UNIVERSIDAD NACIONAL DE CÓRDOBA

FACULTAD DE LENGUAS

Construir el diálogo científico en la Matemática: la búsqueda del equilibrio entre símbolos y palabras en artículos de investigación sobre Teoría de Juegos

Constructing the Scientific Dialogue in Mathematics: The Search of an Equilibrium between Words and Symbols in Game Theory Research Articles

Maestría en Inglés con Orientación en Lingüística Aplicada

Trabajo de tesis presentado por: Graciela Lucero Arrua
Directora: Dra. Mariana Pascual

Abstract

Most scientific communication is conducted in English, which may be a difficult task and a source of obstacles for researchers whose primary language is not English (Bitchenera & Basturkmen, 2006; Borlogan, 2009; Duff, 2010; Matsuda & Matsuda, 2010). As a matter of concern for language scholars, this situation requires at least two actions: (1) the development of research focused on the problems faced by researchers when writing in a foreign language, and (2) the design and implementation of pedagogical and didactic programmes or services aimed at providing researchers with the tools to enhance their linguistic and rhetorical skills. In both cases, the ultimate objective of these lines of action is to help researchers integrate into and interact with their knowledge communities in an independent, active and successful way. Considering those needs and the emerging interest in English as a *lingua franca* or as an international language, many scholars have devoted to studying the features of writing and language use across the world and across disciplines (Hyland, 2004; Matsuda & Matsuda, 2010; Mercado, 2010). However, few have explored the case of Mathematics (Lemke, 2002; Morgan, 2008; O'Halloran, 2005; Schleppegrell, 2007), and even fewer have investigated the discourse of scientific research articles (SRA) in this discipline (Graves & Moghadassi, 2013, 2014). In view of this situation, investigation of the discourse of science in the field of Mathematics (Game Theory - GT) as used in the Institute of Applied Mathematics (IMASL), at the National University of San Luis (UNSL), becomes both an answer to local researchers' needs and an attempt to contribute to current research in writing, evaluative discourse and use of English as an international language for the communication of science. Thus, the main objective of this work is to conduct a comparative description between unpublished GT SRAs written in English by IMASL researchers and published GT SRAs written in English by international authors, in terms of linguistic features used to build authorship and authorial stance. The exploration of the genre is made from the perspective of the system of Appraisal (Hood, 2010; Martin & White, 2005; White, 2000), with the aid of Corpus Linguistics (CL) tools (Cheng, 2012; Meyer, 2002; Tognini-Bonelli, 2001). The results of this research are expected to be useful for the enhancement of knowledge of language professionals devoted to the teaching of writing as well as translation, proofreading, editing and reviewing services. A further goal is to lay the foundations for the production of didactic material which can potentially be incorporated into writing courses or professional writing, translation, reviewing and proofreading training programmes.

Acknowledgements

My special thanks go to Mariana Pascual, who has been a supportive and enlightening supervisor, as well as a patient and dedicated teacher, a great colleague, and a mentor throughout more than 10 years. I would also like to thank my workmates from *Instituto de Lenguas* (*Universidad Nacional de San Luis*), for their teachings, their unconditional support and their faith in me. I also thank those teachers at the master's programme who inspired me to write this work, each of them from their particular perspective.

This work would have neither been possible without the inspiration triggered by the productions of all the authors who studied the language of Mathematics from different angles, and for different purposes. Without ignoring the importance of all those founding works, which are fairly cited across this study, I want to thank Heather Greaves, who also contributed valuable findings to the field, and kindly shared her knowledge and expertise to guide my own reflections.

I have neither enough, nor proper words to thank Laura and Carolina, for their acute observations and comments about my work. I am also deeply indebted to all the members of the *Instituto de Matemática Aplicada* (IMASL), at *Universidad Nacional de San Luis*, who patiently guided me into the basics of Game Theory and offered me an invaluable help by giving me the chance to work in the context of their exceptional research team. I am equally thankful to all of them for having shared their knowledge and letting me observe and analyse their work. None of the results in this work would have been possible without them.

Last but not least, I would like to express my loving gratitude to Elva and Antonio, my parents, who have stood by my side, even during their agony. Likewise, reaching the final stage of this master programme would have been impossible without the support and love of Pablo, Francisco and Joaquín, the three of whom have given me the energy to continue and face difficult times. My special thanks also go to Lyllan, Lola and Jorge, for the indulgent meals, the cosy bed and the comfort of feeling at home in Córdoba.

Table of Contents

Abstract I
Acknowledgements II
Table of Contents III
List of Figures and Tables VI
List of Abbreviations Used in this Work VIII
Chapter 1. Introduction to the Study 1
1.1. Motivations for Research 1
1.2. Reconciling Language and Field of Knowledge 2
1.3. Context of the Study 5
1.3.1. The Instituto de Matemática Aplicada and the Game Theory Group 5
1.3.2. The Gabinete de Asesoramiento a la Escritura Científica (GAECI) 6
1.3.3 The Relationship between Natural and Mathematical Language 9
1.4. Objectives of the Study 10
1.4.1. General Objective 10
1.4.2. Specific Objectives 10
1.5. Organization and Overview of the Chapters in this Work 11
Chapter 2. Previous Studies on Scientific Writing 12
2.1. Approaches to the Study of Writing 12
2.2. Approaches to the Study of Mathematical Discourse 13
2.3. Approaches to the Study of Introductions as a Genre in Scientific Articles 15
Chapter 3. Theoretical Background 21
3.1. The System of Appraisal 21
3.2. Architecture of the System of Appraisal 23
Chapter 4. Methods 28
4.1. General Description of the Research Design 28
4.2. Data for Analysis 31
4.2.1. The Corpus 31
4.2.2. Main Corpus: Description, Selection and Building Criteria 32
4.2.3. Contrast Corpus: Description, Selection and Building Criteria 33
4.2.4. Observation of IMASL Internal Seminars 34
4.2.5. The Interviews 35
4.3. The Software 36
4.4. Data Analysis 37

4.4.1. Qualitative Phase: Manual Analysis 37
4.4.2. Quantitative Phase 39
4.4.3. Qualitative Phase: Interpretive Reading, Interviews and Observation Sessions 41
4.5. Special Considerations in the Process of Identifying Appraisal Elements 42
Chapter 5. Results 44
5.1. The Structure of GT SRA Introduction Sections 45
5.1.1. Analysis of the MC 45
5.1.2. Analysis of the CC 49
5.2. Description of Appraised Entities in the GT SRA Introduction 51
5.2.1. Distribution of Entities across the Introduction Section 53
5.2.2. Entities Addressed at the JUSTIFICATION Stage 54
5.3. Patterns of Appraisal across Introductions 55
5.4. The Combined Effect of ATTITUDE and ENGAGEMENT 61
5.5. Extrinsic and Intrinsic Evaluation 63
5.6. Other Linguistic and Discourse Items Considered as Appraisal Resources 64
5.7. The Role of Footnotes 68
Chapter 6. Discussion of Results 71
6.1. General Characteristics of the Structure of GT Introductions 71
6.2. Appraising Entities across the Introduction 72
6.3. Authorship as the Combined Effect of Introduction Structure, Distribution of Entities and Evaluative Resources 74
6.4 Triangulation with Observation Sessions and Interviews to IMASL Researchers 75
6.4.1. Interview Responses in Relation to Introduction Structure and Entities 75
6.4.2. Interview Responses in Relation to Evaluative Elements 77
6.4.3. Report on the Observation Sessions 78
Chapter 7. Conclusions 79
7.1. Theoretical Implications 79
7.1.1. Structure 79
7.1.2. Entities 79
7.1.3. Appraisal Patterns 80
7.1.4. Methods 80
7.2. Pedagogical Implications 81
7.2.1. Pedagogical Implications for Writing Mathematics as a GT Researcher at IMASL 81
7.2.3 Pedagogical Abstractions Derived from the Analysis of GT Introduction Sections 82

7.3. Suggestions for Further Research 83
7.4. Ongoing and Future Work Incorporating Contributions Derived from this Study 84
REFERENCES 86
APPENDICES 95
Appendix 1: Corpora 95
Appendix 2: Complete Software-Assisted Analysis, Manual Analysis Sample 164
Appendix 3: Interview and Observation Grid Templates 177
Appendix 4: Consent Form Used to Access Research Articles Written at IMASL 179
Appendix 5: Interview Transcripts 180

List of Figures and Tables

Figures

Figure 1.1: Authorship ……. 3
Figure 1.2: Screenshot of the form used to schedule a session at GAECI …….. 7
Figure 1.3: Workflow at GAECI ……. 7
Figure 1.4: Screenshot of the form used to record work during revision sessions at GAECI 8
Figure 3.1: Location of Appraisal in discourse semantics (Martin, 2019) …….. 22
Figure 3.2: Discourse semantics in relation to register and lexico-grammar (Martin, 2019) 23
Figure 3.3: An outline of the System of Appraisal (Martin & White, 2005, p. 38) …….. 24
Figure 4.1: Data for analysis ……. 31
Figure 4.2: The three-layered analysis using UAM Corpus Tool ……. 37
Figure 4.3: Appraisal categories considered in corpora analysis …….. 39
Figure 4.4: Lexico-grammatical realisations considered during identification of Appraisal items …….. 40
Figure 5.1: Proposed Introduction structure for GT SRAs …….. 46
Figure 5.2: Appraised entities within GT SRA Introductions ……. 51
Figure 7.1: Contextualizing research as seen by IMASL researchers ……. 82

Tables

Table 2.1: Comparison between Swales' and Lakic's proposals for the structure of the Introduction section …… 17
Table 2.2: The Introduction section as a research warrant (Hood, 2011) …….. 18
Table 2.3: The Introduction section in medical SRAs (Boccia, 2010) …… 19
Table 4.1: Methodological design …… 30
Table 4.2: Example of analytical matrix used in the manual analysis ……. 38
Table 5.1: Summary of chapter sections and relation with objectives of the study ……. 44
Table 5.2: Introduction components in the MC (per 1000 tokens) …….. 48
Table 5.3: Introduction components in the CC (per 1000 tokens) …… 49
Table 5.4: Lexico-grammatical realisations of each entity in the MC (per 1000 tokens) .. 52
Table 5.5: Lexico-grammatical realisations of each entity in the CC (per 1000 tokens) .. 53
Table 5.6: Entities in GT SRA Introductions across corpora …… 53
Table 5.7: Evaluative density …… 56
Table 5.8: Appraisal instances in GT SRA Introduction sections (normalized values per 1000 tokens) 57

Table 5.9: Differences in use of Appraisal resources between MC and CC …… 58
Table 5.10: Evaluative dimension constructed in relation to each entity ……. 59
Table 5.11: ATTITUDE and lexico-grammatical realisations in the MC and the CC … 61
Table 5.12: Types of footnotes found in the corpora, functions associated to them and frequency of occurrence ……..... 69

List of Abbreviations Used in this Work

AEJ (American Economic Journal)

CC (Contrast Corpus)

CL (Corpus Linguistics)

CR (Current research)

EC (Established concept)

ET (Economic Theory)

FCFMyN (Facultad de Ciencias Físico-Matemáticas y Naturales)

GEB (Games and Economic Behaviour)

GT (Game Theory)

IJGT (International Journal of Game Theory)

JET (Journal of Economic Theory)

JME (Journal of Mathematical Economics)

MC (Main Corpus)

OS (Object of study)

PR (Previous research)

Rr (Researcher)

SCW (Social Choice and Welfare)

SFL (Systemic Functional Linguistics)

SRA (Scientific Research Article)

UNSL (Universidad Nacional de San Luis)

Chapter 1. Introduction to the Study

The existing bibliography on how to produce mathematical scientific research articles (SRA) focuses on technical writing, points to features of proofs and theorems, and warns on the perils of using notation excessively (Halmos, 1970). However, all authors coincide on one fact: An Introduction cannot start with a list of notations or definitions, such as "*Let $R-k$ be the preference profile of agents after removing the preferences of agent k"* (Higham, 1998; Knuth, 2009; Thompson, 2011). The first sentence should entice the audience into reading the whole paper, and a list of mathematical notations will not achieve this aim. All authors agree on the fact that an Introduction must carry essential information - like the problem under consideration - and at the same time lead the reader into the body of the article.

This chapter aims at describing what writing SRA Introductions is like for Game Theory (henceforth, referred to as GT) mathematicians. It focuses on showing the challenges faced by researchers in trying to meet the requirement to place their work in the context of the existing literature and describe their main findings at the same time. It tries to raise consciousness about the difficulties that authors must overcome in coping with the combination of conciseness and attention to details of their work and plan of development. Understanding the reasons why writing Introductions is so difficult is expected to show the relevance of the selection of introductory sections for analysis in this work.

1.1. Motivations for Research

The initial interest in mathematical language for this thesis arose from a practical concern with a group of researchers attending the Scientific Writing Center (known by its Spanish acronym, *GAECI – Gabinete de Asesoramiento a la Escritura Científica*), an initiative born out of a research project financed by the National University of San Luis whose work and objectives will be developed later in this chapter. We observed that mathematicians often came searching for ways to improve their writing, but understanding what they meant by *good writing* was an intricate endeavour, at least without knowledge of the discipline basics. Our lack of mathematical training made the tutoring sessions a time-consuming and strenuous process. Referees commenting on UNSL mathematicians' articles pointed to a *better* use of the language, but we found it difficult to find the mistakes, apart from minor spelling or grammar issues. We started considering that the real problem was a more delicate or hidden aspect of scientific writing that was beyond knowledge of the science itself or the English language. Communicating effectively in the highly demanding context in which they are rigorously assessed on the basis of their written productions involves much more than that. We hypothesised that the source of the problem might not be exclusively related to the syntax and grammar of mathematical language – which they master -, but rather to the appropriate combination of symbols (mathematical language) with natural language.

Similarly, in line with research on World English (WE) and academic writing (Celce-Murcia, 2014), we also considered other possibilities in trying to understand the origin of written productions' failure to be accepted. We departed from the idea that the required *good writing* and *better language use* claimed by reviewers might be connected with the struggle to achieve clear meaning in the context of introducing new information by using varieties of English. This idea is based on the notion that a negotiation between readers and writers is required in order to find a balance between standardized norms and language diversity, if we expect WE to be accepted in scientific/academic settings. As part of the processes intended for helping researchers to write for publishing in English, we cannot underestimate this viewpoint, even when it is connected to a Sociolinguistic issue which we will not deal with in this thesis.

As writing tutors, we believe it is important to consider teaching what Matsuda and Matsuda (2010) call dominant and non-dominant language forms (standard English and WE, respectively) to make writers aware of the differences and warn them about the risks of using *deviational* features as a way to resist the current domination of Inner Circle English (Kachru, 1990). We also agree with the idea of promoting a language that reflects the sociolinguistic reality of modern higher education and values "clarity, effectiveness and contextual appropriateness of communication" (Strauss, 2017) over familiar, standard forms. Resisting the supremacy of English as the language of scientific communication is not so much about rejecting it, ignoring or underestimating the value of learning, or creating alternative spaces for dissemination in other languages, but rather about accepting the diversity of English around the world, accepting differences and conducting research on features of specific varieties. Thus, this work is an attempt to work in this direction.

Last, but not least, we deem important to make a statement about the reference to the *search of an equilibrium between words and symbols* in the title of this work. We do not intend to study GT discourse from a multimodal approach, as this has already been extensively dealt with by O'Halloran (2005) and will not be the focus of this thesis. Rather, we aim to show that we acknowledge the validity of these studies and understand the construction of an authorial voice as both a combination of words and symbols, and as the fusion between words and information structure. The way in which those words are used and how they are conceived from the perspective of the Appraisal System will likewise be the core of this study.

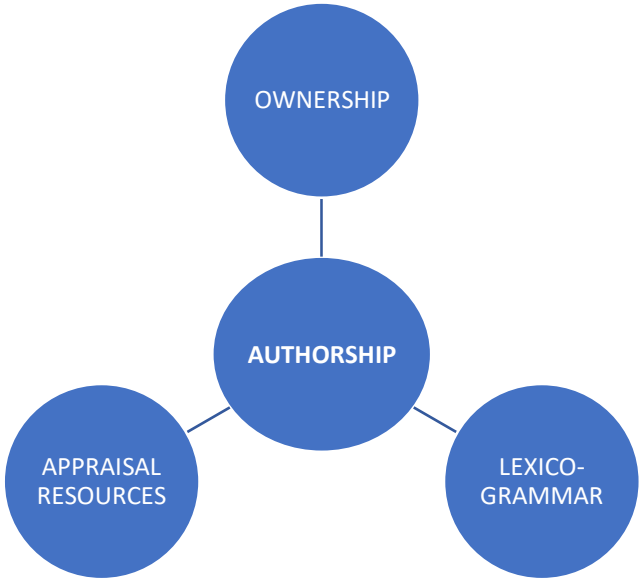
1.2. Reconciling Language and Field of Knowledge

The book *Mathematical Discourse: Language, Symbolism and Visual Images* (O'Halloran, 2005) was also an inspiration to start analysing mathematical texts and looking for those features that enabled authors to state their position, or make their subjectivities visible, in the complex world of scientific communication. We adopted the term *authorship* to name the main aspect that we will be analysing in this thesis. By *authorship* we refer to a notion based on a discursive and socio-linguistic construction based on

the application of the Appraisal framework to identity and affiliation studies. The idea of authorship is similarly connected with O'Donnell's work (2014), who purports that writers/speakers assume an identity when they produce a text, and that, in turn, they assign an identity to other voices that participate in a text by means of evaluative language. So, authorship in this work will be used as a concept embracing the degree to which the information presented in a SRA can be attributed to the writer (ownership), the resources used to do it (lexico-grammar), and the values attributed to both conceptual knowledge and sources (Appraisal resources) (Figure 1.1). It is not the goal of this thesis to delve into an exhaustive discussion of how the term identity has been defined by different fields of human practice, but rather to pinpoint some linguistic features through the analysis of GT texts in terms of Appraisal Theory.

Figure 1.1

Authorship



Unlike the case for other sciences, previous studies indicate that Mathematics exhibits an absence of lexical items expressing evaluative, interpersonal meanings, and that lexical choice is more focused on experiential or logical meaning, rather than on interpersonal ones (O'Halloran, 2005). This apparent lack of interpersonality makes it difficult to analyse Mathematics SRAs under the perspective of the Appraisal System (White, 2000; Martin & White, 2005). Nonetheless, this does not mean that mathematicians do not make evaluations, but rather that they presumably do so in a manner still unexplored from a linguistic point of view, or else, from the multisemiotic view, as proposed by O'Halloran.

Several authors agree on the idea that Mathematics is a linguistic activity (Morgan, 1998; O´Halloran, 2005). However, they also argue that lexico-grammar in Mathematics embraces much more

than language; it encompasses three systems, since the construction of the message depends on the successful combination of language, symbolism and visual images (O'Halloran, 2005). Each system has its own grammar and their successful combination results in preciseness of communication. Nonetheless, this thesis explores how authorship is construed in SRAs by exclusively analysing natural language, through which most knowledge is constructed in GT.

GT ideas are not inherently mathematical, but Mathematics is used to express them formally. GT is either treated as a branch of Mathematics and as a social science in the scientific community. As a social science within the realm of Mathematics, knowledge is built out of agreement among members of a community, in line with a social constructivist approach to the philosophy of Mathematics (Ernest, 1998; Hersh, 1997). Mathematics is, as Hersh and Umland (2006) state, a social entity that explains the laws underlying all the natural phenomena of the universe. Although more recently, it also accounts for social, economic and cultural phenomena. This view of Mathematics as a social fact is best represented by GT.

Taken as a branch of Mathematics, research articles in GT, thus, turn into material eligible for analysis due to the presence of potentially rich elements and characteristics that are unique, different from those found in other subdisciplines or branches. One of these special features is the profuse use of natural language, especially in Introductions, where authors are expected to do their best to persuade the audience of the importance of their contributions (Hood, 2011; Osbourne & Rubinstein, 1994; Swales, 1990;).

Many of the ideas in this thesis were also inspired by the work of Candia Morgan (1998), and her concept of *discourse*, which we have adopted to refer to the combined study of texts (lexis and grammar) and the wider set of social and linguistic practices within which the texts are situated (meanings constructed in the context of social and institutional practices). Along this thesis, the term *discourse* will be used to refer to an entity whereby lexis, grammar and organization of GT SRAs are studied together with "the concepts and categories that participants in the discourse have available to construct meanings, as well as the possible positions that participants may adopt as they compose or receive texts" (Morgan, 1998, p. 15).

Throughout this thesis, the term "scientific research article" (SRA) will be used to differentiate articles written in research settings related to Mathematics as a science from those written in research settings related to teaching practices, what Morgan (1998) calls "professional discourse" (p. 68). The use and meaning of other terms related to GT will also be dealt with in order to provide an appropriate context in which such terms should be interpreted.

1.3. Context of the Study

The following section describes the context in which the interest for studying mathematical discourse in GT SRAs emerged. It starts by giving some details of the *Instituto de Matemática Aplicada* (IMASL), where part of the SRAs under analysis were produced. Next, we provide a brief description of how writing is approached by scientists in this setting. The aim of this recount is to offer an overview of the reasons why Introductions were chosen for the analysis. Finally, we provide a summary of the work conducted at GAECI, the writing centre where we made the first contact with IMASL researchers.

1.3.1. The Instituto de Matemática Aplicada and the Game Theory Group

The *Instituto de Matemática Aplicada San Luis (IMASL*) was created by Professor Ezio Marchi on August 30th, 1982. It was the result of an agreement signed between the *Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET)* and the UNSL. One of its initial objectives was forming a research group where Mathematics and all the disciplines and areas of investigation studied at the Institute met at one single point. It currently incorporates scholars from the fields of Mathematical Analysis, Environmental Research, Game Theory, Structural Bioinformatics, Linear Algebra and Matrix Analysis.

The study of Game Theory at the UNSL began in the 1970s, thanks to the pioneering work of Professor Marchi. Several of his students engaged in Game Theory research later on, and today an ever-increasing number of researchers and doctoral fellows belong to the Game Theory Group, which actively participate in a wide international collaboration network. The results of the research undertaken by members of the Game Theory Group are published in prestigious Game Theory and Economics journals, such as *Journal of Economic Theory, Theoretical Economics, Games and Economic Behavior, Social Choice and Welfare, International Journal of Game Theory* and *Mathematical Social Sciences*.

Since 2014, IMASL researchers have organized seminars as a way to promote and keep a study routine within the research group. A schedule is fixed at the beginning of each year, and every member of the Game Theory Group has to commit to at least one presentation. Meetings usually last a maximum of two hours and are held every Thursday at 10 a.m. The initial idea of the seminars was to follow the activities undertaken at the Economics departments in universities from around the world to discuss new ideas for future projects, to share articles, to present potential new ones and to open the discussion among peers with a view to improving written productions, or to rehearse oral presentations for scientific events.

Even when the preferred language during the delivery of seminars is Spanish, the writing of articles and other activities, like reading and design of PowerPoint presentations, are produced in English. IMASL researchers acknowledge the role of English for their professional development (Mirallas, 2017), and several actions have been conducted to cater for their needs from GAECI, like writing courses and oral presentation workshops. This study is likewise intended to work in this line.

