **133**, No. 11 (2007), 1466-1469.

[6] E. Ibraim, A. Diambra, D. Muir Wood, A. R. Russell, Static Liquefaction of fibre reinforced sand under monotonic loading, *Geotextiles and Geomembranes* **28** (2010), 374-385.

[7] K. S. Heineck, M. R. Coop, N. C. Consoli, Effect of Microreinforcement of Soils from Very Small to Large Shear Strains, *Journal of Geotechnical and Geoenvironmental Enginnering* **131**, No. 8 (2005), 1024-1033.

[8] A. Diambra, E. Ibraim, D. Muir Wood, A. R. Russell, Fibre reinforced sands: Experiments and modeling, *Geotextiles and Geomembranes* **28** (2010), 238-250.

[9] R. L. Michalowski, J. Cermák, Triaxial Compression of Sand Reinforced with Fibers, *Journal of Geotechnical and Geoenvironmental Enginnering* **129**, No. 2 (2003), 125-136.

[10] V. A. Rinaldi, J. J. Clariá, Low strain dynamic behavior of a collapsible soil, *Proceedings of the XI Panamerican Congress of Soil Mechanics and Geotechnical Engineering*, Foz de Iguazú, Brasil (1999), Vol. 2, 835-841.