1.3.2. The Gabinete de Asesoramiento a la Escritura Científica (GAECI)

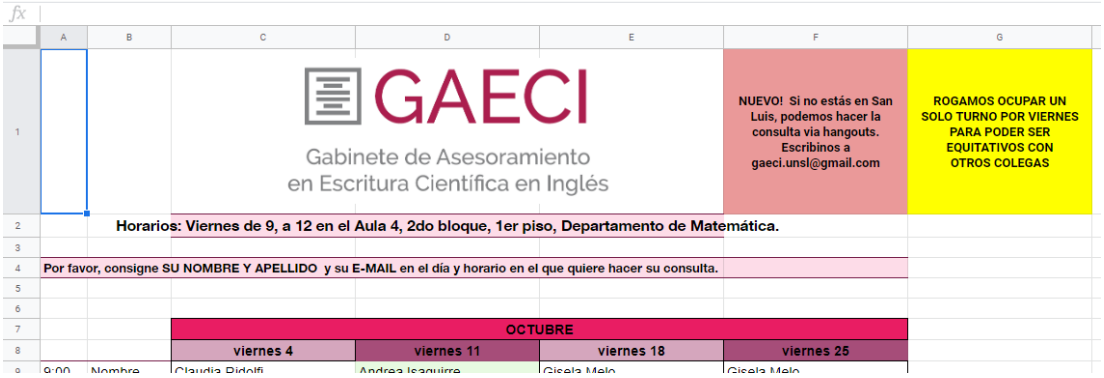
GAECI was born at the National University of San Luis out of the research project *Discourse Analysis (DA): Perspectives, Resources and Contributions for Institutional Discourse Practices*. It is the product of study and research activities conducted by its members, all of whom are linguists. It is an outgrowth of its Director's initiative and vision, Mariana Pascual, PhD in Linguistics, who has closely followed up local researchers' needs in terms of writing. GAECI's main aim is to make scientific writing in English less difficult, and to help our scientists publish their articles in international journals[1]. To this end, GAECI's members (all of whom have a university degree in English) have studied the discourse of science in various fields of knowledge for several years. They have devoted to studying different aspects of written discourse, and most analyses have been based on SFL approaches (Mirallas, 2017; Mirallas & Lucero Arrúa, 2016; Pascual, 2015; Waicekawsky, 2015). Moreover, four out of eight professionals have completed postgraduate studies in Linguistics (Mirallas, 2017; Waicekawsky, 2015), thus covering topics closely related to the activities developed at GAECI.

GAECI operates as a writing tutoring centre where researchers attend to read and revise their papers. Tutors, in turn, produce and write descriptions of the errors and difficulties encountered during the sessions. These records turn into data used to develop strategies to enhance writing, such as courses intended to cater for the different needs observed during sessions. Focus has especially been put on general aspects of writing, structure and organization, grammar and language. However, the tools and strategies designed at GAECI have also been thought of as a response to specific groups of researchers' demands (particular production situations, like writing abstracts or whole articles for a specific community of researchers, dealing with syntax, etc.).

[1] Further information about this writing centre may be found at http://webfcfmyn.unsl.edu.ar/?p=3762

Figure 1.2

Screenshot of the form used to schedule a session at GAECI



Thus, tutors' work at GAECI considers ways to tackle the difficulties encountered in the papers revised during the tutoring sessions. Likewise, the courses offered by GAECI seek to provide solutions to the problems that researchers often face when writing. The tutoring sessions last one hour, they are offered every Friday, and are scheduled via an online form, as shown in Figure 1.2. After a session has been agreed, researchers go to the classroom specially assigned to GAECI in the Mathematics Department building.

Figure 1.3

GAECI's workflow

Tutors and researchers spend the one-hour session reading the article that researchers are interested in improving or correcting, and searching for and discussing alternatives for especially problematic sections or fragments. Discussions often turn around lexico-grammatical features or generic aspects. Figure 1.3 shows the cycle of tutoring sessions at GAECI. Once each session finishes, tutors use the data to produce an annual report to be submitted to the Faculty of Physical, Mathematical and Natural Sciences (FCFMyN), which is the sponsoring institution. The template used to produce the report is illustrated in Figure 1.4. This template (accessible at https://forms.gle/t3onnMxXv2SmoMnN8) also collects information about the text under revision, researcher background, type of difficulties encountered during revision, among other factors.

Knowledge necessary to improve scientists' writing does not solely lay on English, and tutors' language education is often not enough. Interviews with scientists are thus obligatory to understand the idiosyncrasy of their fields of study and the topics they are working on. Consequently, tutoring sessions are in fact an opportunity for knowledge exchange, where tutors and researchers benefit from providing one another with data relevant for writing the articles.

Figure 1.4

Screenshot of the form used to record work during revision sessions at GAECI

Studying the discourse of Mathematics may provide linguists and mathematicians with tools and strategies to approach the communication of results and developments in the science with more accuracy and security, and with a renewed awareness of intention and purpose in each section of SRAs.

1.3.3 The Relationship between Natural and Mathematical Language

We have already referred to the importance of using natural language in Mathematics. However, it is necessary to expand on this aspect, where the discipline itself and philosophical issues intersect. Natural language cannot be considered peripheral to the main mathematical content, as it serves both in "*the construction of the identities of the author and reader, and the epistemological and ontological assumptions underlying the writing*" (Burton & Morgan, 2013, p. 430). Thus, our focus in this work is the natural language within which the symbolic or special vocabulary and structures particular to Mathematics (mathematical language) are embedded. We expect to understand the role played by language and the particular meanings that it can create in the specific situation of writing GT SRAs. Taking Kress' words, we believe that this understanding will provide an insight into "*the social needs and the cultural values and meanings of its users*" (as cited in Burton & Morgan, 2013, p. 23). We further expect that this work can be a stepping stone on the way to helping mathematical researchers to write more effectively.

Our assumption that natural language is a key element in mathematical discourse is not, of course, original. In fact, scholars devoted to studying mathematical communication have emphasized its role (Burton, 1984; Burton & Morgan, 2013; Morgan, 1998) and some have even provided useful guidelines for authors (Halmos, 1970; Higham, 1998; Sollow, 2014). Halmos (1970) produced an essay suggesting a series of tips for writing successfully. Among those more related to natural language, he included: (1) say something, (2) speak to someone, (3) organize first, (4) write in spirals, (6) write good English, (8) use words correctly (especially technical ones), (9) resist symbols, and (10) [*remember that*] all communication is exposition. We believe the suggestion related to symbols particularly illustrates how laborious communication can be in Mathematics without using symbols, as their meaning is universal, and its use saves writers many words. In fact, some authors consider *good writing* in Mathematics to those texts that are planned to be spoken (Halmos, 1970), which limits or excludes the use of symbols. Halmos (1970) also states that pretending to be explaining "the subject to a friend on a long walk in the woods, with no paper available" (p. 16) is the best way to make mathematical discourse clear. He further contends that symbolism should only be used when it is really necessary.

Likewise, Higham (1998) supports the idea of carefully combining symbols and words to produce clear writing. He suggests that a paper dominated by symbols may be more difficult to read than one dominated by words, and he provides a series of tips intended to guide authors as to (a) when to use symbols ("*if the idea is too cumbersome to express in words, or if it is important to make a precise mathematical statement" p.* 25); and (b) when to use words ("*as long as they do not take up much more space than the corresponding symbols*" p. 26). The other tips and recommendations point to the correct use of English.

Natural language (English in this case) is as important as symbols to communicate effectively. That is the reason why this work focuses on it, apart from highlighting the notion that striking a balance between

language and symbols can be the secret to good writing. In relation to the specific realm of language (natural), which is our field of study, we expect that this work helps researchers using language deliberately to create the particular meanings that they intend. By analysing language in the context of GT SRA Introductions, we aim at finding ways of teaching the features, tendencies, strategies and resources to enhance writers' opportunities for creativity and self- expression. We believe that the information potentially derived from this work will not just help writers to conform to conventional expectations, but also empower them to make informed choices to break the conventions in order to achieve deliberate effects, including to imprint their own style and defend it, as Halmos suggests (1970, p.20), through their writing.

1.4. Objectives of the Study

1.4.1. General Objective

The primary objective of the present study is to analyse, interpret and compare how authorial voice is constructed in published and draft versions of Game Theory SRAs written in English. We especially focus on how interpersonal meanings are deployed across articles. To this end, a total of 22 SRAs were analysed so as to throw light on how interpersonal meanings construe the textual voice of the writer. The analysis draws upon the framework provided by the Appraisal System (Martin & White, 2005; White, 2000), within Systemic Functional Linguistics (Halliday, 2014).

1.4.2. Specific Objectives

1. To explore the Main Corpus (MC) [published SRAs] in terms of its internal features and genre structure.
2. To identify linguistic and discursive resources typical of SRA in GT that help to build authorial voice in the MC.
3. To explore the Contrast Corpus (CC) [draft SRAs] in terms of its internal features and genre structure.
4. To identify linguistic and discursive resources typical of SRA in GT that help to build authorial voice in the CC.
5. To identify and contrast what entities are presented and evaluated by authors in both corpora, analyse what values they are assigned and what lexico-grammatical resources are associated with those meanings, following the taxonomy proposed by White (2000).
6. To suggest lines of work in relation to the design of didactic material to be used in writing courses and/or revision sessions by drawing on the results of the analysis proposed in this thesis.

1.5. Organization and Overview of the Chapters in this Work

This thesis is divided into seven chapters. Chapter 1 introduces the topic and outlines the problem. This chapter also explains the significance of the topic chosen for the work and describes the motivations and the context out of which the investigation arose. Chapter 2 reviews previous studies which were considered as a basis for this thesis. Chapter 3 describes the main theoretical principles underlying this research. Chapter 4 explains the methodology followed in the study. We explain the research design, provide details on how the corpora were collected, processed and analysed, and make reference to the tools used to process the corpora. Chapter 5 presents the main finding. Chapter 6 discusses the similarities and differences found in the analysis of the 3 dimensions explored in the corpora: structure, entities and evaluation. Finally, Chapter 7 elaborates on the main conclusions and pedagogical implications that emerged from this study, and suggests lines of further research and work to design didactic material.

Chapter 2. Previous Studies on Scientific Writing

This section includes references to the literature on the study of science genres and mathematical discourse. We have organized the contributions into three parts: (a) approaches to the study of writing; (b) approaches to the study of mathematical discourse, and (c) approaches to the study of introductions as a genre in scientific articles.

2.1. Approaches to the Study of Writing

The emergence of Applied Linguistics as a discipline, in the 1950s and 1960s, gave rise to three large domains of research that have focused on the analysis of written text and discourse (Hinkel, 2002): (1) Contrastive rhetoric (CR), which examined Second Language (L2) text and discourse paradigms in English as a Second Language (ESL) student writing and academic texts; (2) Text linguistics (TL), in the 1960s, 1970s, and 1980s, which contributed data about the way discourse is organized and text is constructed; and (3) critical discourse analysis, which focused on the relationship between the spread of the teaching of English around the world and the social, cultural, and economic contexts in which English is used (Hinkel, 2002). These approaches or perspectives have all contributed to the study of how written text is produced in academic contexts, by different groups of writers, especially in English.

The interest in the way academic texts are produced lies on the fact that an increasing number of international students in most colleges and universities around the world has spotlighted English as a *lingua franca* - the language used among speakers lacking a common first language (L1) (Celce-Murcia, 2014) - and a world language (the language used to communicate in international contexts among speakers of various languages, English as L1 included). This circumstance has originated at least two trends: first, those approaches aiming at problematizing the quality of writing and the characteristics of L2 and those approaches that seek to dive into the even more complex issue of the social construction of language. While the first ones devoted much attention, time, effort, and resources to analysing and teaching L2 academic writing and its conventions (lexico-grammatical features, patterns of usage across disciplines, etc.) (Hinkel, 2002), the second ones focused on the connections between language, discourse, and power in society (Pennycook, 2016). However, in both cases, emphasis remains to be placed on differences between texts produced by *non-native speakers* (NNSs) and texts written by native speakers (NSs) of similar academic standing (Perez-Llantada, 2012).

Undoubtedly, the teaching of text features, discourse, and writing is an important focus (and challenge) in university contexts. And although the successful performance of scholars in academia may not depend exclusively on acquiring high-level writing skills, it is often considered an asset, and this is not different at the UNSL.

Our informants, who have different language backgrounds and training, acknowledged the aspects that they believed good writing depends on: (a) a socializing process, which involves systematically

exposing themselves to academic text and discourse (in English), and (b) a process of formal acquisition of syntactic and lexical features of academic discourse, which they associate with taking courses or attending tutoring sessions in order to receive thorough and explicit instruction. We agree on those views, and - as teachers, instructors or tutors -, we need to achieve an advanced level of professional knowledge of the language as it is used by expert writers. One way to do so is by exploring discourse, as has been shown by previous studies, though in different disciplines (Anglada & Gaido, 2010; Anglada et.al, 2010). This study was conducted with the aim of identifying the specific and subtle area of evaluative language in mathematical texts. We hope the results help in the development of potential didactic strategies or guidelines for a group of international learners of English, who belong to this highly specialised scientific community.

2.2. Approaches to the Study of Mathematical Discourse

Mathematical discourse has recently been studied from several perspectives. Most of the existing bibliography focuses on the language used in Mathematics textbooks and classrooms, and the most important goal in this literature has been the improvement of communication of ideas between teachers and students (Morgan, 1998, 2005; Lövstedt & Rose, 2015; Schleppegrell, 2007). O'Halloran's seminal research (1999, 2006) points to the multisemiotic nature of Mathematics, as she states that knowledge is constructed through the successful combination of natural language and symbolism. She has even developed a special approach to study mathematical discourse that combines the principles of Systemic Functional Linguistics (Halliday, 2014) and Multimodal Analysis (Kress & van Leeuwen, 1996): The Systemic Functional Approach (SFA) to Multimodal Analysis (MDA). Like her, other authors (Kress & van Leeuwen 1996, Lemke 2002) have argued that mathematical texts are multimodal, consisting of "semiotically rich configurations of images, diagrams and physical activity as well as language" (Veel, 1999, p. 187). This makes the meaning potential of multimodal mathematical texts far greater than that of any single element considered in isolation.

Other linguists who take a functional perspective on language have described the grammatical patterns through which mathematical knowledge is construed (e.g., Lemke, 2002; O'Halloran, 1999; Veel, 1999). Like O'Halloran, these authors have especially focused on the communication of mathematical knowledge in school settings. Each of them has contributed different aspects of mathematical discourse, but all of them agree on the fact that the language of Mathematics fascinates due to its uniqueness, as it is a language that blends symbols and images with spoken and written English to construct ways of modelling the world with seemingly endless applications.

Veel (1999) discusses distinctive linguistic features of mathematical discourse which he classifies into (a) the predominance of teacher spoken language; (b) the predominance of distinctive patterns of spoken language interaction; (c) the technical fields of knowledge construed through spoken and written language;

(d) the hierarchical ordering of mathematical concepts through language; and (e) the gap between student use of mathematical language and teacher/ textbook use of mathematical language. He further draws on the tenets of SFL, and Bernstein's sociology of education to make explicit the kinds of language employed to construe the technical meanings of Mathematics. Such characterization focuses on classroom language, and it includes - but does not limit itself to - the identification of technical lexis and phrases, grammatical structures, and the identification of spoken and written genres.

Lemke (2002) delves into a series of questions, such as the scope to which Mathematics can be considered a language, or a part of language, and how far it goes beyond language in its resources for making meaning. By resorting to a historical and semiotic explanation, he argues that Mathematics has evolved historically to help us make "topological" meaning, which natural language cannot successfully achieve, and that Mathematics always transforms and functions in close conjunction with natural language, as well as with other semiotic resources, such as visual representations.

Likewise, Schleppegrell (2007) refers to the linguistic challenges of Mathematics, which include multi-semiotic formations, dense noun phrases that participate in relational processes, and the precise meanings of conjunctions and implicit logical relationships that link elements in Mathematics discourse. She further states that research on pedagogical practices is necessary to develop mathematical knowledge through attention to the way language is used.

Gutierrez de Piñeres Reyes and Díaz Frías (2011) similarly acknowledge the multisemiotic nature of mathematical discourse, which they refer to as Informal Mathematical Discourse (IMD). In their work, they take a step further by drawing on Computational Linguistics, and review the three major areas of work from which IMD has been analysed in order to automatize writing in textbooks, research articles and proofs: (a) automated proof checking; (b) automation of linguistic phenomena occurring in mathematical proofs; and (c) mathematical knowledge management on large corpora of mathematical texts. This work opens interesting avenues of research by suggesting an approach that combines Corpus Linguistics, Discourse Analysis and Computational Linguistics. In this sense, they lay the ground for developing a promising method to help teachers, students and researchers in the process of writing.

Likewise, though based on an SFL perspective, Oliveira and Cheng (2011) analysed how language is used to construe different mathematical meanings. Even when their research focused on primary-school Mathematics, it is worth mentioning, as it studied some of the linguistic challenges that the discipline poses for English Language Learners (ELL). The implications of their investigation for teaching and doing Mathematics are valuable, as some of them suggest that certain language features, such as noun groups, in early elementary textbooks make texts difficult to understand. Similarly, it is shown that the construction of mathematical information is the result of the various semiotic systems used in texts, and that ELLs may have problems when switching from one semiotic system to another. Thus, by highlighting the potential

difficulties that students may have when reading and doing Mathematics, they also emphasize the multisemiotic nature of the discipline, in tune with the existing literature.

Finally, Burton and Morgan (2000) produced an interesting investigation of how language in research papers helps to build the identities that mathematicians present to the world and the ways in which they represent the nature of mathematical activity. Their work drew on notions of SFL to analyse mathematical research articles from the perspective of the interpersonal function. These authors classified the representation of identity into: (a) a sense of identity; (b) a sense of community membership; and (c) a sense of authority. They further sought to answer the following questions:

> How is the author's identity constructed as an authority in the field of research mathematics? To what extent and how is the author positioned as a member of the community of mathematicians? How is the author's relationship to the subject matter of mathematics constructed? In particular, to what extent does the author appear to claim ownership of the subject matter through demarcation of knowledge or territory within the field? (p. 435)

We believe that these questions are closely related to the objective of this thesis. We have very similar concerns and expect to contribute to this discussion. Burton and Morgan's study managed to identify linguistic means for achieving various types of authority, significance, interest, among other values. However, they acknowledged the need to further research on the characterizations of the various forms used by mathematical writers. We hope that our investigation helps in providing hints in this direction.

2.3. Approaches to the Study of Introductions as a Genre in Scientific Articles

The SRA, its structure, lexico-grammatical features and its social and interactive dimension has generated interest in the area of linguistics. The development of theoretical notions and analysis tools resulting from various research works have translated into useful didactic support material for writing texts. Swales (1990), for example, proposed and developed the notion of *move* to refer to a structural segment that has a specific communicative function and purpose. He acknowledges the difficulty of writing Introductions for both native and non-native speakers of English, and offers a move-based model according to which information is normally organized. Swales' model has certainly been a lighthouse in the world of academic writing, as it has offered a precise description of how most Introductions are organized across disciplines and a useful guide for writers.

Graves et al (2014) and Graves and Moghaddasi (2013) studied how and to what extent Swales' proposal (1990) for SRAs structure is actually realised in Mathematics SRA. They observed a prevalence of Introduction sections in the corpus, and suggested that writers of Mathematics SRAs begin by creating a rhetorical space for their research, in line with writers in other disciplines. Graves and Moghaddasi (2014) go a step further in the characterization of the structure and the language used, and make an interesting

contribution in interpreting what lies behind the order of information within the section. They state that Introductions do not only highlight the importance of the research and introduce new results, but also contribute to the construction of an argument. They contend that, in Mathematics, persuasion only results when readers and writers share understanding about the concepts being discussed; thus, all introductory sections include clauses that define mathematical concepts and introduce symbols. This is a fundamental description, and it agrees with Hood's (2001) observation that definitions or descriptions are rhetorical. In fact, descriptions and definitions play the "hidden" roles of identifying topic details, clarifying gaps in knowledge being addressed, and creating shared knowledge between writer and reader from which the results arise, among other functions.

Other authors (e.g. Lakic, 1997) note that Swales' model cannot be strictly applied to all disciplines/fields, as there is much variation, both in terms of how a topic is dealt with in the sciences and in terms of how the writing style and conventions change across time, communities and journals. Lakic (1997) proposes a model slightly different to the one proposed by Swales (1990) for Economics SRAs, by adding a fourth *move* between the first and the second ones in Swales' structure. We consider this model closer to that observed in GT SRAs, since GT is one of the key theoretical frameworks used in Economics. A comparison between Swales' and Lakic's models is depicted in Table 2.1. We find Lakic's contribution interesting, as he also adds two different steps in Move 1 (*Stating key characteristics* and *Stating current knowledge*), which is in line with the data provided by our informants in interviews about writing introductory sections. Lakic similarly defines the Move *Summarizing previous research*, as a separate one, and not as a step serving a specific function within a Move. However, we find this observation debatable. A summary of previous research is certainly and obligatorily included in most GT SRAs, but its function may not be limited to acknowledging the existence of interest in the topic within the scientific community. Rather, it may be linked to the author's need to claim centrality of the research topic, establish the research territory, or define a research niche out of the identification of previous works' weaknesses or gaps.

Table 2.1

Comparison between Swales' and Lakic's proposals for the structure of the Introduction section

Swales (1990)	Lakic (1997)
Move 1: Establishing a territory	**Move 1**: Establishing a territory
Step 1- Claiming centrality and/or	*Step 1*- Showing centrality:
Step 2- Making topic generalization	a- By topic prominence
Step 3- Reviewing items of previous research	b- By interest
	c- by importance
Move 2: Establishing a niche	d- by standard procedure
Step 1a- Counter claiming or	
Step 1b- Indicating a gap or	**Move 2**: Summarising previous research
Step 1c- Question-raising or	
Step 1d- Continuing a tradition	**Move 3**: Stating current knowledge
	Step 1- Indicating gap
Move 3: Occupying the niche	*Step 2*- Question-raising
Step 1a- Outlining purposes or	*Step 3*- Hypothesis-raising
Step 1b- Announcing present research	*Step 4*- Question validity
Step 2- Announcing main findings	*Step 5*- Airing a problem
Step 3- Indicating research article structure	
	Move 4: Occupying the niche
	Step 1- Describing the present research
	Step 2- Outlining purpose
	Step 3- Announcing principal findings
	Step 4- Extending a finding
	Step 5- Indicating RA structure

From the perspective of SFL, Hood (2010) sees Introduction sections as a macro-genre (see Table 2.2). Her position and her notion of genre are, thus, markedly different from Swales' and several other authors following his work (Graves et al, 2013; Lakic, 1997) on the issue of genre. Whereas the analytic construct of 'moves' is arrived at intuitively, and it can then be described with reference to the distribution of certain "syntactic features that are interpreted as doing the pragmatic work assigned to a move" (Hood, 2011, p 31), analyses of genres in SFL result from an exploration of the meaning potentials realised in language choices in instances of discourse. This means that the relationship between language and genre is theorised rather than intuited. In this study, genre is approached from the perspective of SFL, which defines genres as "recurrent configurations of meanings (...) that (...) enact the social practices of a given culture" (Martin & Rose, 2008, p. 6). Consequently, following Hood (2011), describing Introductions as a genre is the result of an analysis that considers the realisation of ideational, textual, and interpersonal meanings.

Table 2.2

The Introduction section as a research warrant (Hood, 2011)

The Introduction Section

Surface genres	Metaphorical context
Descriptive report of object of study Descriptive report of research that contributes to knowledge of the object of study Descriptive report of the writer's own study	Evaluative discourse may be present at any of these stages to serve a "legitimation" function (van Leeuwen, 2007): a. legitimating by reference to discourses of value b. legitimating by reference to the writer's own results c. legitimating by reference to existing literature

In her book, *Appraising research: Evaluation in academic writing,* Hood (2011) explores how writers take an evaluative stance in academic writing, across a range of disciplines, from the point of view of the Appraisal System. This book provides a detailed description of academic writing and makes special emphasis on the challenges that authors face in trying to strike a balance between the expected objectivity for the presentation of knowledge and the requirement to engage critically or to persuade readers. This situation is, thus, presented as problematic, especially for novice writers, since it requires that researchers command the language, and that they are familiar with the genre and the social conventions. The relevance of such aspects for international and novice writers has also been acknowledged by Duff (2010).

Hood's contribution is especially interesting because she analyses evaluative discourse in academic RAs in English with the aim to "make the nature of the discourse more apparent and accessible" (2011, p. 16). Drawing on Duff's concept of *socialization* (2010), it could be said that Hood's work does not simply dissect texts in order to make their discourse more transparent for novice writers and facilitate academic texts reproduction; this contribution also represents a fundamental work, as it shows how academic argument and academic knowledge are socially constructed, how researchers engage in a dialogue with other colleagues through writing. An understanding of the meanings and values at stake in academic writing can be a precious tool for writing instructors, writers and writing tutors or reviewers.

Also from the perspective of SFL, Boccia (2010) conducts a rhetorical and lexico-grammatical analysis of the Introduction section in medical SRAs. Boccia views Introductions as a *macro-proposal* - in terms of Martin's interpersonal notion (1992) -, through which researchers seek to offer their work as a response to weaknesses or gaps in existing literature and expect, in exchange, the acceptance of such work. She explains that *proposal* and *proposition* are the semantic categories of the interpersonal function, and that these categories embrace the roles of the participants in a communicative act and the nature of the exchanged object: whereas information is exchanged in a proposition, goods and services are exchanged in

a proposal. However, she contends that in the Introduction sections that she studied, researchers perform an indirect speech act, or, in terms of the SFL, a "contextual metaphor" (Martín 1997, p. 31). That is, they announce their research apparently in a propositional context of information exchange. However, the rhetorical-linguistic characteristics detailed throughout the analysis suggest that, on a deeper level, the text makes a macro-proposal, as meanings are not expressed in a "natural" or "congruent" way, but "indirectly" or "metaphorically" as a consequence of tensions that occur at the contextual level between the genre -the Introduction- and characteristics of the register, fundamentally of the tenor, that is to say, of the roles and characteristics of the relationships between the participants.

Thus, Introductions have a double interpersonal function: personal– the attitude of researchers towards the information they present -, and interpersonal– the relationship between researcher and audience. The latter function, in turn, embodies a double function: *acting* on the world (through commands offers, questions and answers -, and *reacting* to what others say or do. Boccia proposes a characterisation (Table 2.3) derived from the analysis of introductory sections considering language as a way to enter a communicative event, express opinions, show attitudes, react, evaluate and establish relations between the producer of the text and the audience.

Table 2.3

The Introduction section in medical SRAs (Boccia, 2010)

Boccia's proposed model for Introductions	
Justification I	1. Indicating the relevance of the object of study/phenomenon/model, or the problem related to that object of study/phenomenon/model
	2. Making generalizations about the topic/providing definitions or explanations
	3. Reviewing related previous research
	4. Highlighting the need to consider the topic as an object of study
	5. Identifying limitations/gaps/weaknesses in the existing literature about the object of study
	6. Indicating the need to confirm or contest existing findings/results
Justification II	1. Indicating limitations in relation with the "state of the art" in the discipline
	2. Indicating the need to confirm positive tendencies or revert negative ones through further studies
Offer	1. Offering work through which the researcher proposes to fill in a gap*solve a problem/make up for weaknesses, identified in Justification II

The investigation of how language is used is important and many authors have attempted to account for what happens at the heart of texts from perspectives other than SFL. Anthony (2013), for example, investigated the structure of research article writing in the field of Mathematics and compared it to that in Mechanical Engineering. His investigation was based on a corpus of 410 refereed journal articles covering one complete year of publications in a high-impact Mathematics journal. The results showed that the structuring of Mathematics papers varied considerably from article to article and that few consistent patterns

in the choice of sectioning could be found. This author also analysed the style of Mathematics research articles, and concluded that the researcher often takes the reader on a journey through various theorems and lemmas to arrive at a proof or new model. In doing so, he coincides with Halmos (1973) in stating that mathematicians are prone to sacrifice formality in exchange for clarity. An interesting aspect of Anthony's work is that he refers to the use of "vague terms such" as "easy", phrasal verbs, and connectives "and" "but", and "so" as *informal style*. Anthony further states that this feature has "traditionally been considered to be inappropriate for a formal academic writing style" (p. 20). Consequently, he suggests that rather than proscribing that language, ESP teachers need to be aware that informal expressions can be used in some disciplines, and inform students of this fact in the writing classroom.

Anthony's work is valuable in terms of discourse practice description, as he successfully manages to describe the language of Mathematics and the structure of SRA, but he barely contributes to the understanding of how information is interwoven to achieve peers' recognition, to persuade readers of the importance of a work, or to justify the need for research on a specific topic. His work is an excellent example of an investigation in Corpus Linguistics.

The results deriving from all the above referenced works help to develop an awareness of both dominant and non-dominant language. This can contribute in making writers become aware of the differences and warn them about the risks of using what Matsuda and Matsuda (2010) call *deviational features - i.e.* language forms which can be understood and achieve their communicational function, but that are not the usual ones, or the traditionally accepted by the community- as a way to resist the so called domination of Inner Circle English (Kachru, 1990). Resisting the supremacy of English as the language of scientific communication is not so much about rejecting it, ignoring or underestimating the value of learning, or creating alternative spaces for dissemination in other languages, but rather about accepting the diversity of English around the world, accepting differences and conducting research on the features of specific varieties valuing, promoting or privileging particular kinds of academic literacy practices over others in the context of constructing academic knowledge. A secondary goal of this work is, thus, the promotion of a language that reflects the sociolinguistic reality of modern higher education and values "clarity, effectiveness and contextual appropriateness of communication" (Strauss, 2017) over familiar, standard forms.

Our work seeks to understand how meanings are built to achieve persuasion, and thus construct authorial voice. It is also an attempt to describe the textual structure of GT SRAs and the lexico-grammatical elements that shape it from the perspective of the interpersonal metafunction, more specifically from the viewpoint of Appraisal.

Chapter 3. Theoretical Background

3.1. The System of Appraisal

Linguists and language researchers have historically been interested in the way language works. One of the main functions that language has been attributed to is the expression of feelings and judgements, which has been given various names, such as *expressive function* (Bühler,1939), *emotive function* (Jackobson, 1960), and *interpersonal metafunction* –among others- according to Hallidayan perspectives (Halliday & Matthiessen, 2014). Despite the differences lying behind these labels, these approaches intersect at the definition of *evaluation* -broadly speaking– as the speaker's/writer's expression of attitude, judgement, stance, or feelings in relation to, either their listeners/readers, or to the ideas they deal with; or in Hood's words (2010) the "valuing and taking (of) a position in relation to both entities and propositions" (p.13).

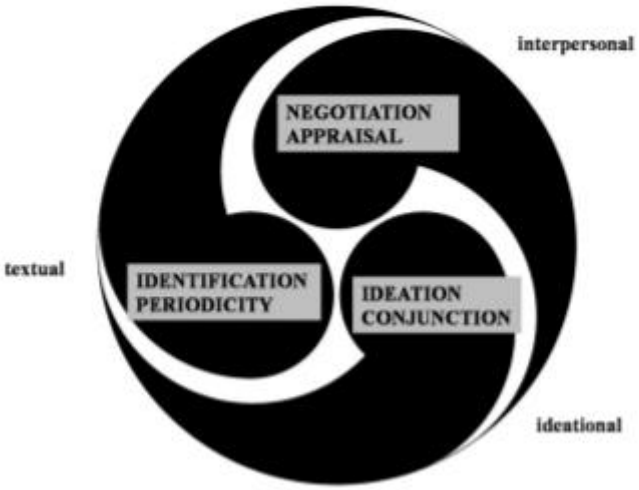
Since the 1980s three theoretical trends based on the conception of language as a semiogenic system have led research works on this topic: (a) the American School (Biber et al., 1999; Biber & Finnegan, 1988) known as *stance studies*; (b) the English School (Hunston, 1989, 2000; Hunston & Sinclair, 2000; Hunston & Thompson, 2000), known as *evaluation studies*, and (c) the Sydney School, whose studies are framed in the Appraisal System (Martin, 2000; Martin & White, 2005; Martin & Rose, 2007; White, 2002).

Appraisal is located within the framework of Systemic Functional Linguistics (SFL) (Halliday & Matthiessen, 2014), and it is based on Bahktin's (1981) notions of dialogism and heteroglossia. Contextualizing Appraisal within the broader theory of SFL is fundamental to understanding this system's potential as a research tool. Next, we will briefly refer to the way in which Appraisal connects with some key dimensions of the architecture of an SFL model of language.

SFL models language as a tripartite system (Halliday & Matthiessen, 2014) which constructs meaning simultaneously across three strata, namely experiential, interpersonal and textual. These three strata are in a relationship of realization across relative levels of abstraction. This means that patterns in language at the level of text (discourse semantics) are realised in patterns of language at the level of clause (lexicogrammar), which are in turn realised as expressions in sound or writing systems (phonology/graphology). Interpersonal meaning can be explored across all strata. However, reference to Appraisal means that evaluation is approached from discourse semantics (Martin & White, 2005), the stratum of meaning interfacing lexicogrammar with context (register and genre) (Figure 3.1).

Figure 3.1

Location of Appraisal in discourse semantics (Martin, 2019)



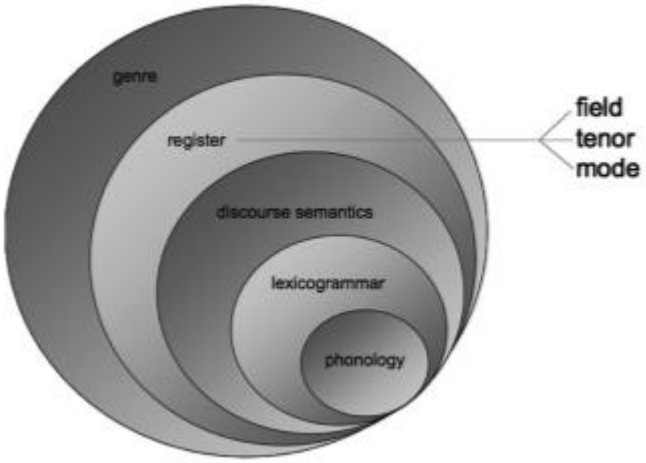
Appraisal is, thus, one of the two systems comprising the interpersonal metafunction within the realm of discourse semantics. Together with that of Negotiation, Appraisal's function is to enact the register variable Tenor, where Tenor is concerned with the relations of power and solidarity whereby speakers position themselves as interlocutors in discourse (Martin, 2019) (Figure 3.2). However, whereas Appraisal models the **-personal** in interpersonal meaning, the system of negotiation models the **inter-** of the interpersonal. This implies that the full responsibility to account for interpersonal meaning potential does not solely rely on Appraisal. Each system has a particular responsibility and complements each other in creating relations of solidarity and power (Hood, 2019).

Appraisal offers a systematic framework for the study and understanding of linguistic phenomena related to ideological positioning construction. In this work, we seek to unveil the linguistic mechanisms through which researchers construct their image as authors in a discipline that has traditionally been labelled as devoid of subjectivity. Thus, this is one of the main reasons why Appraisal has been chosen as an analytical-interpretive framework for this work. This system is a valuable theoretical and analytical tool useful to conduct research on evaluation in language from a holistic perspective.

Appraisal System studies the linguistic resources by which texts/speakers express, negotiate and naturalise particular inter-subjective and ideological positions. Within this broad scope, the theory is more particularly concerned with the language of evaluation and emotion, and "with a set of resources which explicitly position a text's proposals and propositions interpersonally" (White, 2000).

Figure 3.2

Discourse semantics in relation to register and lexico-grammar (Martin, 2019)



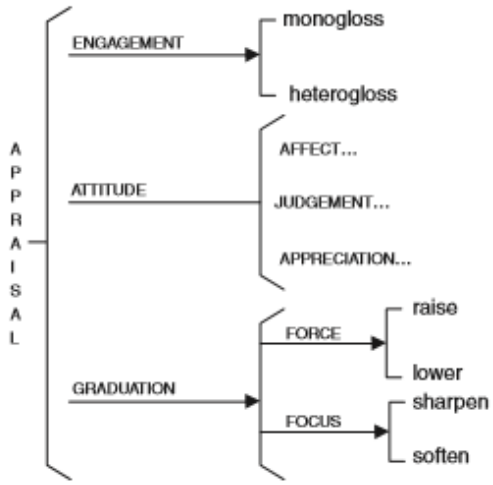
3.2. Architecture of the System of Appraisal

The framework of Appraisal organizes meanings in three major semantic domains: ATTITUDE[2], ENGAGEMENT and GRADUATION. The system proposes not only the understanding of feelings, but also a wide range of valuations, such as aspects related to behaviours and preferences, among others. An overview of the System of Appraisal is offered in Figure 3.3.

[2] *In this work, following SFL tradition, the names of the theory systems are written in sustained capital letters, whereas the second level of delicacy of each subsystem is written in initial capital letter. From the third level of delicacy onwards, the categories are written in lower case (Pascual, 2015; Quiroz, 2013)*

Figure 3.3

An outline of the System of Appraisal (Martin & White, 2005, p. 38)



ATTITUDE concerns the "values by which speakers pass judgements and associate emotional/affectual responses with participants and processes" (White, 2000). These evaluations are subcategorised according to: (a) the expression of one's feelings or emotions, (b) a normative or moral judgement of peoples' behaviour and (c) the evaluation of objects, artefacts or situations (Folkeryd, 2006). Each of these parameters corresponds to Affect, Judgement and Appreciation, respectively. Expressions of ATTITUDE, in turn, can be either inscribed (explicit) or evoked (implicit), and this distinction is important when it comes to ascertaining the degree of a text's persuasiveness. Whereas inscribed expressions of ATTITUDE are encoded into the text at the lexico-grammatical level, evoked expressions are not, and their meaning must be analysed as the text unfolds cumulatively (White, 2000).

Within the subcategory of Affect, the feelings expressed are categorised according to happiness, security and satisfaction (in both positive and negative terms). Judgement of human actions can be expressed positively or negatively according to social esteem or social sanction. Social esteem concerns judgements of an individual according to his/her capacity, aptitude or temperament in relation to perceived norms. Judged under social esteem are: normality (what is singular/individual about a person), capacity (the extent to which a person is capable) and tenacity (the degree to which a person is dependable). Social sanction, on the other hand, is concerned with judgements based on laws of morality, ethics and legality, and includes veracity (the extent to which a person is truthful) and propriety (how ethical a person is).

Within the third subcategory Appreciation, an accomplishment, artefact or state of affairs is evaluated. It does not concern human behaviour but rather "things, including natural phenomena and semiosis (as either product or process)" (Martin & White, 2005, p. 36). Evaluation in terms of Appreciation

involves reaction (the degree of one's emotional response to something), composition (the degree to which something has been structured and organised as a coherent whole) and valuation (how far something is worthwhile, significant or useful).

The following examples[3] illustrate how researchers value the contribution of other authors in relation to the topic they introduced (1) and how the emotions of researchers arise in relation to an unsatisfactory previous result (2):

> [1] [CC. Txt 7] One of the most **[GRADUATION>Force>quantification]** significant **[ATTITUDE: Appreciation:Valuation:reaction]** results in the matching literature is the one establishing that the set of stable matchings has a lattice structure.
>
> [2] [MC. Txt1_AEJ] Therefore, it is <u>important</u> **[ATTITUDE>Appreciation>reaction] to have an incentive-compatible mechanism**, allowing agents to reveal their preferences truthfully to implement an ex post stable matching. <u>Unfortunately</u>, **[ATTITUDE>Affect>dissatisfaction] this** is <u>impossible</u> **[ATTITUDE>Appreciation>reaction]**

GRADUATION is concerned with values which act to provide grading or scaling, either in terms of the interpersonal force which the speaker attaches to an utterance, or in terms of the preciseness or sharpness of focus. These two dimensions are labelled Force (variable scaling of intensity) and Focus (sharpening or blurring of category boundaries) (White, 2000): (a) Force includes values like intensifiers, down-tones, boosters, emphasisers, emphatics. This category's most obvious mode of expression is through adverbs of intensification – "slightly", "a bit", "somewhat", "rather", "really", "very", "completely" etc. White (2000) states that "somewhat more problematically, this principle of scaling also applies to those values which act to measure quantity, extent, and proximity in time and space – small, large; a few, many; near, far etc.", which makes it difficult to analyse this resource is the hard sciences, where these types of items are extensively used, and might well be fused with some ideational meaning (i.e. items conveying purely factual meanings); (b) Focus covers those meanings which are typically analysed under the headings of 'hedging' and 'vague language' (White, 2000). Typical values are, *he <u>kindlv</u> admitted it; he <u>effectively</u> admitted it*. According to the view of the Appraisal System, values which *sharpen* rather than *blur* the focus are also included – *a <u>true</u> friend,* for example (White, 2000).

ENGAGEMENT encompasses a wide-ranging subsystem through which speakers/writers manage to negotiate positions and engage into dialogue with their audience (Martin, 2000). ENGAGEMENT covers all those resources by which the textual or authorial voice is positioned inter-subjectively. When considering the relationship of ENGAGEMENT resources to those of ATTITUDE and GRADUATION, Appraisal

[3] *All the examples included in this study were extracted from the studied corpora. They have been identified with MC and CC to refer to Main Corpus and Contrast Corpus, respectively.*

scholars who align with Stubbs (1996) consider that ENGAGEMENT serves the purpose of codifying their point of view towards what they say/write. Additionally, this perspective conceives the communicative act as an exchange of voices that people reproduce, quote and manipulate, as suggested by Bahktin's dialogic view of language.

White (2000) provides a list of the diverse array of lexico-grammatical resources encompassed by this subsystem:

- projection and related structures of attribution/reported speech;
- modal verbs;
- modal and comment adjuncts and related forms;
- reality phase (verbal group elaboration);
- negation;
- conjunctions/connectives of expectation and counter-expectation.

In contrast to what may be assumed about Mathematics texts, ENGAGEMENT (which is the most evident resource for flagging author presence as suggested by the pilot analysis) is one of the dominant strategies found in the corpora, as evidenced by the following examples:

> [3] [MC. Txt2_ET] Except for [ENGAGEMENT>Contract>Disclaim>Counter] particular situations where the payoffs are determined by a constant split of a shared surplus, this kind of alignment of the two sides' preferences seems [ENGAGEMENT>Expand>Entertain] unlikely [ATTITUDE>Appreciation>Valuation], and assortative matching fails to arise [ENGAGEMENT>Monogloss + ATTITUDE>Appreciation>Valuation] purely [GRADUATION>Focus>Sharpen] from using delays as a signal of type.
>
> [4] [MC.Txt2_ET] I consider **[ENGAGEMENT>Contract>Proclaim>Pronounce]** the case of "coarse matching" (McAfee 2002; Damiano and Li 2007), where the market is split into a finite number of sub-markets which meet at discrete dates. **[ENGAGEMENT>Expand>Attribute>Acknowledge]**

These examples further show that constructing authorship is a complex fabric where a wide range of resources are put together and combined to create a specific effect. Notice how in (1), the dialogical space is closed by bringing "*the particular situations where the payoffs are determined by a constant split of a shared surplus"* into the discussion, and then opening it by "considering" the textual voice of the researcher, who invokes other points of view different from the first one. In doing so, researchers indicate that although they are committed to their statement, it is only one among several other possible options in that communicative context. Researchers, thus, expand or open the dialogic space for those alternative views.

Our work focuses on the semantic analysis of Appraisal, specifically on the identification of ATTITUDE and ENGAGEMENT resources, and the corresponding lexico-grammatical realisations in Introduction sections. By focusing on these evaluative elements, we seek to show that Introduction sections establish a negotiation whereby researchers try to gain their scientific community's acceptance and approval.

Chapter 4. Methods

This chapter describes the methodology used for this study. It begins with a general description of the investigation design, the corpus, and the criteria for its selection. Additionally, we offer details about an observation phase and interviews with local IMASL researchers, used as data sources to triangulate results.

4.1. General Description of the Research Design

The design of this investigation adopts a combined or mixed approach involving both quantitative and qualitative research tools. We believe that this approach is suitable for this study as it uses the strengths of both qualitative and quantitative methods in the hope of providing the study with stronger results. Likewise, in agreement with Hernández Sampieri et al. (2010) we trust the idea that mixed research methods may help us to obtain a more complete "photograph" of the phenomenon under study. Following Bednarek (2009) and Baker (2006), we also believe that triangulation or the use of a variety of methods may allow us to cross-check and verify the reliability and the validity of the data collected.

As already stated in Chapter 1, our investigation started as the result of our work as writing tutors at GAECI with a group of GT researchers. Thus, through the joint reading of texts with those researchers, we managed to define the objectives of this study. Part of the corpora selected for analysis was also obtained at GAECI sessions. At this stage, we counted on the valuable collaboration of IMASL informants. They provided us with basic knowledge to understand the discipline and the context of SRA writing in GT by exchanging information and reading material at GAECI sessions, and also by allowing us to participate in internal seminars and conferences (Lucero Arrúa, 2018), where we had the opportunity to observe how work is conducted in this community and to talk with some of its members.

This phase in our study had ethnographic elements (Sampieri et al., 2010), like observation and interviews, which allowed us to describe and analyse ideas, beliefs, meanings, knowledge and community practices related to scientific writing. This immersion into the IMASL culture was fundamental to help us in the building of the corpus and also in its analysis.

During this stage, we further conducted a pilot, manual study on a sample of 3 texts (2 in the MC and 1 in the CC, see Appendices 2 and 4), which we took as a reference for establishing a point of departure in the characterization of the Introductions' structure, evaluative features and lexico-grammatical realisations.

This first qualitative phase was followed by the inclusion of a key quantitative tool: the use of the free downloadable version of UAM CorpusTool (O'Donnell, 2008), available at http://www.corpustool.com. With the help of this software, we annotated evaluative elements, entities, their corresponding lexico-grammar realisations, phases and stages. The UAM CorpusTool assisted us in the

identification of patterns of ATTITUDE and ENGAGEMENT resources in GT SRAs, and guaranteed consistency and preciseness in the annotation procedure.

A purely quantitative software-assisted approach would have not suited the needs arising from the nature of our work, the participants chosen for interviews (and the resulting emergence of epistemological issues), and the complexity and analysis of the data. In the same vein, probably choosing an exclusively quantitative approach would have not done justice to the nature of the objectives of this research – identify linguistic and discursive resources typical of SRA in GT that help in the building of authorial voice in both corpora; identify and contrast what entities are presented and evaluated by authors in both corpora, analyse what values they are assigned and what lexico-grammatical resources are associated with those meanings; and suggest lines of work in relation to the design of didactic material. Thus, we considered analysing corpora mainly using the UAM software because it helped in systematizing findings and drawing patterns which would have otherwise been considerably difficult to elucidate. However, we also needed to combine this work with a more qualitative one, by adding elements of ethnographic designs, such as observations and interviews with local researchers (Appendix 3). These provided us with tools to triangulate, cross-check and validate results. Interviews were likewise useful to understand how social and scientific communicative processes operate in the specific setting of IMASL researchers, who are trying to make their way in the world of writing and publishing.

Our methodological design is illustrated in Table 4.1. This illustration tries to capture the process of studying Introduction sections in GT SRAs from the beginning of our research: the objectives that we pursued and the actions conducted to make them possible.

Table 4.1

Methodological design

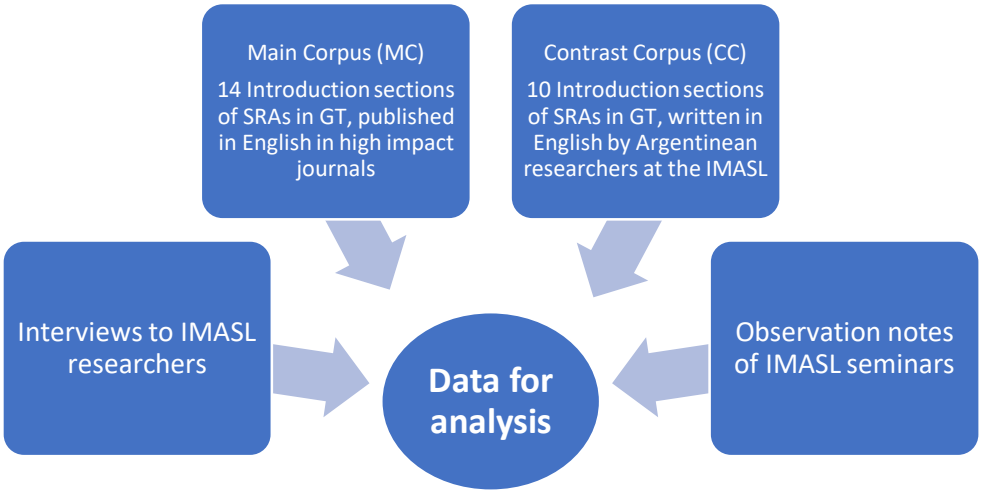
Specific objectives of the study	Research method, source of information and data collection tool		
	Qualitative	**Quantitative**	**Qualitative**
1.To explore the MC in terms of its internal features and genre structure.	Joint reading and discussion of Introductions with IMASL researchers Pilot manual analysis		
2.To identify linguistic and discursive resources typical of SRA in GT that help to build authorial voice in the MC.		Software-assisted analysis: annotation and counting of features	
3.To explore the CC in terms of its internal features and genre structure		Software-assisted analysis: annotation and counting of features	
4.To identify linguistic and discursive resources typical of SRA in GT that help to build authorial voice in the CC.		Software-assisted analysis: annotation and counting of features	
5.To identify and contrast what entities are presented and evaluated by authors in both corpora, analyse what values they are assigned and what lexico-grammatical resources are associated with those meanings, following the taxonomy proposed by White (2000).		Software-assisted analysis: annotation and identification of features	Establishing relations between software-generated data and data from interview responses and observation notes
6.To suggest lines of work in relation to the design of didactic material to be used in writing courses and/or revision sessions by drawing on the results of the analysis proposed in this thesis	Establishing relations between software-generated data and data from interview responses and observation notes		

4.2. Data for Analysis

As already mentioned in the previous section, this study involved a mixed approach. Our focus was on the analysis of GT SRA Introduction sections, and therefore, specialized corpora were built, namely Main Corpus and Contrast Corpus (MC and CC, respectively). However, the consideration of how authorship is constructed was also the result of our analysis of the observation of IMASL internal seminars and interviews with some of the researchers from that institution. Thus, other sources of data for analysis also include (a) the interviews, and (b) observation notes taken during the seminars (Figure 4.1).

Figure 4.1

Data for analysis



4.2.1. The Corpus

The concept of corpus used in this study is based on the one adopted by the EAGLES (1996): "A corpus is a collection of pieces of language that are selected and ordered according to explicit linguistic criteria in order to be used as a sample of the language". As the main objective of this study is to compare how authorship is constructed across Introduction sections in published and unpublished GT SRAs, we have further enhanced the notion of corpus adopted here with that of Laviosa' *comparable corpus* (2002). According to this author, a comparable corpus is a collection of similar texts in terms of their genre, content, form, date, and other features, in different languages or in different varieties of a language whose aim is to study differences and/or similarities presented in similar circumstances of communication. Thus, two different sets of Introduction sections were collected with the purpose to compare ATTITUDE and

ENGAGEMENT elements, and rhetorical components between them: a Main Corpus (MC) and a Contrast Corpus (CC).

Accordingly, we compiled small-scale, specialized and genre-based corpora (Baker, 2006) to comply with the main objective of this study. Criteria for selecting the texts that comprise each corpus are detailed in the following section.

4.2.2. Main Corpus: Description, Selection and Building Criteria

The term Main Corpus (MC) here refers to the collection of texts which are used in a way similar to that described by Sinclair (1996) for the *Reference Corpus* -one designed to provide information about how language is employed in a particular context. We originally collected 101 texts that were part of IMASL researchers' repository, which they update periodically and use for reference and discussion at local seminars. However, after tutoring sessions and observing mathematicians' work at IMASL, we finally reduced the corpus to 14 articles. The reason underlying this decision is that we limited the exploration to the articles and journals which mathematicians referenced across their own productions. Thus, Our MC may not be large enough to represent the whole of that context, or its characteristic vocabulary but it is expected to represent a sample of how evaluative language, though unintended or traditionally neglected, is used in a branch of Mathematics. The underlying aim of this analysis is that this MC can serve as a basis for reliable language reference materials and practices.

Thus, our MC is composed of 14 Introduction sections of Game Theory (GT) SRAs. All of them were written and published in a group of 7 prestigious, high impact journals, as informed by IMASL members during tutoring sessions and observation of referenced works in their manuscripts. Based on EAGLES (1994), our MC can be defined as specialized, since it has been designed for the specific purpose of studying the presence of evaluative language in GT SRA Introductions. Similarly, it did not intend to be representative of the general linguistic use, but rather of the linguistic use of a specific community. Also, it was selected for having characteristics, such as the combined use of symbols and language, which distance it from other texts. Thus, the corpus was gathered on the basis of the external criteria proposed by Sinclair (2005), which include mode, domain, location and date of texts and text-type related to communicative function, representativeness in relation to language variety of the authors, and corpus balance and size. Next, we provide information about the selection criteria:

a) All texts were written in English and published in a group of 7 prestigious, high impact journals, as informed by IMASL members during tutoring sessions and observation of referenced works in their manuscripts.

b) All texts deal with GT-related topics that were included or referenced in the IMASL researchers' productions revised during tutoring sessions, which means that they do not purely deal with Game Theory.
c) The amount of words makes up a total of 21,551 tokens which is almost double in size to the CC. We considered this important, as the MC is intended to be used as a reference in writing.
d) The publication range of the articles is 2010-2018.
e) Titles, subtitles, formulas, equations and other mathematical symbols (if any) were preserved, given the importance of the combination between symbolic and natural language for the construction of meanings, and given the possibility to isolate / neutralize those instances during the analysis.
f) Footnotes included in introductory sections were considered part of the texts under study, and thus, they were included in the analysis.
g) All Introductions belong to original research pieces, which have already been published.
h) The list of journals from which the Introductions were extracted are:
    - American Economic Journal (AEJ)
    - Economic Theory (ET)
    - Games and Economic Behaviour (GEB)
    - International Journal of Game Theory (IJGT)
    - Journal of Economic Theory (JET)
    - Journal of Mathematical Economics (JME)
    - Social Choice and Welfare (SCW)
i) Following Nwogu's criteria of reputation and accessibility (1997), we considered the esteem which IMASL members hold for the publications that we used in this study to guide our choice of the journals. Most articles were easily retrievable from the journals' websites. When access to full articles was restricted to subscribed members, we counted on the generous collaboration of IMASL members, who kindly shared their access permissions.
j) In relation to the SRA section selected for the study, all the Introductions were clearly delimited and identified in the articles.

4.2.3. Contrast Corpus: Description, Selection and Building Criteria

The Contrast Corpus (CC) is made up of 10 draft, unpublished versions of GT SRA Introduction sections, written in English by IMASL researchers who attended GAECI. Here, the term CC is used to refer to the set of texts collected during the period 2016-2018 at GAECI tutoring sessions. The scientists producing the SRAs in the CC are pursuing a career as researchers at IMASL, and they also hold teaching

positions at the UNSL (*Jefes de Trabajos Prácticos* and *Profesores Adjuntos*). They have differing levels of English proficiency, but the common feature among them is that they have just started to write for the highly demanding context of international journals.

Thus, in this context the name "contrast" is given to illustrate the fact that this corpus is employed to analyse the language uses that are different or similar form those found in the MC. The criteria used for the collection of the CC are similar to that of the MC, and we list them below:

a) All texts were revised in the period 2016-2018. The amount of words of the CC makes up a total of 10,647 tokens.
b) The texts comprising the CC are original drafts, unpublished versions.
c) The production range of the articles is 2016-2019. Some of these articles have already been submitted to high-impact journals, and some others are still undergoing internal revision by supervisors and/or colleagues. The intersection point is the fact that none of these articles has already been published.
d) The topics covered by the articles in the CC coincide with those of the MC, and their choice was based on the articles analysed during tutoring sessions, namely assignment models, resource allocation, market allocation and stable matching.
e) Titles, subtitles, formulas, equations and other mathematical symbols (if any) were preserved, given the importance of the combination between symbolic and natural language for the construction of meanings, and given the possibility to isolate / neutralize those instances during linguistic analysis.
f) Footnotes included in introductory sections (if any) were considered part of the texts under study, and thus, they were included in the analysis.
g) Permission to use the texts was obtained through an informed consent that guaranteed both the confidentiality of the data shared and the limitation of use of such data to the completion of this study (See Appendix 4).

4.2.4. Observation of IMASL Internal Seminars

Following Cuevas (as cited in Hernández Sampieri et al. 2010), it was very useful to collect data about phenomena, topics and situations that were complex for us to understand from direct discussions or descriptions of researchers, given our initial lack of familiarity with GT. The "immersion" period during which we observed how seminars were organized and delivered was a fundamental step in this study. It began at the end of 2017 and it extended up to the beginning of 2019, though our contact with the group continued during the writing of this thesis.

We observed a total of 9 presentations, whose development or discussion extended for two days each, due to the complexity of the topics or to the improvements that the audience suggested to the

presenters. The format of the observations was open (Sampieri et al. 2010) and simple in terms of organization, and it basically consisted of completing a template with the following items: date, presenter, presentation title and comments (Appendix 3). The item "comments" included a general description of each session illustrating general organization patterns and personal interpretations. The focus, however, was put on the language used by presenters to refer to their work or intended contribution, the language used by the audience to make corrections, criticisms or comments, and a general description of the atmosphere.

IMASL seminars, which are held since 2014, are organized as a way to promote and keep a study routine within the research group. Participating in seminars is a compulsory activity for researchers of the Game Theory Group, with a fixed schedule from the beginning of each one. Meetings usually last a maximum of two hours and are held every Thursday at 10 a.m.

The seminars were established following the tradition of Economics departments in universities from around the world. The purpose is to discuss new ideas for future projects, to share articles, to present potential new ones and to open the discussion among peers with a view to improving written productions, or to rehearse oral presentations for scientific events.

What researchers present at seminars is not necessarily a finished or ongoing work. It may only be an idea which they need to share and discuss to evaluate the potential interest for research. That is the reason why this space is so important for researchers. It is here where many of the works that are finally published in journals or presented at conferences are born, discussed and improved.

During seminars, an idea becomes a draft from which several cycles of peer review and GAECI interventions follow. Thus, observation of the seminars was key in the understanding of GT SRA writing, as it is a space where authors can detect weaknesses thanks to the comments and corrections made by their audience, which are later captured by their writings.

Seminars are mostly delivered in Spanish, but SRAs on which they are based are written in English. IMASL researchers have occasionally presented in English, especially after a short oral presentation workshop which was organized during the observation period.

4.2.5. The Interviews

We conducted personal interviews to 7 IMASL researchers, who were asked to participate and accepted through a consent form (Appendix 4). The purpose of the interviews was to obtain answers on the issue of writing SRAs in the field of GT in the terms, language and perspective of the interviewees themselves. We were interested in the content and narrative of each answer, and they were also analysed in terms of Appraisal (ATTITUDE and ENGAGEMENT) to study the impact of the contributions, criticisms and debates that arise in oral presentations on the final writings that researchers produce in that context

(Lucero Arrúa, 2018). The interviews were also considered to triangulate the results obtained through the analysis of the SRA corpora.

The researchers who accepted to be interviewed are part of a larger group of mathematicians and economists that constitute the Game Theory Group, one of the 5 lines of research that conform IMASL, where Mathematics is the point at which all the disciplines that are pursued and studied converge. The need to conduct personal interviews arose out of the observation of seminar presentations and writing concerns. We had observed that most of the research topics dealt with in SRAs coincided with those approached at seminars, and we understood that these seminars acted as the cradle of such SRAs. Thus, we considered that accessing researchers' perceptions of how writing is done, the role of seminars in the process of writing, the values expected to be expressed in the writing and the potential reasons for acceptance/rejection of their papers would be a useful source of data to complement the analysis of GT SRA Introduction discourse.

As mentioned above, we interviewed 7 researchers. All of them were teacher assistants at the UNSL and were also on a CONICET research grant. Six of them held a PhD in Mathematics, including the only economist in the group, and one was working on his doctoral thesis. One of them was working with three IMSL members on research articles intended to be submitted to a journal, or presented at an international congress. The rest of the members were working on their own with the same objectives of publishing or presenting.

Following Mertens' classification (as cited in Sampieri et al., 2010), the type of questions included in the interviews were (a) opinion (those aimed at eliciting researchers' point of view in relation with the role of seminars in the process of writing and in their training, in general); (b) expression of feelings (those questions aimed at enquiring into researchers' perceptions in relation with their experience as presenters and writers); and (c) knowledge (those intended to collect data about seminar organization and rules of participation, language education linguistics challenges, fundamentals of the discipline in relation with the world of writing and publishing). The list of questions is illustrated in Appendix 3.

We conducted 3 in-person, individual interviews, and 4 online ones through Google forms, as researchers expressed that their workloads made it difficult to conduct face-to-face interviews. In this way, this group of 4 interviewees were able to respond at their own pace and in their available times.

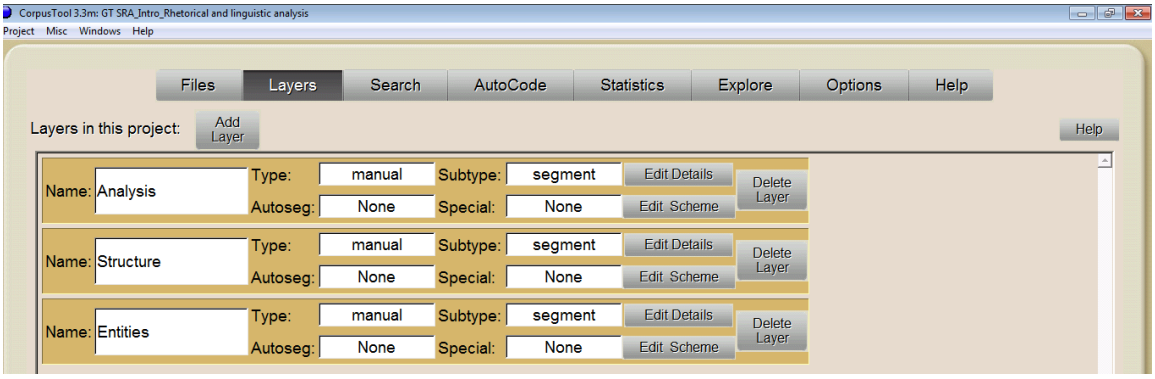
4.3. The Software

The UAM CorpusTool was developed by Michael O'Donnell (2008) at *Universidad Autónoma de Madrid*. It can be freely downloaded from http://www.corpustool.com, and we used it for the manual annotation of segments in each corpus. Manual annotation in this context means that we tagged texts using annotation schemes which were previously designed by us. Those schemes, though based on previously established theories in relation to Appraisal, genre components and evaluated entities, were especially

devised considering our research interests. Our schemes were labelled: Analysis (which included Appraisal categories), Entities and Structure. As the software enables users to load more than one corpus, we had the possibility to compare the features under study in both the MC and the CC. It also enabled us to conduct a multi-layered annotation procedure, which is based on the schemes mentioned above. For example, we could annotate ATTITUDE and ENGAGEMENT (Analysis layer) elements in one Introduction and then carry out an analysis of evaluated entities (Entities layer) and rhetorical components (Structure layer) on the same Introduction (Figure 4.2).

Figure 4.2

The three-layered analysis using UAM Corpus Tool



4.4. Data Analysis

In this section we describe how the processing of the texts in each corpus was carried out. We also refer to the path that we travelled in articulating qualitative and quantitative methods along the study.

4.4.1. Qualitative Phase: Manual Analysis

The process of studying the discourse of SRA GT Introduction sections began with the tutoring sessions at GAECI, as already mentioned in Chapter 1. We also worked closely with mathematicians at IMASL. We pursued several purposes: continuing the tutoring process and making up for the limited time available to address the complexities of the task, collecting the corpus and getting familiar with the discipline and the writing context.

We spent over a year (end of 2017- beginning of 2019) assisting researchers in their writing of SRAs (which were part of our CC) and attending internal seminars and meetings where published articles were discussed (and were part of our MC) to promote the production of new papers. This stage in the investigation was particularly useful and fruitful, as it allowed us to study GT discourse from an ethnographic perspective (Sampieri at al., 2010). It provided us with an overview of the relationship between

the specific GT community and scientific writing. It also contributed to our understanding of the discipline, which helped us in the rhetorical and linguistic analyses.

The pilot manual analysis that we conducted consisted of:

(a) Carefully reading of 2 texts (1 MC and 1 CC);

(b) Identification of stages and phases as part of the rhetorical organization of the genre

(c) Identification and classification of ATTITUDE and ENGAGEMENT resources, and their corresponding lexico-grammatical realisations. The focus of this exploration was on the subsystems of ATTITUDE and ENGAGEMENT, given the trend identified in previous studies (Lucero Arrúa, 2013a, 2013b; Lucero & Laurenti, 2015).

Following Rose (2014), stages are defined as the steps that organise the global structure of a genre, while phases organise how each stage unfolds. Stages are predictable for each genre, but phases vary considerably. Nonetheless, they play an important role in organising the internal structure of stages. Their identification is key to understand how information develops and help other writers to produce new texts that use similar patterns (Martin, 2014). Stages will be written in block capital letters, whereas phases will be written using lowercase.

The identification of evaluative meanings was not the result of the identification of isolated elements embodying a value, but rather the result of the intersection of linguistic choice and prosodic flow (Hood, 2010). In relation to entities, we identified those that were affected by – or were themselves - evaluative elements, following the taxonomy designed by Mirallas (2019, forthcoming), and their corresponding lexico-grammatical realisations. Entities were analysed in terms of whether they were evaluated (or not) and how, by means of the resources provided by the Appraisal System (ATTITUDE and ENGAGEMENT).

We designed and used an analytical matrix that suited the purpose of the pilot analysis (See Table 4.2). The results of this first approach to the selected discourse helped us to guide the annotation procedure using UAM CorpusTool (O'Donnell, 2008) in the rest of each corpus.

Table 4.2

Example of analytical matrix used in the manual analysis

Clause	Appraised Entity	STAGE ^Phase	Appraisal Type	Comments

4.4.2. Quantitative Phase

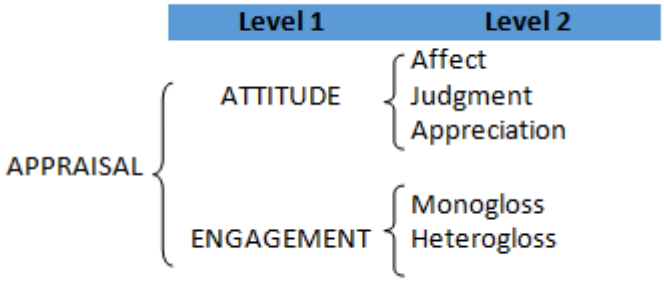
Both in the MC and in the CC, we first identified the Introduction sections of every SRA in PDF format. This step involved no difficulty, as the section was clearly delimited and identified by the word "Introduction" in all the articles. Then, we isolated and converted them to .txt format, which was particularly useful for software-assisted analysis.

The analysis of the MC and the CC consisted in a text-based research, that is, an approach in which texts are analysed using a previously established theory (Bednarek, 2009). We conducted a three-layered analysis (Figure 4.2) using the UAM Corpus Tool involving all the instances of ATTITUDE and ENGAGEMENT (based on the taxonomy proposed by Martin and White [2005]), the appraised entities (following the work of Mirallas [upcoming]) and the section components (following the work of Boccia, 2010).

Appraisal elements in both MC and CC were first analysed. We considered all the instances of ATTITUDE and ENGAGEMENT up to the second level of delicacy (Figure 4.3)

Figure 4.3

Appraisal categories considered in corpora analysis



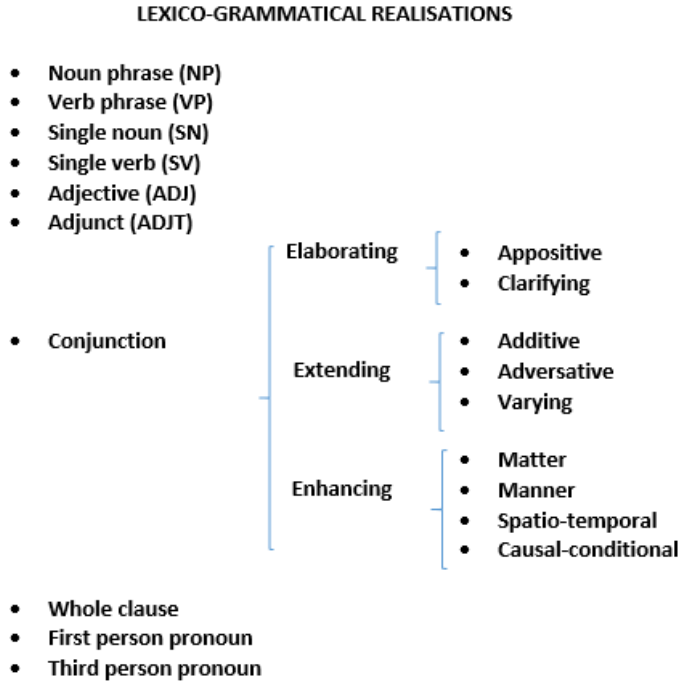
Once identified, we also determined their lexico-grammatical realisations (Fig. 4.4). The criteria for identifying those elements included:

a. The context in which the identified evaluative language functions, i.e. in a scientific context and within a specific discipline. This means that although some wordings in the texts may seem to be evaluative for everyday discourse, such language was not analysed as evaluative in cases in which fields of study have devised a specific set of technical terms. Details about this aspect will be developed later in this section.

b. When referring to the statistical relationship between and among variables, lexical items such as "significant", "negligible", "different" and "dependent" were not considered evaluative.

Figure 4.4

Lexico-grammatical realisations considered during identification of Appraisal items



Instances of ATTITUDE and ENGAGEMENT were automatically quantified by the software after manual annotation. The software offers users to select how to perform and visualize the counting (Global; Local; per 1000 tokens). We chose to specify the counting as expressed per 1000 words to normalize data, given the fact that we worked with different-sized corpora. After this counting was completed, we determined the Evaluative Density (ED) of each corpus, following Shiro (2003). According to this author, ED is an estimate of the frequency of use of evaluative terms in a text. We believe that it is a particularly useful notion, as it helped us in comparing positioning intensities between different corpora. It may not be a valuable indicator if used in isolation, but it can contribute to the analysis of evaluative intensities like the one that we present here.

ED in this study was calculated as the ratio of evaluative items to the total number of words in a corpus (MC and CC, respectively). The formula for calculating ED in our corpora was:

$$\textbf{ED} = \frac{\textbf{Number of appraising instances} \times 1000}{\textbf{Number of words}}$$

where the number of evaluative instances corresponds to the total of appraising elements in the corpus under analysis; the number of words is the total number of words in the corpus; and the number 1000 is the normalisation value that makes comparison between texts possible.

The second layer of analysis was the one corresponding to entities, which we conducted simultaneously with Appraisal tagging. This was helpful to decide whether an instance was evaluative or not. As entities, we considered semiotic objects that were appraised by the writer (Thetela, 1997). These elements may be real or they may be abstractions or mental constructions of the writer, and included mathematical models, theorems, previous studies, elements within models, potential results, researchers' actions (decisions, work, findings, contributions). Thus, participants, processes and circumstances were targeted as lexical strings aiming at individualizing appraised entities.

The third layer of analysis consisted of studying texts in terms of its dynamics and structure. Thus, following the notion of interpersonal analysis (Martin, 2000), Boccia's proposal for the structure of Introductions in medical SRAs, and considering the existing literature on the function of introductory sections (Hood, 2011), we studied the texts in terms of its generic structure. Based on the analysis of the two previous layers, we identified stages and phases.

4.4.3. Qualitative Phase: Interpretive Reading, Interviews and Observation Sessions

In this third phase, we performed an interpretive close reading of RA Introductions in both the MC and the CC with the purpose of elucidating the rhetorical function of the annotated features and investigating their interplay with other discursive resources to construct authorial voice across the section. During this process, we conducted semi-structured interviews with a group of 7 IMASL researchers. These interviews helped us to clarify ambiguities and solve problems derived from our lack of familiarity with GT (See Appendix 3).

At this point, we also compared the results in the software-assisted analysis with the responses provided by IMASL researchers in relation to the values associated to different aspects of the Introduction section. This was particularly important to validate the procedures and the results obtained from the analyses in this study.

Interviewing IMASL researchers was a key action to achieve a comprehensive understanding of: (a) how IMASL works; (b) their writing habits; (c) the nature of the discipline that they study; (d) their expectations regarding their writing; (e) the values that researchers associated with writing (well).

4.5. Special Considerations in the Process of Identifying Appraisal Elements

The following section is devoted to making special clarifications of the meaning of some terms used in the specific setting of GT as an IMASL researcher. We believe this is key to understand the way in which we worked, and the inclusions and the omissions that we made in the process of classifying terms as loaded with evaluation or not. We consider this important to prevent ourselves from incorrectly labelling terms as evaluative when they are, in fact, typical of the discipline. The following aspects of GT language, thus, have been especially considered in the analysis.

Game Theory is a set of analytical tools designed to help mathematicians study and understand the phenomena that they observe in the interaction among "decision makers" (Osborne & Rubinstein, 1994). The basic assumptions underlying the theory are that decision makers pursue well-defined exogenous objectives (they are rational) and take into account their knowledge or expectations regarding the behaviour of other "decision makers" (who reason in a strategic way). The models studied in game theory are highly abstract representations of different real-life situations and they offer researchers the possibility of applying the observed principles and responses to a wide range of phenomena.

It is important to note that GT ideas are not inherently mathematical. GT makes use of Mathematics to express ideas formally. Thus, GT is treated both as a branch of Mathematics and as a social science in the scientific community. As it was previously mentioned, the high abstractness of the models of different real-life situations that it represents makes GT suitable for studying a wide range of phenomena. However, in this study GT is considered a branch of Mathematics, given the fact that this is the predominant context in which it is used at IMASL.

GT SRAs, thus, tell stories about real-life situations that, put very simply, refer to problems of allocation or distribution of resources. Basically, GT SRAs are concerned with rational choice, in which the outcome of the choice depends on the choices of other rational agents. The problems, situations and solutions presented in GT SRAs are represented by a range of terms which are not necessarily (or absolutely) loaded with evaluative meanings. Thus, before introducing the results of the analysis of the entities identified in Introductions, we deem imperative to offer a description of some of the participants involved in the stories of GT SRAs. As mentioned above, Game Theory is full of terms and expressions which can mistakenly be labelled as evaluative by the untrained reader, making the analysis complex and time-consuming. The following description - based on the work of Osborne and Rubinstein (1994) - is not exhaustive, and many other terms could be added, but our purpose in including these notions is to illustrate that they are discipline-related and do not imply any type of evaluation whatsoever. A **solution** is a systematic description of the results that may arise in a family of games. It is not a positively (or negatively) evaluated notion per se. A solution is part of a game in GT. In GT, games can be cooperative or non-cooperative. These "features" do not refer to words used by researchers to evaluate a game, but rather to discipline-related terms referring to:

(a) those models in which there is competition between individual players (non-cooperative), and (b) to those in which alliances can only operate if self-enforcing (cooperative). In the same vein, games can be *strategic* and *extensive*, and this features are not subjective values assigned to a situation or its doers. A game is strategic when it models a situation in which players must simultaneously choose their action plan once and for all; and it is extensive when it specifies the possible orders of events - players can consider their action plan, not only at the beginning of the game, but also every time they have to make a decision.

The words *perfect* and *imperfect* are also discipline-related. A perfect game is the one in which each player is fully informed about the possible movements of the other players; whereas this is not the case in an imperfect information game. Finally, when researchers write about *rational* behaviour, they are not judging a colleague. Instead, they use this expression to refer to a decision maker, or agent who is aware of her/his alternatives in a situation (game), generates expectations regarding unknowns, has clear preferences, and deliberately chooses an action.

As it may be observed, several terms could be regarded as evaluative, but a closer reading and the analysis of the texts reveal that many are solely disciplinary. Without proper guidance, for those not familiar with GT, it may be difficult to distinguish between a term or an expression used to simply describe or name an aspect related to the research and a term or an expression meant to evaluate an object of study or a colleague's work. We have tried to bear this in mind along the whole process of analysis, and we have heavily relied on our informants' support and supervision to categorize expressions according to the Appraisal framework. Nonetheless, we acknowledge that there might be some degree of inaccuracy, which may be mainly due to the combination of the fact that we are not GT experts and our informants are not familiar with subtle aspects of the language.

Chapter 5. Results

The following chapter gathers the main findings of this research in terms of the description of the Introduction section in GT, evaluated entities and type of evaluation. For the sake of organization and clarity, results have been organized per aspect studied (structure, entities and evaluation), and each section within the chapter has been related to the corresponding objective (Table 5.1). Thus, this chapter has been organized in the following sections, which attempt at offering answers to the issues raised by the objectives of this study:

Table 5.1

Summary of chapter sections and relation with objectives of the study

Section	Related objective
5.1. Preliminary Considerations in relation to the Structure of GT SRA Introduction sections 5.2. Structure of GT SRA Introduction sections in the MC 5.1.2. Contextualizing the study in GT: JUSTIFICATION [J] 5.1.3. Negotiating acceptance of a study: OFFER [O] 5.3. Structure of GT SRA Introduction sections in the CC5.3. Triangulation with observation sessions and interviews with IMASL researchers	1.To explore the MC in terms of its internal features and genre structure. 3.To explore the CC in terms of its internal features and genre structure.
5.4. Patterns of Appraisal across Introductions, evaluated entities and lexico-grammatical resources used in the construction of authorship in the MC	2. To identify linguistic and discursive resources typical of SRA in GT that help to build authorial voice in the MC.
5.4.1. Entities across Introductions in the MC 5.4.1.1. Distribution of entities across the Introduction section in the MC 5.4.1.2 Appraising entities in the MC 5.4.2. Entities across Introductions in the CC 5.4.2.1. Distribution of entities across the Introduction section in the CC 5.4.6. Appraising entities in the CC	4. To identify linguistic and discursive resources typical of SRA in GT that help to build authorial voice in the CC 5. To identify and contrast what entities are presented and evaluated by authors in both corpora, analyse what values they are assigned and what lexico-grammatical resources are associated with those meanings, following the taxonomy proposed by White (2000).

5.1. The Structure of GT SRA Introduction Sections

In line with objective 1 of this study, we explored the corpora in terms of its internal features and genre structure. Such exploration revealed aspects of the Introduction section organization which may be considered different from previous models (Hood, 2011; Swales, 1990). Even when these contributions mentioned before were taken as a point of departure, we mainly drew on Boccia's model for Introductions (2010), as it considers the section's organization as a macro proposal (following Martin, 1992), where something (a solution) is offered after providing justifications. In this study, we also understand the Introduction section of GT SRAs as a macro proposal. However, in the process of applying Martin's ideas (1992) and adapting Boccia's model to our context, we considered several factors typical of Mathematics. One of these factors was the primary goal of texts, as described by Halmos (1970), namely following the principle that a text should assert, using carefully constructed logical deductions, the truth of a mathematical statement.

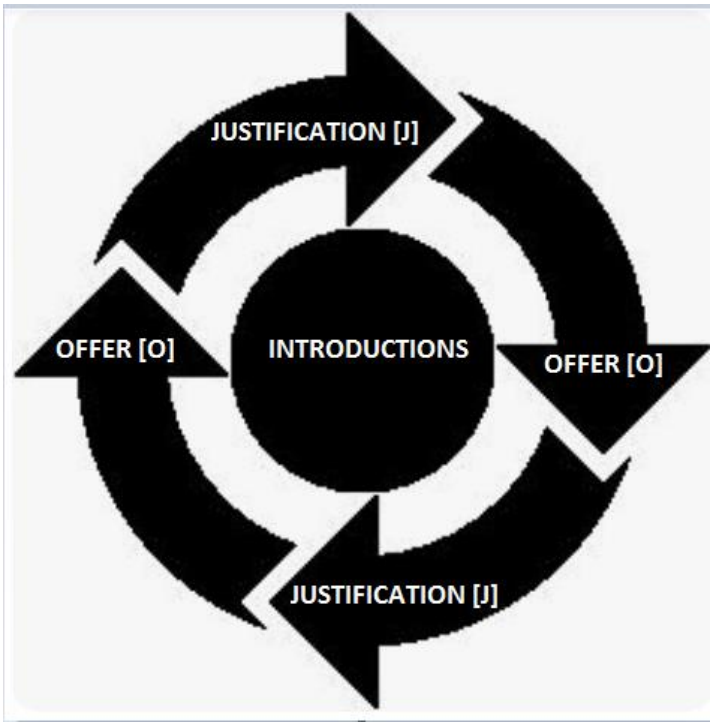
Another aspect that we considered when analysing the corpora was the fact that GT mainly seeks to understand the behaviour of interacting decision-makers and to find how to optimize decision-making processes in the hope of finding the most appropriate strategy to solve a problem. In doing this, GT uses Mathematics as a resource to facilitate the definition of concepts in a precise manner, *"*verify the consistency of ideas, and to explore the implications of assumptions*"* (Osbourne & Rubinstein, 1994, p. 2). Thus, part of making ideas, models, strategies, solutions and behaviours clear is achieved through logical reasoning mechanisms. The details of our findings in each corpus are developed in the following 2 subsections.

5.1.1. Analysis of the MC

As we have already mentioned, Boccia (2010) considers introductory sections in medical articles as a macro proposal, following Martin's model (1992). She contends that Introductions offer a solution after providing justifications. We observed that there is also an exchange of information in GT Introduction sections, and can thus be conceived as a macro proposal. This exchange of information is realised through stages: (a) the JUSTIFICATION, whereby researchers offer information related to the state of the art, weaknesses of the model under study or of previous research; (b) and the OFFER, whereby a solution or alternative based on that information is provided. In general terms, information was found to be presented in a cycle of logical thought processes or explanations, interspersed with the definition of fundamental notions, or references to previous research (what we later associated with what interviewed researchers called "contextualization") and the formulation of a truth (results, current contribution) (Figure 5.1).

Figure 5.1

Proposed Introduction structure for GT SRAs



Thus, following this observation, we divided GT SRA Introductions into two main stages: JUSTIFICATION [J] and OFFER [O] (Figure 5.1). This was combined with Boccia's view of Introductions as macro-proposals (2010) in terms of Martin's interpersonal notion (1992), through which researchers seek to offer their work as a response to weaknesses or gaps in existing literature and receive the acceptance of such work. Introduction sections operate as research warrants (Hood, 2011) in any discipline, and so they do in Mathematics, as well. However, the way in which they are organized in GT SRAs follows the natural dynamics of Mathematics, where a cycle of stages involving "JUSTIFICATIONS" [J] and "OFFERS" [O] is repeated along the articles (at least one phase of each stage), resulting in the J^O^J^O structure.

This coincides with Burton's observation of the mathematical thinking processes, leading to the construction of convincing arguments (1984). Burton's notion of mathematical writing as a loop involves four central processes, namely (a) specializing, (b) conjecturing, (c) generalizing, and (d) convincing, in which specializing, conjecturing and generalizing could be grouped into the [J] stage and convincing could be embraced by the [O] stage. The individual components of Introductions' structure, as observed in the MC, are described in the following section.

a. **The JUSTIFICATION [J]: Contextualizing the study in GT**

This stage is the opening of GT Introduction sections, in which the object of study, the analysed model or the theorem under scrutiny are announced, explained and contrasted with previous research. The JUSTIFICATION construes a contextualizing and convincing process in the sense that it is the stage at which researchers try to make readers familiar with the proposed topic and persuade them of the importance or relevance of the topic or object under study. We observed that researchers introduce their work by giving information about the phenomenon under investigation, justifying the need to carry out the research and referring to the relevance of the phenomenon. In both corpora, JUSTIFICATIONS also include explanations of established concepts or logical statements, which contribute to the building of a context expected to favour the success of what they intend to present. Introductions have been found to be developed across the following steps or phases:

a) Announcing the topic of research/object of study [at]
b) Indicating relevance of object of study [ir]
c) Reviewing previous research [rpr]
d) Identifying gaps/weaknesses [ig]
e) Stating need for current research [sncr]
f) Indicating objective [io]

b. **The OFFER [O]: Negotiating Acceptance of a Study**

The OFFER [O] is the stage in which authors present their contribution, evaluate its aspired relevance and locate it in the context of the whole paper and previous research. This function is realised through statements which are carefully presented after having displayed a whole set of references to previous research in which the problem has not been approached in a particular way, or for finding a solution by means of a different perspective. OFFERS are often accompanied by an explanation or the introduction of a logical statement whereby authors support their contribution. OFFERS also develop through phases, which were identified as:

a) Announcing contribution [ac]
b) Appraising contribution [app_c]
c) Indicating structure [is]

But what we found novel in relation to how reference to results is made in the Introduction section of SRAs of other disciplines was the fact that OFFERS (any phase) are often interspersed with the phase Reviewing research, from the JUSTIFICATION stage, as it may be observed in the following example:

[5] [MC: Txt2_SCW_Strategy-proof and anonymous rule in queueing] [O>ac]

In our setting, Pareto-efficiency is decomposable into two conditions of efficiency: queue-efficiency (minimization of the total waiting cost among agents) and budget-balance (zero-sum transfers)[3].

Dolan (1978) has provided a rule that satisfies strategy-proofness and queue-efficiency. The class of equally distributed pairwise pivotal rules (rules assigning an efficient queue and transfers considering each pair of agents in turn, making each agent in the pair pay the cost he imposes on the pair, and distributing the sum of these two payments equally among the others) proposed by Suijs (1996) satisfies not only strategy-proofness and queue-efficiency but also budget-balance (that is, Pareto-efficiency). [J>rpr]

In the MC, all the phases within each stage have been observed to be distributed in a relatively even way, thus suggesting a balance in the information presented (Table 5.2):

Table 5.2

Introduction components in the MC (per 1000 tokens)

Feature (STAGE>Phase)	MC (per 1000 tokens)
JUSTIFICATION [J]	2.93
Announcing topic [at]	0.22
Indicating relevance [ir]	0.26
Stating established concept [sec]	0.39
Reviewing previous research [rpr]	0.99
Identifying gaps or weaknesses [ig/w]	0.52
Stating need for current research [sncr]	0.09
Indicating objective [io]	0.09
OFFER [O]	3.32
Announcing contribution [ac]	1.59
Appraising current contribution [app_c]	0.52
Indicating SRA structure or organization [is]	0.52

5.1.2. Analysis of the CC

Introductions in the CC texts were observed to follow the same organisation than in the MC texts. Information is similarly presented in the cycle of stages J^O^J^O, in which the formulation of truths or results are followed by logical thought processes or explanations and interspersed with the definition of fundamental notions, or references to previous works, either of their own or others'.

The [J] phases Reviewing previous research [rpr], Stating established concept [sec] and the [O] phase Announcing contribution [ac] were found to be profusely used along the introductory sections, as revealed by Table 5.3.

Table 5.3

Introduction components in the CC (per 1000 tokens)

Feature (STAGE>Phase)	CC (per 1000 tokens)
JUSTIFICATION [J]	5.4
Announcing topic [at]	0.4
Indicating relevance [ir]	0.4
Stating established concept [sec]	1.2
Reviewing previous research [rpr]	2.5
Identifying gaps or weaknesses [ig/w]	0.3
Stating need for current research [sncr]	0.08
Indicating objective [io]	0
OFFER [O]	4.1
Announcing contribution [ac]	1.7
Appraising current contribution [app_c]	0.8
Indicating SRA structure or organization [is]	0.7

This might suggest a coincidence with local researchers' responses as to the importance of what they call contextualization in their work. They openly communicate their results, but put a considerable effort in acknowledging others' contribution to mark their point of departure. They also emphasize the introduction of established concepts before taking the risk of announcing a result. Interestingly, no reference to indicating objectives was found (Example 5). Appraising contributions in the [O] stage [app_c] is observed to be used in a low percentage.

[5] [CC: Txt3_Convergence of the Approximate Cores to the Aspiration] STAGE>Phase

In this paper, we study and compare two non-empty extensions of the core that give alternative solutions to the restrictive condition established by Kaneko and Wooders (1982). One of the solutions is the approximate core which proposes the replication of games to obtain non-empty e—cores if the number of replications is sufficiently large. This idea has been introduced by Wooders (1981-1983)2 and studied in Kaneko and Wooders (1982), Kovalenkov and Wooders (2003) and Wooders (2008), among others. In this approach, the existence results are based on the fact that, with a finite number of types of players and bounded basic group sizes, large games have non-empty approximate cores. [J>at] [O>ac] [J>rpr]

The other solution concept is the aspiration core which proposes that the cooperation (or negotiation) of the players can be supported by overlapping structures of coalitions (not just the grand coalition) called balanced families. The aspiration core has been introduced by **Bennett** (1983) (see also, **Cross** (1967), **Albers** (1979)) and recently, studied by **Bejan and Gomez** (2012), **Cesco** (2012) and **Arribillaga** (2013), among others. [O>ac] [J>rpr]

Although the approximate core and the aspiration core are two solutions that have the same motivation -to give an answer to (partitioning) games with an empty core- they have not yet been compared and linked in the literature. **The main contribution of this paper** is to show different relations between the approximate core and the aspiration core in partitioning games. First, we show that the cores of the replicated games, in a subsequence of the replica games, are equal to the aspiration core of the (original) game. Second, we prove that the collection of e—approximate cores converges to the aspiration core when e tends to zero. All the obtained results are completely independent of the set T of feasible coalitions and the payoff functions. [O>ac]

The paper is organized as follows. In the **next section**, preliminary definitions and notation are introduced. In **section 3**, approximate core and aspiration core definitions are presented. In **section 4**, we present the main results.) O>is

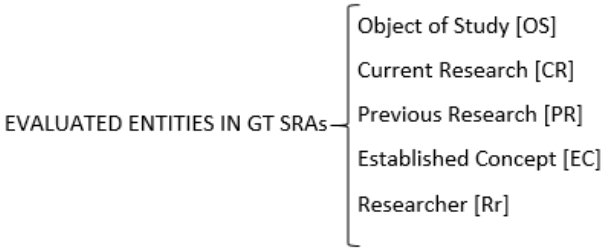
We see how, in agreement with the responses provided by IMASL researchers as to the aspects considered key in introductory sections, emphasis in the JUSTIFICATION stage in CC texts is placed on strengthening the context from which the current research is written. Authors in CC texts clearly devote a substantial effort in setting the stage for the topic they present.

5.2. Description of Appraised Entities in the GT SRA Introduction

In this study, evaluated entities were identified as the semiotic objects that were appraised by the writer. Following objective 5 in this study, the targeting of entities was also useful to help us define stages in the development of Introductions, as bearing in mind what researchers deal with is key to understand how information unfolds. These elements, which may be real situations, abstractions or mental constructions of the writer, included mathematical models, theorems, previous studies, elements within models, potential results, and researchers' actions (decisions, work, findings, contributions. We analysed the MC first, and identified 5 main categories, namely Object of study [OS]; Current research [CR]; Previous research [PR]; Researcher [Rr] and Established concept [EC] (Figure 5.2). Then we compared those findings with the analysis of the CC, and observed that Introductions address the same categories.

Figure 5.2

Appraised entities in GT SRA Introductions



The [OS] can be a model, a variable or element in a specific model, or a problem; the category [PR] mostly refers to similar or same models or problems dealt with from different perspectives, under different conditions/contexts, considering other variables; the category [CR] often refers to the current work in general terms, parts or sections in the study, a specific result, variables, contexts or circumstances which have not been considered in previous works, as shown in the following example (my bolds), which illustrates how authors openly refer to their own contribution:

[6] **Our first result** [CR. MC Txt] complements these findings by showing that there is no incentive-compatible and ex post budget-feasible mechanism that produces ex post stable matchings in a market with transfers (Theorem 1).

The category [EC] refers to theoretical frameworks, models or specific notions fundamental for the understanding or development of a certain work. ECs also refer to notions whose definition or explanation

is often necessary, not so much because the targeted audience may not know it, but rather because defining or explaining an EC has the discursive power to help construct authorship by showing knowledge of the discipline, and thus, community membership. The following example illustrates how this category is realised (my bolds):

[7] [EC. MC Text] (…) ***a matching mechanism*** is ex ante stable if agents get nonnegative utilities for all outcomes and there exists no firm-worker pair who could match with each other at a particular wage and both get higher expected utilities at the ex ante stage.

Example 7 above presents an established concept by means of a definition, thus excluding this sentence from the realm of argumentation or negotiation with the rest of the scientific community.

Other entities can also be the author(s) of the current paper, or authors of cited literature [Rr] and/or previous works in general [PR]:

[8] [Rr. MC Text] A special case of this result, in which both matchings are stable, was obtained in Gale and Sotomayor (1985) for the Marriage model and called by these authors Decomposition lemma for all outcomes and there exists no firm-worker pair who could match with each other at a particular wage and both get higher expected utilities at the ex ante stage.

[9] [Rr. MC Text] (…) **we** then show how several results known from population games carry through to our setting.

Example 8 refers to other authors' contribution, by pointing to their works, whereas Example 9 highlights the agency of the analysed text's authors. In both cases, entities are realised through Noun Groups (NG) at the lexico-grammatical level, which is also the tendency for [OS], as revealed by the exploration conducted with the help of the UAM Corpus Tool. In the case of [CR], reference to researchers within the SRA is mainly made through first person pronoun (FPP) "we", whereas reference to other authors is mainly construed through NGs corresponding to the names of the cited researchers (Table 5.4).

Table 5.4

Lexico-grammatical realisations of each entity in the MC (per 1000 tokens)

Entity	L-G feature	MC
OS	NG	1.63
CR	NG	7.24
PR	NG	8.1
Rr		
Current	FPP	5.43
Previous	NG	3.19
	TPP	0.56
EC	NG	2.39

The same entity categories were also found in the CC texts, and the most frequent lexico-grammatical realisations were similar to those found in the MC, as shown in Table 5.5:

Table 5.5

Lexico-grammatical realisations of each entity in the CC (per 1000 tokens)

Entity		L-G feature	CC
OS		NG	1.33
CR		NG	5.70
PR		NG	8.03
Rr			
	Current	FPP	6.23
	Previous	NG	3.07
		TPP	0.63
EC		NG	

5.2.1. Distribution of Entities across the Introduction Section

Now that the corpora analysis has revealed that the same entities exist both in the MC and the CC, we will refer to the way in which they are distributed across MC and CC Introductions in the same chapter section. We will deal with the similarities and differences found during the analysis.

In line with objective 5 in this thesis, one of our contributions consists in identifying how entities are distributed across the Introduction section in GT. We believe that this analysis can help both in determining the different stages through which information is developed and how authorship is constructed. The frequency of occurrence of each category reveals the importance attributed to specific entities, and gives us information as to the type of evaluation assigned to them. Table 5.6 illustrates what entities are given more prominence at each stage in each corpus. A brief discussion of how we interpreted these data is provided below.

Table 5.6

Entities in GT SRA Introductions across corpora

Entities	MC (per 1000 tokens)	CC (per 1000 tokens)
Object of Study (OS)	2.25	1.46
Current Research (CR)	8.01	6.21
Previous Research (PR)	8.66	8.75
Researcher (Rr)	9.36	10.1
Established Concept (EC)	2.89	1.56

This table also shows some interesting differences. The first one is the reference to the OS, which is markedly more frequent in MC than in CC texts. Again, this difference might reinforce local researchers' perception of the aspects on which Introduction sections should focus. Once more, the idea of

contextualization that they emphasized during interviews is supported by the data found in the corpus, where the OS is extensively developed. The other two entity categories which feature visible differences in occurrence, CR and EC, are found in the MC. The numbers suggest that, in contrast to the evidence in the MC, authors of the CC texts tend to recoil from exposing themselves when writing about their own work. The difference in the frequency of ECs may show an agreement with Burton and Morgan's observation (2000) that an important part of constructing an identity as a SRA writer is generalizing, which can be done through making reference to an EC. ECs are the bases from which new contributions are developed, and authors are aware of that fact, as evidenced by the information found in the MC. The implication of this last statement is not that authors in CC texts ignore this idea, but rather that they rely more on reference to PR, as also shown in Table 5.6.

5.2.2. Entities Addressed at the JUSTIFICATION Stage

The prominence of the entities CR and PR in both the MC and CC implies that this stage is clearly the one in which researchers try to make readers familiar with the proposed topic and persuade them of the importance or relevance of the topic or object under study. This is consistent with the values revealed by the interviews, where researchers highlight the importance of contextualizing research before taking the risk of announcing their own contribution. This process is achieved by focusing on [PR], which is also addressed by explicitly mentioning the names of researchers whose work has laid the bases for further developments (Rr), as shown in the following examples:

[10] [CC: Txt3_Convergence of the Approximate Cores to the Aspiration]

> The **partitioning games** [OS] have been introduced by **Kaneko and Wooders** (1982), [PR] and recently studied by **Solymosi** (2008), and **Auriol and Marchi** (2009), [PR] among others. **These games** [OS] are useful in modeling situations with restricted cooperative possibilities between the players, and therefore, only some coalitions may be formed. (…). But even if all coalitions are allowed, it may still happen that only small coalitions play essential roles, because **the game** [OS] has some special structure, as in **the bridge game of Shubik** (1971) [PR], and **the assignment games of Shapley and Shubik** (1972) [PR].

[11] MC: Txt2_SCW_Strategy-proof and anonymous rule in queueing

> In **our setting**, [CR] Pareto-efficiency is decomposable into two conditions of efficiency: queue-efficiency (minimization of the total waiting cost among agents) and budget-balance (zero-sum transfers). **Dolan** (1978) [PR] has provided a rule that satisfies strategy-proofness and queue-efficiency. **The class of equally distributed pairwise pivotal rules** [EC] (rules assigning an efficient queue and transfers considering each pair of agents in turn, making each agent in the pair pay the cost he imposes on the pair, and distributing the sum of these two payments equally among

the others) **proposed by Suijs (1996)** [PR] satisfies not only strategy-proofness and queue-efficiency but also budget-balance (that is, Pareto-efficiency). Moreover**, this class** [EC] is the only one of rules that satisfy strategy-proofness, equal treatment of equals in welfare, and Pareto-efficiency (**Kayı and Ramaekers 2010**) [PR]. Equal treatment of equals in welfare requires that the rule assign an allocation for which the welfare levels of agents are equal whenever their unit waiting costs are the same. **Pareto-efficiency [EC]** is a desirable but demanding condition for not only queueing problems but also allocation problems of indivisible objects and money [see Holmström (1979). Thus, it is interesting **to search rules without Pareto-efficiency [ES/EC]**.

5.2.3. Entities Addressed at the OFFER Stage

The results derived from the corpora analysis using the UAM Corpus Tool show an almost exclusive focus on CR and Rr in the [O] across both corpora. Reference to CR is made in the context of announcing a contribution, evaluating it, putting it to test in the community. Reference to Rr, in turn, is related to an overt evaluation of their CR, since the entity Rr is mainly expressed through clauses headed by first person pronouns, as may be observed in example 12. The latter is a significant difference with the way in which this entity is realised in the JUSTIFICATION stage at the lexico-grammatical level. Whereas reference to Rr in the JUSTIFICATION is related to PR, and it is instantiated through NGs or single nouns corresponding to the names of other researchers, Rr in the OFFER accounts for the authors of the articles in each corpus (MC and CC), as illustrated in example 12. These results coincide with the preliminary observations that we made in the pilot analysis, with the description of the function entailed by the OFFER in Introduction sections, and with IMASL researchers' responses during the interviews:

[12] [MC] We [Rr-CR] analyze the rules satisfying strategy-proofness and anonymity in welfare and show that under strategy-proofness, anonymity in welfare implies queue-efficiency. As equally distributed pairwise pivotal rules satisfy anonymity in welfare, by combining the result of Kayı and Ramaekers (2010) [Rr-PR] with ours [Rr-CR], we [Rr-CR] also give another characterization of the class of equally distributed pairwise pivotal rules as the only one of rules that satisfy strategy-proofness, anonymity in welfare, and budget-balance.

5.3. Patterns of Appraisal across Introductions

In this section, we illustrate how the ENGAGEMENT and ATTITUDE frameworks can be used to reveal the linguistic resources that enable authors to present a stance, or an opinion towards the entities identified through the analysis of the corpora. We also expect to make those linguistic resources explicit in ways that can inform writers about how to develop and present an argument, or to position themselves and their research across Introductions. In this way, we managed to fulfill objectives 2, 4

and 5 - to identify linguistic and discursive resources typical of SRA in GT that help to build authorial voice in the MC and the CC; and to identify and contrast what entities are presented and evaluated by authors in both corpora, analyse what values they are assigned, and what lexico-grammatical resources are associated with those meanings, following the taxonomy proposed by White (2000).

5.3.1. Appraisal Values in Introductions: General Characteristics

We have already analysed what researchers write about (entities), and where in Introductions they tend to do it. This relates to objectives 2, 4 and 5 (page 19 in this study). We now turn to the evaluations that they assign to those entities, which is also a concern of objective 5. Objective 5 in this study aims at finding what values are at stake when researchers write about each entity and through which resources, at the lexico-grammar level and from the point of view of the Appraisal system.

The construction of authorship is a complex process, which unfolds as the text develops. Thus, no single strategy or resource can be attributed to such a construction. In addition to using linguistic resources that enable the kind of positioning expected in much academic writing, projecting an effective authorial voice also calls for the management of prosody, known as the collection of values to strengthen an argument (Hood, 2011), or a flow of evaluative patterns which accumulate along the text (Hunston & Thompson, 2000). We view authorships as a construct comprising the linguistic resources that can be explicitly linked with the genre components and the entities dealt with at each stage. Our analysis draws on the ATTITUDE and ENGAGEMENT frameworks (Martin & White, 2005) in Systemic Functional Linguistics. It furthers links the linguistic resources identified through those frameworks with the genre components identified throughout corpora analysis, in the hope of providing information about how researchers achieve an authoritative voice.

In her study of academic writing, Hood (2004) used the Appraisal system to show that published writers used more linguistic resources for evaluating concepts and findings (ATTITUDE: Appreciation), while student writers used more resources for presenting feelings and opinions about people (ATTITUDE: Affect and Judgement). In our study, the distribution across texts in both corpora of evaluative elements, as a whole, is quite similar (Table 5.7).

Table 5.7

Evaluative density in the MC and the CC (normalized values per 1000 tokens)

	MC	**CC**
EVALUATIVE DENSITY	57.07	52.2

A more in-depth analysis as to the type of Appraisal used reveals that ATTITUDE resources are evenly used in both corpora. However, there is a marked tendency to openly state a position in the MC in

the form of ENGAGEMENT (Table 5.8). This situation could relate to a more cautious use of language on the part of CC text authors when committing to opinions or evaluating a contribution (as revealed by interviews).

Table 5.8
Appraisal instances in GT SRA Introduction sections (normalized values per 1000 tokens)

	MC	**CC**
ATTITUDE	18.2	18.1
ENGAGEMENT	34.3	28.7

The following examples illustrate the tendency revealed by the figures in the table above:

[13] [MC. JUSTIFICATION stage] The theoretical literature on two-sided matching began with the **simple** (ATTITUDE> Appreciation) one-to-one (marriage) model of **Gale and Shapley** (1962), (ENGAGEMENT>Expand) in which agents on opposite sides of a market (men and women) seek to match into pairs.

[14] [MC. OFFER] This model **subsumes** all classical matching models (ATTITUDE>Appreciation + ENGAGEMENT>Monogloss), and its **generality** (ATTITUDE> Appreciation) **allows us** (ENGAGEMENT>Contract) to make two novel theoretical contributions.

A direct implication deriving from these results is that taking responsibility for what is written and calling other voices into play when introducing a notion or a result is not alien to Mathematics. The evidence provided by the analysis shows a tendency to include evaluative features, which enable authors to build an identity. To this end, we need to examine in closer details how ATTITUDE and ENGAGEMENT resources are used. To begin with, we illustrate differences in use and frequency, which are most notable in the second level of delicacy of the system, as revealed by Table 5.9.

Table 5.9

Differences in use of Appraisal resources between MC and CC

Feature	MC per 1000 tokens	CC per 1000 tokens
ATTITUDE		
Affect	0.99	0.32
Judgement	0.17	0.16
Appreciation	16.5	17.34
ENGAGEMENT		
Monogloss	0.81	2.54
Heterogloss	26.22	32.01
Contract	6.05	10.21
Expand	20.17	21.8

Following a previous work (Lucero Arrúa, 2013), Appreciation is one of the most frequent types of evaluation in both corpora. As an evaluative resource, Appreciation refers to "the negative or positive evaluation of social products, conditions, activities, processes or phenomena" (Coffin & O´Halloran, 2006, p.83), and its use in GT SRA Introductions seems to be no exception. The following examples illustrate this type of meaning:

[15] [MC. Txt 1_AEJ] The second strand considers incentive-compatible **core** concepts for markets with transfers, following the **seminal** work of Wilson (1978).

[16] [CC. Txt 1] We have four **main substantive** findings.

Example 15 illustrates that authors value concepts considered fundamental for the development of the discipline and further studies, and the work of other authors who have contributed to the understanding of a model, a problem, a context. Likewise, Example 16 shows how authors appraise their own results, thus positioning them as a valuable contribution to the study under discussion.

Both MC and CC texts show a profuse use of ENGAGEMENT resources, with heteroglossic elements as a salient characteristic. However, how voices are brought into play is apparently different in each corpus, as shown in Table 12 above. Whereas MC texts exhibit a more frequent use of expansive items, CC texts seem to resort more frequently to contractive resources. A deeper analysis may certainly reveal details which can be useful to consider during text analysis with a view to making writing strategies visible. Our preliminary conclusion is that, as a tool to introduce other voices into the text to make them part of a discussion within the scientific community, heteroglossic resources are fundamental. The prominent presence of this resource in the MC leads us to think about the need to highlight this feature as a valuable aspect, key to enter the discussion with colleagues. By contrast, the high frequency of contractive items in the CC may be indicative of an unnecessary source of conflict with other authors. Contractive options might

represent an active challenge of the views of others, and they are indeed necessary to argue for one's own contribution or results, but the apparent overuse observed in the CC may not be desirable in the context of academic argumentation. The following example partly illustrates this:

> [17] [MC] The outcome of such coalitional interactions **should** then be a stable matching, **if it exists**. **However**, such predictions **should** be revised in the cases in which preferences are not necessarily strict.

Authors in this text introduce the outcome of specific coalitional interactions as an arguable possibility by using "should" and the conditional clause at the end of the sentence. Discussion over the possibility of coalitional interactions being a stable matching is heteroglossic in the sense that it depends on factors that the authors cannot control (or do not even know), but whose existence they acknowledge. Contractive options (constructed through the "if" clause and the conjunction "however") serve the purpose of arguing for the notion of a stable matching as a possibility by deterring readers from not considering other factors affecting the situation described.

5.3.2. Appraisal Values and Entities across Introductions

An analysis of what values are assigned to entities across the Introduction section might further clarify how authorship is constructed. These results, which are shown in Table 5.10, were manually calculated due to software failure during processing.

Table 5.10

Evaluative dimension constructed in relation to each entity

Entity	Appraisal instances + Entities									
	ATT:Aff		ATT:App		ENG:Cont		ENG:Exp		ENG: Mon	
	MC	CC	MC	CC	MC	CC	MC	CC	MC	CC
CR		0.1	0.3	1.3	0.4	0.3	0.5		0.3	
PR			0.3	0.4	0.1	0	0.5		0.09	
Rr		0.09	0.1	0.3	0.2	0	0.1		0.04	
EC	0.04		0.1	0.1	0.2	0.3	0.1	0.3	0.09	0.1
OS		0.09		0.4	0.2				0.1	

In line with Hood's findings (2011), our corpora also show differences in favour of the use of Affect resources in CC texts. Interestingly, no Judgment instances were found in either corpora. CC texts exhibit a higher frequency of Affect instances, and they are linked to the OS and to the Rr, as shown in table 5.10. Affect in MC texts is not completely absent, but its use is scarce, and limited to the expression of dissatisfaction with a fact that cannot be changed (EC), thus showing the need to search for other possible ways to tackle the problem on which it is based, as in the following examples:

> [18] [MC] Therefore, it is important to have an incentive-compatible mechanism, allowing agents to reveal their preferences truthfully to implement an ex post stable matching.

Unfortunately, this is impossible: Roth (1982) shows in a matching problem without transfers that there is no incentive-compatible mechanism that produces stable matchings.

[19] [MC] Namely, he addresses the question of whether a two-agent coalition can circumvent stable mechanisms via pre-arranging, whereby both of them are at least weakly better off while at least one of them is strictly better off. Then, **unfortunately**, he shows that no stable mechanism is immune to pre-arrangements.

Appreciation values are most visibly used in relation to CR in both corpora, although a higher percentage of instances is observed in CC texts. Clearly, the need to depict a contribution as necessary, important or novel is more evidently focused on this resource. We believe that this is a positive feature of CC texts, but it reinforces our observation that authors of these texts tend to centre on a more limited range of resources to show their position in relation to - for example - CR. Texts in the MC show a wider variety of Appraisal elements, as evidenced by table 5.10.

5. 3. 3. Lexico-grammatical Realisation of Appraisal Instances

The analysis further showed us how ATTITUDE is realised through lexico-grammar, which we consider important to record the actual ways in which positioning is achieved. We believe that this would eventually be the data that will be shared with by author-researchers through course, didactic material or tutoring sessions. As evidenced in Table 5.11, single adjectives are the most frequent element used in relation with Appreciation, which, in turn, is more frequently associated with the evaluation of CR. Some examples are presented below:

[20] [CC texts, in relation to CR] We term the **new** solution as the balanced core. As the balanced core needs to consider the distribution of payoff, x, and the balanced family, B, which guarantees the assignment x, the elements in the balanced core are pairs (x, B).

[21] [CM text, in relation to CR] Our paper is **important** not only for theoretical purposes, but also for practical issues.

Notice how in both examples, authorship is not only achieved through the use of Appreciation, but also through the use of the first person pronoun, which turn the clauses into monoglossic options. It is important to note the relevance of ENGAGEMENT will be referred to a few paragraphs below for the projection of an authorial voice.

Similarly, Appreciation of other authors' work (PR) is also noteworthy in both corpora, but a closer evaluation of the frequency of other Appraisal items suggests that authorial voice is a joint construction achieved through ATTITUDE and ENGAGEMENT resources.

Table 5.11

ATTITUDE and lexico-grammatical realisations in the MC and the CC

ATTITUDE	Adjectives MC	Adjectives CC
AFFECT	0.2	0
JUDGEMENT	0.07	0.06
APPRECIATION	8.44	7.7

5.4. The Combined Effect of ATTITUDE and ENGAGEMENT

In line with objectives 2, 4 and 5 in this study, we now discuss the resulting patterns of Appraisal in Introductions. At the JUSTIFICATION stage, ATTITUDE is mainly found in the form of Appreciation, and it affects the way OSs are presented, as well as how or to what extent the current research connects with PR, and therefore it is also evaluated, as in the following examples:

[26] Although desirable [ATT:App], **ex post stability** [OS] may not be necessary [ATT: App] for the success of some matching markets, especially when the outcome of the mechanism can be enforced.

[27] Following high-profile applications of matching in labor markets and school choice programs, the **foundational** [ATT: App] work on matching [PR] has been **extensively** [ATT: App] generalized.

In line with the communicative focus of the OFFER stage, Appraisal values were observed to be exclusively focused on the Rr and CR in the MC. As we mentioned earlier in this study, CC texts exhibit more variation in the type of entities that are addressed at the OFFER, but a higher number of instances of CR and Rr are observed, and consequently, the same is observed in the case of Appraisal values.

The following examples illustrate how resources for ENGAGEMENT contribute either to show attitudinal values by entertaining alternatives and possibilities as claims open to question (Expand), or to promote and/or emphasize researchers' perspectives and assertions (Contract):

[28] [MC] First, **we show** [ENGAGEMENT: Contract] that both acyclicity and full substitutability are **necessary** [ATTITUDE: Appreciation] for classical matching theory. **If either condition is violated**, [ENGAGEMENT: Expand] then stable allocations **cannot** be guaranteed [ENGAGEMENT: Contract]

We also show how monoglossic propositions are presented as taken-for-granted facts and/or as assumed by the reader to be shared with the researcher's position (Monogloss).

[29] [MC] Pareto-efficiency **is** an efficiency condition. [ENG: Mon] An allocation is Pareto-efficient if there is no other allocation which makes each agent at least as well off and at

least one agent better off. A rule is Pareto-efficient if it assigns, for each unit waiting cost profile, a Pareto-efficient allocation

In all cases, instances of ATTITUDE in the OFFER stage are followed by an ENGAGEMENT instance in an interplay of Contract and Expand options, as shown in examples 30 to 32 below:

[30] [CC] In this section **we** also **prove** [ENGAGEMENT: Contract] that this set has a lattice structure.

[31] [MC] **Our contribution** [ATTITUDE: Appreciation + ENGAGEMENT: Contract] is to **propose** [ENGAGEMENT: Expand] a perspective on designing for school choice with consent, and to design new mechanisms and interpret existing mechanisms based on that perspective.

[32] [CC] **Our characterization** [ENGAGEMENT: Contract + ATTITUDE: Appreciation] gives an alternative prove for this two result, for the school choice set-up **due to Schlegel [17]** [ENGAGEMENT: Expand] is **straightforward [**ATTITUDE: Appreciation], and for the marriage marker **due to Roth et al. [15]** [ENGAGEMENT: Expand]**,** its **necessary** [ATTITUDE: Appreciation] only to set all quotas of all firms equal to one.

This pattern, which can be observed in both corpora, to a greater or lesser extent, is a fine strategy to present results, as it guarantees the introduction of a positively evaluated outcome (ATTITUDE: Appreciation) in a way that it is neither too humble, nor to bold: authors make exhaustive statements when introducing a result, therefore closing exchange and negotiation options. However, statements are "softened" by the use of the attribution resources "due to ...", placing all responsibility for the statement on the authors whose contribution is acknowledged as fundamental.

The ATTITUDE>ENGAGEMET pattern is also observed at the JUSTIFICATION. Nonetheless, the communicative function of such a combination may be different in this case, as depicted in the following example:

[33] Unraveling—contracting long before the job begins and before all the relevant information is available—has been one of the **most prevalent phenomena** [ATT: App] in entry-level labor markets. **Most existing studies on unraveling** [ENG: Exp] offer a number of driving factors behind it on multiperiod models with uncertainty as to relevant information. In such models**, it has been shown** [ENG: Cont] that unraveling is undesirable, as it causes Pareto-inefficient outcomes to arise.

In contrast to [EXP_ Cont] the above-cited works, **Sönmez (1999) approaches** [ENG: Exp] the unraveling phenomenon from the manipulation point of view in centralized matching markets without uncertainty. **Namely**, [ENG: Cont] he addresses the question of whether a two-agent coalition can circumvent stable mechanisms via pre-arranging, whereby both of

them are at least weakly better off while at least one of them is strictly better off. Then, **unfortunately**, [ATT: Aff] **he shows** [ENG:Cont] that no stable mechanism is immune to pre-arrangements.

This fragment, which belongs to the JUSTIFICATION stage in an MC text is a sample of how the evaluative pattern is developed across Introductions. With very subtle differences, the same has been observed in CC texts: first, authors introduce a problem (hypothetical or real) as something known and troublesome which has not had a solution, through a monogloss/contract-expand interplay. Then, the OS or an EC is introduced through monoglossic resources, often flagged with attitudinal options depicting their centrality in the discussion of the topic under discussion. Previous studies are cited to show the importance of the problem. This is done through the interactive action of expansive and contractive options. In this way, authors subscribe to the seriousness or importance of finding a solution for the problem under study (ATTITUDE: Appreciation + ENGAGEMENT: Expand), but close the space for discussion by distancing themselves from the assertion of the existence of a gap. Rather than questioning the gap or need for research by resorting to contractive options in discourse, authors "protect" themselves from potential challenges or deter colleagues from questioning their arguments and motives. This marked opposition between the unsolved problem and the proposed solution contributes to building a positive authorial image of the researcher.

5.5. Extrinsic and Intrinsic Evaluation

As we have already mentioned in the previous chapter, GT SRAs tell stories about real-life situations that, in very simple terms, refer to problems of allocation or distribution of resources. Basically, GT SRAs are concerned with rational choice, in which the outcome of the choice depends on the choices of other rational agents. The problems, situations and solutions presented in GT SRAs are represented by a range of terms which are not necessarily (or absolutely) loaded with evaluative meanings, or have a connection with the authors' subjectivities. This can be illustrated in the following example:

[24] [MC] A matching rule is individually **rational** if an agent is never assigned to a partner to whom the agent **prefers** being unmatched. Individual rationality is **necessary** for agents to participate voluntarily in matchings.

Words like "rational", "necessary" and "prefer", which would be tagged as evaluative in other contexts, operate like disciplinary terms. They are part of the definition of a key concept in matching theory[4]. Furthermore, they have been individualized as ATTITUDE: Appreciation (*rational/necessary*) and

[4] In Economics, matching theory is a mathematical framework that allows analysing the formation of mutually beneficial relationships over time (Han, Gu & Saad, 2017)

ATTITUDE: Affect (*prefer*), but in cases like these ones, we have decided to divide resources into *extrinsic* and *intrinsic* evaluation. This differentiation between types of evaluation attempts at covering objective 2 in this study.

We have termed *extrinsic* those items referring to the evaluation or positioning that is external to the explanations or definitions of disciplinary concepts, as illustrated in Example 10. In other words, extrinsic are those resources by which the researchers' position towards a specific entity within the research is evident and determine how this entity is presented to the reader. *Intrinsic* evaluation refers to those items that are evaluative in essence but allude to elements that are part of a definition, or an illustration of a situation or model under study. Example 25 shows this (my underlining):

[25] In many matching markets, using monetary transfers to allocate partners **would** [ENG: Exp] appear **uncouth** [ATT: Judg], if not **morally repugnant**. [ATT: Judg]

Notice that this example exhibits an example of ATTITUDE (Judgement), which has not been reported as salient or significant in the software analysis. However, we considered it interesting to show because it is an instance of both intrinsic and extrinsic evaluation: the situation that the authors describe is apparently "normal", but it leaves the possibility of being considered as potentially inconvenient open to discussion. The authors do not position themselves as in line with this perception but acknowledge that it is feasible.

Authorship in these cases is not constructed at this level of identification of evaluative resources, but rather at a more comprehensive level, like the introduction of a clause through a monoglossic or heteroglossic statement, thus acknowledging or disregarding alternative positions and contributions, aligning or distancing themselves from other authors.

5.6. Other Linguistic and Discourse Items Considered as Appraisal Resources

This section retrieves the notion of *special expressions* already dealt with by Halliday and Martin (1993). Following their work, we have identified items that can be compared to the so-called *technical grammar*, which they contend to be more common in Mathematics than in other sciences. This feature of mathematical discourse, they state, is not particularly problematic once it has been explained. Analysing how this grammar is constructed and used is important for this study, as (we believe) it is coherent with the orientation of this master programme. We hope to understand how GT discourse develops as a means to help researchers write successful papers; we aim at analysing and deconstructing GT discourse so as to gain knowledge on the ways it is organised and used, and eventually apply this information to the design of didactic material or tutoring sessions. The following is a list of the technical grammar features that we have observed to contribute to the construction of authorial voice:

a. Conditional clauses

Conditional clauses are one of the expected features of mathematical discourse. They are obligatory constructions in thought processes leading to the understanding of a particular context or problem, and eventually, to the acceptance of a proposed solution. However, the exploration of the corpora has led us to conclude that conditional clauses are also used to introduce definitions, as in the following case:

> [37] In other words, a matching mechanism is ex ante stable if agents get nonnegative utilities for all outcomes and there exists no firm-worker pair who could match with each other at a particular wage and both get higher expected utilities at the ex ante stage.

We believe that conditional constructions are not intrinsically evaluative. However, at a more general level, in discursive terms, conditional clauses contribute to the building of mathematical thinking processes, which involve the four central processes described by Burton (1984): specializing, conjecturing, generalizing and convincing. Conditional clauses could be considered part of the first three ones. Thus, they make a key contribution to reaching the final stage: convincing.

This use of conditional clauses in definitions was only observed in the CC, which may indicate that researchers may be, in fact, flouting the traditional function of conditionals of considering imagined or uncertain situations and the possible results of these situations. Higham calls this "*false If*" (1998, p. 47), and suggests avoiding it. This would probably be one of the points to keep in mind when instructing or guiding researchers into writing SRAs.

b. Definitions

Definitions are part of a group of typical rhetorical functions used in academic writing (Leech, 2013). Definitions are a key resource in GT SRAs. They have been found to be part of both JUSTIFICATION and OFFER stages, as indicated above. They are important devices in Mathematics-related disciplines intended to create a sense of community membership and a sense of authority, as including definitions not only helps readers understand the proposed topic, but also reveal the authors' knowledge of the object of study or the supporting ideas. In all the cases, definitions have been tagged as monoglossic statements, since no place for dialogic debate is possible, and sometimes no recognizable authors can be attributed to them, as they are part of what we have termed ECs. Observation of the corpora suggests that a definition can be:

- A statement constructed with (a) the verb *to be* in the present; (b) a statement constructed as a conditional clause:

> [38] [CC] Two-sided matching models are used to study assignment problems in which agents can be divided into two disjoint subsets from the very beginning;

[39] [MC] (…) a matching mechanism is ex ante stable if agents get nonnegative utilities for all outcomes and there exists no firm-worker pair who could match with each other at a particular wage and both get higher expected utilities at the ex ante stage.

- An expansion of the elements that form the definition, which can include a sequence of steps indicating a thought/logical process:

[40] [MC] In a school choice problem, students have preferences for schools and in turn, schools ranklist students by their priorities. An allocation mechanism matches students with seats at schools. The Gale and Shapley (10) student-proposing deferred acceptance algorithm (henceforth, DA) always selects the optimal stable matching for students;

[41] [MC] In this model the Blocking Lemma says the following. Fix a responsive preference profile. Suppose that the set of workers that strictly prefer an individually rational matching to the workers-optimal stable matching is nonempty. Then, we can always find a firm and a worker with the following properties: (a) the firm and the worker block the individually rational matching, (b) the firm was hiring another worker who strictly prefers the individually rational matching to the workers-optimal stable matching, and (c) the worker (member of the blocking pair) considers the workers-optimal stable matching to be at least as good as the individually rational matching

- A restatement of the main definition headed by a conjunction indicating (a) consequence; (b) agreement; (c) addition:

[42] [CC] For a Marriage Market, Roth et al. [15] define a strongly stable fractional matching as a stable fractional matching that fulfill a non-linear equalities that represent this non-blocking condition mentioned above. **In other words** [addition], a stable fractional matching that fulfill the non-linear equalities from Roth et al. (15), is a strongly stable fractional matching.

As evidenced by the previous examples, definitions are also an important part of the argumentative side of discourse, as their function is to persuade the reader of the validity of what is introduced.

d) Giving Examples

Another function associated to academic writing is giving examples (Leech, 2013). References to examples are also powerful devices to mark the authors' presence, especially to clarify or show veracity in a point that they are trying to make, or to support a statement with reference to a previous work. We illustrate this feature with the following examples:

[43] Although desirable, ex post stability may not be necessary for the success of some matching markets, especially when the outcome of the mechanism can be enforced. **For example,**

in the National Resident Matching Program, which assigns medical school graduates to residency programs, the matching outcome is binding.

[44] Second, as pointed out by (citation), even if in a labor market most workers are employed by one firm, the presence of a few workers with multiple employers can make a crucial difference. Precisely, Example 2.2 showed that the presence of only one worker with two part-time jobs can already change the stable outcome for all other agents.

Examples have been tagged as ENGAGEMENT>Expand instances. No difference between MC and CC texts have been found regarding this feature. Examples are included in both corpora approximately with the same frequency (MC 1.4%; CC 1.08%).

Examples can also be introduced through parenthetical clauses/phrases, although this resource is mainly used to add a clarification or explanation. In either case, authors may express their involvement with a previously introduced idea (ENGAGEMENT>Contract). The main proposition would not be the same without the clarification of the parenthetical clause or the exemplification. The presence of the author is unquestionably found in these instances:

[45] The first result says that if in centralized markets (like entry-level professional labor markets or the admission of students to colleges) a mechanism selects for each preference profile its corresponding workers-optimal stable matching then, no group of workers can never benefit by reporting untruthfully their preference relations.

Notice how the parenthetical clause (which the author could have omitted) models the readers' attention to specify the type of market in which the described situation operates, and thus closes the space for dialogic exchange (ENGAGEMENT> Contract).

In other cases, the use of parenthetical clauses to introduce examples may open the space for debate, and thus be considered an ENGAGEMENT resource (Expand> Entertain). Some of these examples, in turn, can be intravocalized (without citations) or extravocalized (with citations). Depending on how citations are introduced, the latter case could also be considered an instance of ENGAGEMENT> Expand> Attribute> Acknowledge, through which authors distance themselves from the idea; or it can also be considered an instance of ENGAGEMENT>Contract> Concur, through which authors openly show agreement with the idea previously presented, as may be observed in the following segment (my underlining):

[46] The interest of the Blocking Lemma lies in the fact (1) that it is an instrumental result to prove key results on matching. For instance, (2), the fact that in the college admissions problem the workers-optimal stable mechanism is group strategy-proof for the workers (Dubins and Freedman, 1981) (3).

In this example, authors close and open the space for discussion through the text, first by presenting the interest in the Blocking Lemma as a (1) fact (ATTITUDE>Appreciation>Valuation) and then by leaving

that consideration to the evaluation of readers by (2) introducing an example which is further supported by a reference to a previous study (3) (ENGAGEMENT> Expand>Entertain + ENGAGEMENT>Expand>Attribute>Acknowledge). We can also say that the explanation of the Blocking Lemma notion is introduced as an unquestionable truth (ENGAGEMENT; Monogloss), therefore closing the dialogic space. The example that follows opens the discussion by showing that the idea has been applied before in other contexts, by other researchers, thus reinforcing the current authors' intention to present the above mentioned model as a reference. In playing the game of closing and opening the dialogic space, researchers guarantee an equilibrium between what they present and believe as a fact and what supports their idea.

e) Imperatives

Although it is a characteristic feature of mathematical discourse (Halmos, 1970), imperatives are not always used to introduce a thought process, as in:

> [47] Suppose that the set of workers that strictly prefer an individually rational matching to the workers-optimal stable matching is nonempty. Then, we can always find a firm and a worker with the following properties (...)

However, it can also be used to introduce the authors' voice as engaging the reader in a conversation in which the proposition headed by the imperative form has the objective of persuading the reader of a certain idea, as in:

> [48] Observe that our former result (Martínez et al., 2004) showing that the workers-optimal stable mechanism is group strategy-proof for the workers on the set of substitutable and quota q-separable preference profiles was proved assuming that the Blocking Lemma was true on the set of all these profiles.

We identified this element as an ENGAGEMENT resource (Expand>Entertain), as the dialogic space is opened through an expression of deontic modality. The statement is reinforced by the citation (ENGAGEMENT>Attribute>Acknowledgement).

5.7. The Role of Footnotes

Footnotes were found in 14 out of 14 SRA Introduction sections (100 %) in the MC, whereas 5 out of 10 SRAs (50 %) included footnotes in the CC. We consider that in the discourse of GT SRA the mere act of writing footnotes is evaluative. As an "expansion" of clauses which they enhance, complement or clarify, they provide an unambiguous sign of authority and community membership, two of the values identified as fundamental in the construction of authorial voice.

We analysed footnotes both as part of the clauses they "expanded", and as independent propositions, to conclude that they could be classified into two categories (as summarized in Table 5.12):

Table 5.12
Types of footnotes found in the corpora, functions associated to them and frequency of occurrence

Footnote type	Functions	MC (per 1000 tokens)	CC (per 1000 tokens)
Bibliographical footnotes	• Supporting the idea included in the main body of the Introduction • Tracking a concept back to previous research • Persuading the reader of the validity of the statement in the main body through reference to previous research	2.59	1.08
Non-bibliographical footnotes	• Statements produced by the authors of the SRAs to clarify an aspect introduced in the main body which do not refer to others' works	0.69	0.60

a) Bibliographical footnotes: those including reference to previous research, which may be interpreted as serving the following purposes: supporting the idea included in the main body of the Introduction; showing that the idea introduced can be traced back to previous research; and /or persuading the reader of the validity of the statement in the main body through reference to previous research. These types of footnotes were considered eminently ENGAGEMENT resources. Researchers show alignment or distance from the propositions introduced in the main body through reference to other pieces of research, which reinforces the idea of community membership. Consider the following example:

[49] Therefore, it **may no longer be beneficial** for them to block the mechanism.[2]

(Footnote)2. Similar difficulties arise in auctions with collusion. (See, for example, Laffont and Martimort (1997, 2000) and Pavlov (2008).

In the clause of the main text [49], researchers acknowledge a situation that may constitute an obstacle ("***may no longer be beneficial***") and reinforce it with the comment in the footnote. The use of a footnote in this case is especially telling, as the idea of encountering difficulties is informed by the footnote to have been also experienced by other authors. Thus, by referring to others' work, researchers validate their own statements.

b) Non-bibliographical footnotes: statements produced by the authors of the SRAs to clarify an aspect introduced in the text body which do not refer to other researchers' studies. Consider the following example:

[50] Ex post budget feasibility requires that the mechanism does not need an outside subsidy to run.[1]

1One <u>needs to consider</u> [ENGAGEMENT: Expand: Entertain] a budget feasibility constraint when agents can make endogenous transfers. Without such a requirement, ex post

stability and incentive compatibility may [ENGAGEMENT: Expand: Entertain] be consolidated by subsidizing the system. Of course, [ENGAGEMENT: Contract: Proclaim: Concur: Affirm] no [ENGAGEMENT: Contract: Disclaim: Deny] such condition is needed [ENGAGEMENT: Monogloss] for markets without transfers).

Again, ENGAGEMENT resources are evident in this footnote. Nonetheless, whereas in the bibliographical footnotes evaluative instances tend to be associated with reference to other authors, the presence of the researchers in non-bibliographic footnotes is clear, and their position is openly stated. Notice, however, that no first-person pronouns have been used in this case. Instead, the pronoun "one" has been chosen to signal a certain distance from the propositions presented. This, together with the use of expansive items (*needs to consider/may be*) reinforces the debatable nature of this fragment, in the sense that, even when a position is stated, researchers choose to keep it open to discussion or consideration by readers.

The analysis of the corpora revealed several matches with the perceptions of interviewed researchers as to the elements expected for the structure of Introduction sections. We believe that this awareness of what an Introduction should include is a particularly good sign, as it increases researchers' possibilities of writing successfully. Knowledge of the forms of language that are valued within mathematical discourses and awareness of the effect that such forms may achieve are fundamental for science writers.

Chapter 6. Discussion of Results

This chapter discusses the similarities and differences found in the analysis of the 3 dimensions explored in the corpora: structure, entities and evaluation. This chapter is also expected to provide answers to the main objective of this study, namely analyse, interpret and compare how authorial voice is constructed in published and draft versions of Game Theory SRAs.

6.1. General Characteristics of the Structure of GT Introductions

The analyses in both the MC and the CC exhibited a high degree of agreement with Halmos' idea of mathematical writing (1970). We observed that researchers follow the principle that a text should assert the truth of a mathematical statement using carefully constructed logical deductions. Our analysis of the texts in the corpora has indeed revealed this, and that even when the intended readers are expected to know the object of study and language of the discipline, texts are constructed as a convincing argument that a statement is true. The strength of a work is not assumed, and SRAs in Mathematics are constructed as mathematical proofs. Researchers write to convince their readers that their work is well-founded.

Therefore, just as a mathematical proof starts with what is assumed to be true, and it ends with what is trying to be proved (Sollow, 2014), GT Introduction sections follow the same logic. Like proofs, Introductions are *good stories* (Cheng, 2004) made up of a beginning (what is assumed to be true); a middle (statements, each following logically from the statement before it), and an end (what is trying to be proved). GT SRA Introduction sections are similar to mathematical proofs in that sense. These observations coincide with the idea that, as the opening stage of SRAs, Introductions are expected to achieve a specific social purpose (Swales, 1990; Hood, 2011), in this case, to convince the reader that the presented study offers a solution for a problem.

The way in which GT SRA Introductions are organized, thus, follow the natural dynamics of Mathematics, where a cycle of stages involving "JUSTIFICATIONS" [J] and "OFFERS" [O] is repeated along the articles (at least one phase of each stage), resulting in the J^O^J^O structure. This has been observed in both corpora and coincides with Burton's notion of the construction of convincing arguments through thinking processes involving a loop of four central processes, namely (a) specializing, (b) conjecturing, (c) generalizing, and (d) convincing, in which specializing, conjecturing and generalizing could be grouped into the [J] stage and convincing could be embraced by the [O] stage.

We observe a high degree of coincidence in the results of the analyses and IMASL researchers' awareness of the writing process. This is a very positive sign in terms of possibilities of success in the production of final versions that can finally be published. However, we also believe that the differences revealed in relation with the stages where MC and CC authors put more emphasis in their writing can be a

valuable hint to guide pedagogical interventions, or a deeper awareness of how information is organized for writing tutors, reviewers and translators.

6.2. Appraising Entities across the Introduction

Values of ATTITUDE in the corpora are predominantly expressed in the context of monoglossic propositions, or statements with a polyphonic effect, either with or without attribution, but we could contend that what researchers present in SRAs is always impacted by a considerable degree of ENGAGEMENT. Researchers are involved in and take responsibility for their statements, even when introducing monoglossic clauses (*bare assertions*), which has been interpreted in this study as a mechanism to show a sense of authority, as in the following example (my underlining):

> [34] This paper studies various stability notions in a one-to-one matching problem with transfers. In this problem, the market has two sides, say, firms and workers. An outcome in this setup not only specifies which firm is going to hire which worker, but also the monetary payments. <u>One intuitive solution concept for such a market is ex post stability. Ex post stability requires that each agent end up with nonnegative utility and that no firm worker pair would prefer to match with each other at some wage rather than the current matching [ENGAGEMENT: Monogloss]</u>. Ex post stability was introduced by <u>Gale and Shapley</u> (1962) [ENGAGEMENT: Expand: Attribute: Acknowledge] for a matching market without transfers and was extended to a market with transfers by <u>Shapley and Shubik</u> (1971).” [ENGAGEMENT: Expand: Attribute: Acknowledge]

From the Appraisal framework’s dialogic perspective, *bare assertions* are defined as absolute, declarative statements ignoring the diversity of voices that are naturally put into play in every act of communication (Kaplan, 2004). However, this approach conceives non-dialogic propositions as a way of adopting socio-semiotic positions of great rhetorical and interpersonal force, rather than neutral and non-modalised statements. We believe that bare assertions contribute to consolidating researchers’ identity within the community since they inform readers about the theoretical or methodological approaches that researchers align or disagree with. In the previous example, the notion of *ex post stability* is authoritatively introduced as the solution concept for a certain market. However, the last clause makes it clear that such an assertion is actually the contribution of other authors, with whom researchers align.

Dialogical space is often partially closed, and cyclically interrupted by the introduction of other voices or expansion resources, which clearly reveals researchers’ willingness to act as expected in the scientific community: by sharing findings with a view to inviting others to respond, either by aligning or disagreeing with them, thus creating a space for dialogue, professional growth and scientific advancement. The following example illustrates these features, which help to create a sense of community membership:

[35] Unlike [ENGAGEMENT: Contract: Disclaim: Counter] the marriage model with strict preferences, where there exists a unique concept of stability by pairs of agents, in the marriage model with indifferences there are several concepts of stability. [ENGAGEMENT: Monogloss].

The convincing process embraced by the JUSTIFICATION stage in Introductions displays evaluative elements which might be conceived as a strategy to capture the attention of the readers, create a sense of authority and identity and to highlight the relevance of the contributions, as evidenced in the following example:

[36] Finally, we **show** [ENGAGEMENT: Heterogloss: Contract] that interim stability **can** [ENGAGEMENT: Heterogloss: Expand] be satisfied together with incentive compatibility, efficiency, and ex ante budget balance when there is only one agent on the short side of the market (Proposition 3). This **assumption** [ENGAGEMENT: Heterogloss: Expand] in **not innocuous**, [ATTITUDE: Appreciation] but it is satisfied by some **important** [ATTITUDE: Appreciation] markets, such as auctions and monopolies.

The discussion of the results presented in this chapter attempts at drawing a map of how the interplay of information, organization and evaluation helps in the construction of an authorial voice. By analysing the different data collected for this study, we conclude that at least in this branch of Mathematics - GT -, Introductions unfold in a similar way that information is presented in mathematical texts. As already mentioned, the cycle of JUSTIFICATIONS and OFFERS coincides with the loop of mathematical thinking processes described by Burton (1984) - a loop involving four central processes, namely (a) specializing, (b) conjecturing, (c) generalizing, and (d) convincing, in which specializing, conjecturing and generalizing could be grouped into the JUSTIFICATION stage and convincing could be embraced by the OFFER stage. Software-assisted analysis helped us identify the entities more frequently addressed at each stage and the evaluation associated to them, and revealed that:

1. JUSTIFICATIONS in both the MC and the CC mainly deal with PR, ECs and OSs (in this order). However, Appraisal values mainly affect PR in the MC, whereas evaluation is mostly focused on OSs in the CC. The implication could be related to the aspect that we have already discussed in relation to CC texts, in which authors' tendency is to write longer "contextualisations" to create a space to introduce a solution to a problem.

2. OFFERS deal mainly with CR and Rr, EC, in line with its function as a stage to present researchers' contribution, evaluate its aspired relevance and locate it in the context of the whole paper and previous research. This is the case in both the MC and the CC. Differences are mostly observed in the use of ENGAGEMENT resources. MC exhibit a greater number of monoglossic and contractive expressions, thus implying a more authoritative voice in communicating results. CC texts, instead, display an almost equal number of ATTITUDE and ENGAGEMENT (Expand) options. This may suggest that evaluation of

the contributions or results is tied to attribution to other sources, probably in the hope of increasing their validity.

6.3. Authorship as the Combined Effect of Introduction Structure, Distribution of Entities and Evaluative Resources

The fact that the same entity categories exist both in the MC and the CC is not very revealing. Instead, the way in which those categories are distributed and evaluated across the Introduction and the similarities and differences found during the analysis may help us define how information is developed and how this helps in the construction of authorship. Thus, it might be interesting to consider that:

1. The fact that reference to the OS in the [J] stage is more frequent in MC than in CC suggests that this feature plays a key role in setting the scene for the investigation that is intended to be presented. This is clearly perceived by local researchers during the interviews, but it is not evident in their productions.
2. Interviewed researches did not mention open, explicit evaluation towards their own results or investigation, and this probably reveals that either they are not aware of the importance of this resource to position themselves as authoritative voices in the community, or they ignore how to do it. It is worth considering that entities CR and EC were found to be extensively used in the MC, which suggests that MC authors do not recoil from exposing themselves when writing about their own work.
3. As a base from which new contributions are developed, reference to ECs in the MC was observed to be more frequent in the MC than in the CC. This may reinforce Burton and Morgan's observation (2000) that an important part of constructing an identity as a SRA writer is generalizing, which can be done through making reference to an EC. The implication of this may not be that authors in CC texts ignore the importance of referring to EC. Instead, it may suggest that they rely more on reference to PR, on justifying their results and investigation through research that has already been conducted and proved, thus showing alignment or distance from the notions that they want to support or be different from, respectively.
4. The frequency of evaluative items within Introductions did not vary much between the CC and the MC, and the differences are so small that we consider this aspect of no relevance for deriving conclusions. Nevertheless, it might be interesting to note that ATTITUDE>Appreciation resources are the most frequent items associated to all entities, including reference to Rr. No significant use of Affect or Judgement instances were observed in either corpus, which coincides with previous studies relating the use of Appreciation to concepts in published writers (Hood, 2004) and to products, results and studies in SRAs (Lucero Arrúa, 2013).

5. We observed that ENGAGEMENT resources, either heteroglossic or monoglossic, are associated to ATTITUDE>Appreciation instances, and thus we could say that it is a recurrent pattern, where the entity CR is the most evaluated one in both corpora. These results may not be a revealing finding. Consequently, we believe that probably emphasis on the actual lexico-grammatical realisations and the repertoire of discourse features (which may not be limited to those identified in sections 5.5, 5.6 and 5.7 of this study) entailing Appraisal may be the key to feed didactic proposals aimed at writing. This can also be valuable raw material for translators, writing tutors and reviewers.

6.4 Triangulation with Observation Sessions and Interviews to IMASL Researchers

We have frequently referred to the fact that many of our findings in this study are consistent with the responses obtained in the interviews to IMASL researchers, or with the observations made during the seminars. However, no specific details have been provided yet. Thus, in this section we show the results of those interviews and observation sessions, the aspects that are mainly addressed in the questions, and the way in which interviewees' responses relate to the data obtained through corpora analysis.

We conducted interviews to 5 IMASL researchers and observed 9 seminar interventions, as a way to find out about how information was perceived to be structured across Introductions and the values that researchers conferred to idea of building authorship, as described in Chapter 1.

6.4.1. Interview Responses in Relation to Introduction Structure and Entities

One of the 5 questions in the interview was specifically oriented to knowing about the idea that they had on information organization, their writing habits and "recipes" ("Do you have a writing routine that can be considered part of a process?"). Two of the seven researchers who answered that question provided especially revealing descriptions, the fragments of which are reproduced below (my translation):

> [Researcher 1] "First, there is something different here, perhaps, with respect to writing, that we do as mathematicians: many times, what we have is just a conjecture (hypothesis) about whether something is true or not. If it's true, well… you have to write the proof. In other words, the whole mathematical part comes first. Once we have all the mathematical skeleton of the results, we begin to write in words. You start writing only after you already have all the mathematical structure and the proofs. So, it may seem that you already have a whole path, but the challenge is to generate the mathematical results. Sometimes, the conjectures that you have are false, so you have to start describing the underlying properties a bit, and go on looking for the reasons why they fail. So, maybe you wanted to introduce a certain theorem, but you see that it is not true. However, having a good counterexample is also something that can be published. Then, what you write will depend

> on the mathematical results obtained ... the writing part comes later on ... but at the beginning, you are not quite sure of what is going to happen, or what you can get, right? What is obtained first is all the mathematical results. Then, based on that, you write by "assembling" the review, the literature, the results that you obtained, the reason why they are interesting. You can even present a property that is new in the literature, and you have to explain why it is good, what it is for, what economic application (in general) it has, since our studies are on economic theory in general, but well, writing it would be more or less would like that."

Among many other aspects which are not the focus of this work, the analysis of this answer revealed a high degree of coincidence with the idea that information that is communicated through a SRA should be organized following the values identified by Burton and Morgan (2000) for mathematical texts. Researchers expect their introductions to show: (a) a sense of identity; (b) a sense of community membership; and (c) a sense of authority. They rightfully conceive scientific communication as a strategic "game", in which the writer's choices determine the reaction of peer readers, thus building an identity in which researchers' professional reputation and their place in the community are at stake.

Writing Mathematics is both similar and different to writing in other sciences ("First, there is something different here, perhaps, with respect to writing, that we do as mathematics*"*- Researcher 1*)*. As Halmos (1970) states, writing Mathematics poses the same problems as *"*writing biology, writing a novel, or writing directions for assembling a harpsichord" (p.2). Interview responses revealed that in Mathematics - as in any other science-, communicating an idea implies knowing the language, the audience, the way in which ideas must be organized, and the technique for writing a specific genre.

IMASL researchers are aware of the fact that writing proper English is a must, and its role in achieving clarity and precision should not be underestimated (Halmos, 1970; Knuth, 2009; Sollow, 2014; Thompson, 2011). Writing proofs, for example, involves producing a convincing argument that a statement is true. Proofs are normally expressed in the language of Mathematics, what O'Halloran (2010) terms *symbolism*. However, an amount of natural language is necessary to understand and explain thought processes or the logic of reasoning to solve a problem ("Once we have all the mathematical skeleton of the results, we begin to write in words*.")*.

The writing of Game Theory SRAs at IMASL is mainly intended for Economics journals, and most of those studies are theoretical in nature ("*...* since our studies are on economic theory in general*..."*- Researcher 1*)*. This means that researchers start by producing theorems, i.e. mathematical models which they assume can be potentially applicable to a specific context. However, to persuade the audience of a theorem's validity or applicability, researchers need to contextualize their work carefully. This is a key stage in the writing of Game Theory SRAs, since the success of such contextualization may be one of the factors determining whether a manuscript will be accepted for publication or not, as illustrated in this answer:

[Researcher 1] "You could not present a work only with a mathematical skeleton. In our case, which write on economic theory, it is necessary to put a good economic motivation for all the theorems that are presented, a context, and say why they are relevant ... because a theorem… one can write a theorem of anything, but whether it is interesting or not has to do with the underlying economic motivation and with its relevance. And that is very difficult to do".

Thus, as the interview with IMASL informants revealed, the steps for producing an SRA embrace two key stages: (1) mathematical production of conjectures (hypotheses), reasoning processes, proofs and results. This includes reading extensively, keeping up with recent studies and trends in research, actively participating in conferences and seminars, and sharing thoughts with colleagues to look for inspiration (i.e. posing a problem and a potential solution for it); and (2) the production of a work (heavily relying on natural language) which can clearly account for the motivations and the rationale behind the writing about a specific topic or problem, and the relevance of the theorem presented (which ideally should match the motivation for writing from the perspective of Economics). These two steps coincide with (and materialise in) the stage identified as [J], in which the challenge is justifying why a topic is interesting, important or relevant.

6.4.2. Interview Responses in Relation to Evaluative Elements

Noting the difference between relevant versus irrelevant topics, or else, persuading the audience about the importance of their work is a defying task. In such a context, as Researcher 1 explained, a potential scene is that conjectures may prove false even when the theorem produced may be interesting, as perceived by colleagues during GT group seminars. Mathematicians, thus, have to analyse the properties which can account for this (finding out and expanding on the source of falsity), or presenting a counterexample to demonstrate that they know the proposed theorem is false, but that the assumptions and hypotheses were necessary to prove it. Counterexamples are also ways of presenting an argument, and in any case, could feed an article for publication. However, again, natural language is a key element; it becomes unavoidable, and having an efficient command of English is essential. All these functions described by researchers could be equated with the phases necessary to complete a stage.

The sole organization of information is evaluative itself and helps in the building of authorship. However, researchers acknowledge the difficulty in achieving this, as positioning their work as a valuable contribution in the community goes beyond the sole act of presenting a novel model or theorem. Constructing an authorial voice is a complex process involving awareness of previous works, knowledge of the discipline or the topic under study, and expressing whether they adhere or not with specific lines of research, for example, or showing alignment or distance from specific results, which are addressed earlier in this chapter.

6.4.3. Report on the Observation Sessions

We observed a total of 9 presentations, whose development or discussion extended for two days each, due to the complexity of the topics or to the improvements that the audience suggested to the presenters. The presentations were delivered in the context of IMASL seminars, which are organized on a weekly basis.

What researchers present at seminars is not necessarily a finished or ongoing work. It may only be an idea which they need to share and discuss to evaluate the potential interest for research. The reason why this space is so important for researchers is that ideas or final works presented undergo several sessions of peer review, which enable researchers to detect weaknesses thanks to the comments and corrections made by their audience. This is later (ideally) captured by their writings. An example concerning the questioning of clarity in the results presented or established concepts which are fundamental for the development of the topic under study appears when the seminar's main speaker is interrupted by members of the audience. The interruptions normally point to contents, or issues related to the explanation of processes o justifications for proposed solutions, but they usually end up in revealing that the source of a misunderstanding might be the way a statement is made. Thus, interruptions and interaction with peers during the seminar have the dual function of lending a critical eye to the information presented and the language used to communicate it.

Besides, even when seminars are mostly delivered in Spanish, the SRAs and the Power Point presentations on which they are based are written in English. The audience is also very attentive to visual aids, and comments about slide's organization and discourse are also addressed.

As we have already mentioned in Chapter 4, the objectives of the seminars are to discuss new ideas for future projects, to share articles, to present potential new ones and to open the discussion among peers with a view to improving written productions, or to rehearse oral presentations for scientific events. These presentations play a key role in modelling not only the content of new productions, but also to help researchers in organizing information, which has been identified as a very important feature of author positioning.

Chapter 7. Conclusions

This chapter summarizes the main findings and highlights the main contributions. In line with the application objective in this study, we also suggest lines of work in relation to the design of didactic material to be used in writing courses and/or revision sessions. Finally, some ideas for further research are also discussed, as we believe that even when our results do shed some light on the workings of GT discourse –understood as a branch of Mathematics-, there is plenty of work to do beyond the context in which this thesis was produced.

7.1. Theoretical Implications

7.1.1. Structure

We believe that the main theoretical contribution of this study is the description of a rhetorical structure suitable for Mathematics-related SRAs that is based on an interpersonal perspective. The J^O^J^O structure is consistent with the communicative purpose of the genre. It is also in line with the nature and characteristics of Mathematics and Mathematics-related Introduction sections in SRAs. Similarly, at the level of structure, we consider that the analysis and classification of footnotes as evaluative devices may be a valuable contribution, as it unequivocally flags the presence of the author by indicating the proximity or distance from a notion already dealt with in the article. No other study, to the best of our knowledge, has considered footnotes as interpersonal elements.

Our work provides awareness and understanding of how information is organized and of how each component of introductions adds to the construction of an authorial voice. This can be a helpful and valuable resource for both researcher-writers and writing tutors. Although the analysis is limited to the context of GT SRAs produced and socialized in a specific group of scientist within the UNSL, we believe that the results derived from this thesis can be profitably shared, applied and transformed to nurture both research and didactic initiatives in similar and Mathematics-related settings.

7.1.2. Entities

This study is also expected to provide a guide in the identification of entities and an understanding of what items and evaluations are at stake. We believe that this is a valuable contribution that may be used to design scientific writing guides and software tools. These are, in fact, two projects expected to be developed in the course of the following years.

Our study is intended to guide researcher-writers or writing tutors on how to produce an introduction, but above all, to describe how pieces of research are actually created in the context of a scarcely studied discipline. Our contribution is valuable in the sense that it helps in finding the tip of the iceberg, in clearing up the veil of mystery assigned to the elements, the topics and the concerns dealt with in a discipline

of study traditionally considered as obscure, cryptic and devoid of subjectivity or concrete applications beyond abstract ideas.

7.1.3. Appraisal Patterns

Our analysis of evaluative language, based on the Appraisal model extensively developed by Martin (2000), Martin and White (2005), Martin and Rose (2007), among others, reinforces the idea of cyclicity in the use of ATTITUDE and ENGAGEMENT resources. This contribution is worth noting, as it reveals how authors construct their voice across the scientific dimension or SRAs, and it represents a valuable picture of how ideas are positioned and negotiated across the Introduction section of GT SRAs.

We have followed the Appraisal logic of how notions, opinions and contributions are introduced and exchanged within a SRA, and most of the obtained results are consistent with the lexico-grammatical items commonly associated to the ATTITUDE and ENGAGEMENT axes. However, in considering other discourse features or larger units of analysis, like whole sections deploying a description of a model, we expect to create an awareness of how authorship can be constructed at a more holistic level.

We further attempt at showing that building an authorial stance is a matter of choice, not only at the level of linguistic options, but also at the level of text structure components awareness and selection. In this vein, and in line with the ideas provided by the discipline under study (GT), we dare to conclude that writing an effective Introduction section is also a game, as the author needs to get the readers involved in order to persuade them of the validity of the study.

Even if the aim of Introductions were only to inform the reader about some fact, model, or result, this aim could only be achieved if the reader arrived at the intended interpretation and believed the writer to be trustworthy, informed and reliable. In short, the "success" of a linguistic act as represented by the Introduction section of SRAs depends on the behaviour of both the producer and the recipient. Both language use and awareness of information organization and genre conventions satisfy the abstract characteristics of a game as conceived by GT. In a very general sense, we can say that two agents (researcher-writer and reader) play a game together whenever they each have to choose between several actions, such that their preferences over their own actions depend on which action the other agent chooses (Benz et al., 2011). This idea is further developed in section 7.2 below.

7.1.4. Methods

In relation with the methods used to survey GT discourse, we believe that our ethnographic approach can also be regarded as having a worthy impact in future studies. The understanding of a specific discourse and the path that is followed to write an article can be highly enriched by direct contact with the writer-researcher.

Through analysis and observation, we learned that GT models produced at IMASL revolve around hypothetical ideas which cannot be empirically proved for three main reasons (as derived from interviews): (a) the Institute lacks the resources to conduct such a task; (b) most researchers at IMASL are mathematicians, not applied economists, and thus they are not trained to conduct empirical work; and (c) proving theorems empirically is not among the aims or the interests of the Institute. The work of scientists who belong to this research centre is mainly to develop new ways of thinking about potential solutions for old or new problems out of old or new theorems and/or models.

The construction of authorship may be an even more complex process in this context. It unfolds as the text develops, and no single strategy or resource can be attributed to such a construction. Interviewing IMASL researchers and observing how they worked helped us to achieve a comprehensive understanding of: (a) how IMASL researchers work in the context of scientific production in the field of GT; (b) their writing habits; (c) the nature of the discipline that they study; (d) their expectations regarding their writing; (e) the values that researchers associated with writing (well). The responses obtained during the interviews provided us with information as to the type of difficulties that researchers face during writing, gave us valuable data to understand the logic of the discipline, and showed us a path to study strategies for positioning and authoring that may go beyond the sole identification of individual lexical items.

The discovery of particular features for constructing an authorial stance can also be greatly enhanced by the possibility of involving in the process of scientific production within a community - both as an observer and as a reader/language reviewer.

7.2. Pedagogical Implications

From the pedagogical point of view, this study hopes to provide writing tutors with tools to understand the genre, and researchers with lexico-grammatical and discourse resources that enable them to be more independent and self-confident writers. We also expect this study to help writers, reviewers and translators to gain some knowledge on the types of entities that should be focused on as the Introduction develops, and to achieve an understanding of the particular discourse features carrying evaluative meaning.

7.2.1. Pedagogical Implications for Writing Mathematics as a GT Researcher at IMASL

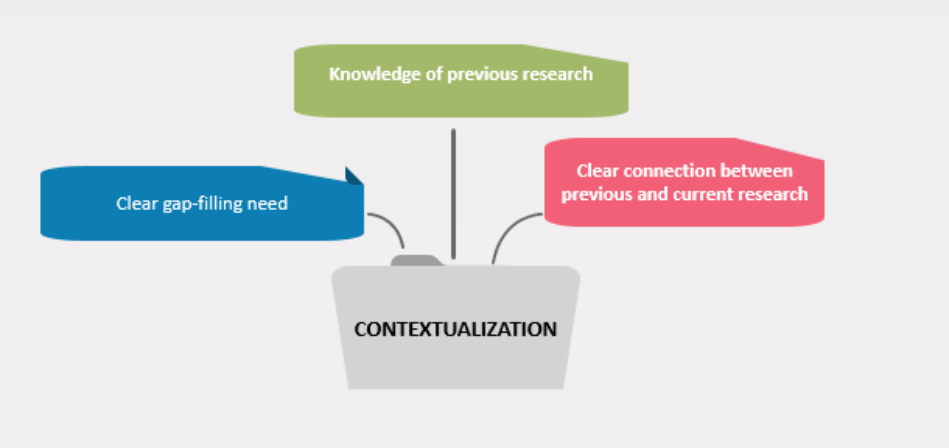
Understanding the special characteristics of how science is pursued at IMASL, and in GT-related contexts contributed to consolidating the already studied idea that the Introduction is one of the most important sections within the SRA. Analysing the writing of published and unpublished GT SRA Introductions helped us to see that writing Mathematics in GT SRAs is like writing any other text in relation to coherence, clarity and precision. But it also raised our awareness of the special features which are mainly

related to the process itself, or how writing is approached, rather than the objectives of a text, which make writing Mathematics in GT SRAs different from other sciences.

One of the main issues that IMASL researchers meet when writing an article is the persuasion, relevance and validity potentials of the arguments that they introduce. IMASL scientists insist on the role of what they call *contextualisation*: the ability to show knowledge of previous related research in depth, to create connections between those previous studies and their contribution, and to make others see that the presented proposal adds to existing theory or fills in gaps left by other authors (Fig. 7.1). Thus, it could be stated that most of the efforts to build authorship in Mathematics, and especially in Game Theory at IMASL, are made when writing Introductions.

Figure 7.1

Contextualising research as seen by IMASL researchers



7.2.3 Pedagogical Abstractions Derived from the Analysis of GT Introduction Sections

IMASL researchers' awareness of what an Introduction should include is a very good sign, as it increases their possibilities of writing successfully. Both knowledge of the forms of language that are highly valued within mathematical discourses and the effect that such forms may achieve are fundamental for science writers. However, knowledge of the principles guiding the organization of information cannot be underestimated.

Though potentially debatable and controversial, the notion of the writing of Introduction sections as a *game* set forward above can be thought of as a corollary of this thesis. It undoubtedly requires much more research and reflection, but a preliminary interpretation is provided below:

1. The construction of meaning -and the building of an authorial voice specifically- is governed by special conventions (related to the features of scientific –mathematical, in particular- discourse).

2. The observation of such conventions can be described as “equilibrium” in a “signalling game”. Signalling games refer to “a class of two-agent games of incomplete information” in which only one of the two agents is informed (Sobel, 2009). In the context of the Introduction section, the informed agent or player would be the researcher-writer, whereas the uninformed one would be the reader. The informed player's strategy set (the tools or resources with which they expect to persuade the reader of the validity and worth of a study) consists of signals contingent on information (knowledge of the genre, knowledge of the discipline and the topic, knowledge of the language), and the uninformed player's strategy set (the tools or resources through which they will be able to interpret the writer’s contribution) consists of actions contingent on signals (the ones used by the writer). Thus, a signalling game includes any strategic setting in which players can use the actions of their opponents to make inferences about hidden information. (Benz et al., 2011). In this context, writing poses a coordination problem, where the writer expects a specific response from the reader. Put it simply:
    a) Writers want to communicate a specific meaning (M).
    b) To this end, they choose particular forms of language and structure.
    c) Readers have to interpret this and choose some interpretation (M’).
    d) Both writer and reader share the common interest to communicate (and interpret) the meaning which the writer has in mind. This coordination problem is solved successfully only if M = M’.

The signalling (“correct” use of language and structure/conventions) and interpretation strategies of writer and reader solve the coordination problem only if writers know and apply the conventions typical of Introduction sections in a particular discipline.

The latter parallelism of a game with writing is merely an illustration to strengthen the importance of knowing what and where to write in Introductions, as this understanding certainly helps researchers conform to conventional expectations. It likewise provides them with empowering tools to make informed linguistic and structural choices. We hope this work makes a contribution in this direction. We have attempted to identify the ways in which the evaluation of a result, or any other entity in an Introduction section is conducted in published works so that similar strategies can be transferred and taught to those aiming to publish. We hope this analysis helps in developing didactic tools aimed at scaffolding writing processes and provide information that can assist reviewers and translators in their work.

7.3. Suggestions for Further Research

The data set in this thesis has been kept small in order to allow for a relatively detailed analysis of structure and meaning-making resources to be analysed mainly qualitatively. This detailed analysis includes meanings within the realm of the subject knowledge, the discourse of the texts and the grammar. Thus, our

study could serve as a point of departure in the analysis of other Mathematics-related scientific texts. Future research could likewise be oriented to corroborate whether our contributions in terms of the identification and categorization of phases within stages with studies can be applied to others the broader spectre of Mathematical science, so that categories can be fully accessible to researchers in the context of reading guides or software tools. Similarly, a finer categorization and systematization of expressions, discourse features and lexico-grammatical items intended for referring to the content of the study presented, addressing the reader and stating a position is necessary to design material that can be openly and easily manageable for researchers writing in English. We also expect this study and others derived or inspired by the present investigation to help researchers, writing tutors and translators in the understanding and use of lexical combinations of scientific discourse in English.

We hope that our study serves the purpose of laying the bases for helping scientists engaged in the communication of research results to face the challenges involved in the preparation of final master's or final degree projects, research articles for publication or doctoral theses. This requires the development of specific skills and competences, and also the accessibility to reliable and practical reference material.

7.4. Ongoing and Future Work Incorporating Contributions Derived from this Study

The preliminary results of this study and the valuable work conducted by Mirallas (2017) have been used to deliver a scientific writing course at the UNSL (Mirallas & Lucero, 2016). Similarly, these contributions are currently being used to produce a practical writing guide, initially intended for UNSL researchers. Following the work of Villayandre Llamazares (2018), we also expect to produce a software-assisted tool derived from this and future investigations that can be used by researchers, writing tutors and translators in the production of academic and scientific articles.

We believe that our study has definitely contributed to materialize the objectives referenced in Chapter 3. Following the line of thought proposed by our study's title, we have managed to analyse and classify the different ways in which authorship is constructed, and provide evidence in relation to the idea that even in Mathematics and Mathematics-related disciplines, like GT, Introduction sections are dialogic constructions. Similarly, we believe that we have successfully managed to show that the construction of an authorial stance, or what we have termed authorship, is the result of the equilibrium created through the interplay among: (a) the type of data provided (entities), (b) organization of data within the section (Stages/Phases), the (c) lexico-grammatical and discursive devices used to write about the scientific contribution (Appraisal resources) and mathematical symbols.

We close this chapter (and this thesis) by insisting on the idea that our contributions could lead to other investigations centred on how research is produced in Mathematics and Mathematics-related fields. We believe that much work remains to be done to find out whether our specific categories of Introduction

structure, evaluated entities and types of evaluation can be generalised to other areas of Mathematics, either applied or pure. We also consider that the observations derived from this thesis in a GT-specific context could likewise be put to test in other study fields where GT notions can be utilised other than Economics. Moreover, the social nature of language, its variation and plethora of linguistic meanings across time, genres and register provides a suitable basis to investigate the aspects explored herein from the same theoretical perspectives, though more in depth. It also offers the possibility of studying Introductions in GT, or any other discipline, from the enlightening angle of knowledge building proposed by Legitimation Code Theory (Maton, 2013). The understanding of how knowledge is built in a discipline makes it easier to grasp, manage, teach and produce the linguistic resources that allow meanings to be made, along with other social semiotic resources including images, symbols and gestures (Hao, 2018). Finally, we consider that it would be equally interesting, though admittedly defying, to analyse the discourse of Introductions (in GT, other Mathematics-related discipline, or any other science) from the looking glass of GT itself, since the perception of language as a game of meaning (Clark, 2012) may contribute valuable findings directed to systematise linguistic resources, without ignoring its social and changing side.

REFERENCES

Anglada, L. & Gaido, A. (2010). Marcas de resguardo de naturaleza epistémica en AI en inglés y español. In Castel, V. & Cubo de Severino, L. (Eds). *La renovación de la palabra en el bicentenario de la Argentina. Los colores de la mirada lingüística* (pp. 125-131). Editorial FFyL, UNCuyo.

Anglada, L.; Ávila, C.; Calvo, A. & Pasquini, M. (2010). Marcas de actitud en AI en las áreas disciplinares de bioquímica y psicología. In Castel, V. & Cubo de Severino, L. (Eds). *La renovación de la palabra en el bicentenario de la Argentina. Los colores de la mirada lingüística* (pp. 133-139). Editorial FFyL, UNCuyo.

Anthony, L. & Bowen, M. (2013). The Language of Mathematics: A Corpus-based Analysis of Research Article Writing in a Neglected Field. *The Asian ESP Journal Special Edition*, 9 (2). http://asian-esp-journal.com/wp-content/uploads/2016/01/AESP-Volume-9-Issue-2-October-2013.pdf

Aumann, R. (1985) *What Is Game Theory Trying to Accomplish?* In: Arrow, K. & Honkapohja, S. (Eds.) *Frontiers of Economics*. (pp. 5-46). Basil Blackwell.

Baker, P. (2006). *Using Corpora in Discourse Analysis*. Continuum.

Baker, P., Costas, G. & KhosraviNik, M. (2008). A useful methodological synergy? Combining critical discourse analysis and corpus linguistics to examine discourses of refugees and asylum seekers in the UK press. *Discourse and Society*, 19 (3). https://doi.org/10.1177/0957926508088962.

Bednarek, M. (2011). Corpus Linguistics and Systemic Functional Linguistics: Interpersonal meaning, identity and bonding in popular culture. In Bednarek, M. & Martin, J. (Eds.) *New Discourse on Language Functional Perspectives on Multimodality, Identity, and Affiliation*. Continuum.

Bednarek, M. (2008, December 5). *Corpora and Discourse: A three-pronged approach to analyzing linguistic data* (Complete paper) 2008 HCSNet Workshop on Designing the Australian National Corpus. New South Wales. http://www.lingref.com/cpp/ausnc/2008/paper2283.pdf

Benz, A.; Ebert, C.; Jäger, G.; van Rooij, R. (Eds.) (2011). *Language, Games, and Evolution Trends in Current Research on Language and Game Theory*. Springer.

Biber, D. & Finnegan, E. (1988). Adverbial Stance Types in English. *Discourse Processes* 11, 1-34.

Biber, D., Johansson, S. Leech, G., Conrad, S. & Finnegan, E. (1999). *Longman Grammar of Spoken and Written English*. Longman.

Bitchenera, J. & Basturkmen, H. (2006). Perceptions of the difficulties of postgraduate L2 thesis students writing the Discussion section. *Journal of English for Academic Purposes*, 5, 4-18.

Boccia, C. (2010). La sección Introduction: una macro-propuesta. In Castel (Ed). *El artículo científico en inglés: aspectos teórico-descriptivos, pedagógicos y computacionales de una modelización*

sistémico-funcional. (pp. 78-127). Editorial de la Facultad de Filosofía y Letras de la Universidad Nacional de Cuyo.

Borlongan, A. (2009). World Englishes and Revision in Academic Writing Instruction. *TESOL Journal*. 6 187-188.

Bühler, K. (1939). *Theory of language: The representational function of language*. (D. Goodwin, Tran.) (1990th ed.). John Benjamins Pub Co.

Burton, L. (1984). Mathematical Thinking: The Struggle for Meaning. *Journal for Research in Mathematics Education*, 15 (1) 35-49. https://doi.org/10.2307/748986

Burton, L. & Morgan, C. (2000). Mathematicians Writing. *Journal for Research in Mathematics Education*, 31 (4), 429-453. https://doi.org/10.2307/749652

Caneti, R. & Alon, R. (2009, November 25). Lecture 6: Basics of Game Theory [PDF file]. http://www.cs.tau.ac.il/~canetti/f09-materials/f09-scribe6.pdf

Cheng, W. (2012). *Exploring Corpus Linguistics. Language in Action*. Routledge.

Clark, R. (2012). *Meaningful Games. Exploring Language with Game Theory*. The MIT Press.

Celce-Murcia, M. (2014). Teaching English in the context of World Englishes. In Celce-Murcia, M., Snow, M. A., & Brinton, D. (Eds.) *Teaching English as a second or foreign language* (4th ed., pp. 63-70). Heinle Cengage.

Cheng, E. (2004, October). How to write proofs: a quick guide [Class handout]. Department of Mathematics, University of Chicago. https://deopurkar.github.io/teaching/algebra1/cheng.pdf.

Doran, Y. J. (2016). The role of mathematics in physics: Building knowledge and describing the empirical world. *ONOMÁZEIN. Número especial LSF y TCL sobre educación y conocimiento*. 209-226 https:// 10.7764/onomazein.sfl.08

Duff, P. (2010). Language socialization into academic discourse communities. *Annual Review hallidayof Applied Linguistics,* 31, 169- 192. https://10.1017/S0267190510000048.

Eggins, S. (2004). *An Introduction to Systemic Functional Linguistics*. Second Edition. Continuum.

Ernest, P. (1998). *Social Constructivism as a Philosophy of Mathematics*. State University of New York Press.

Folkeryd, J. W. (2006). *Writing with an Attitude. Appraisal and student texts in the school subject of Swedish*. [Doctoral Dissertation. Acta Universitatis Upsaliensis. Studia Linguistica Upsaliensia]. Uppsala.

Giannoni, D. (2005). Negative Evaluation in Academic Discourse. A Comparison of English and Italian Research Articles. *Linguistica e Filologia*, 20, 71-99.

Gutierrez de Piñeres Reyes, R. & J. Díaz Frías (2011). Discurso informal matemático: enfoques, métodos, técnicas y tendencias. *Avances en Sistemas e Informática*, 8 (2), 155-164. https://155-164 1909-0056 1657-7663.

Graves, H., Moghadassi S. & Ashim A. (2013). Mathematics is the method: Exploring the macro-organizational structure of research articles in mathematics research articles in mathematics. *Discourse Studies*. 15. 421. https:// 10.1177/1461445613482430 2013

Graves, H.& S. Moghaddasi; A. Hashim (2014). "Let G = (V,E) be a graph": Turning the abstract into the tangible in introductions in mathematics research articles. *English for Specific Purposes*. 36(1):1–11. https:// 10.1016/j.esp.2014.03.004

Han Z., Gu Y., Saad W. (2017) Fundamentals of Matching Theory. In: *Matching Theory for Wireless Networks*. Springer, Cham. https://doi.org/10.1007/978-3-319-56252-0_2

Halmos, P. (1970). How to Write Mathematics. *L'Enseignement Mathématique*, 16(1), 123-152. http://retro.seals.ch/cntmng?type=pdf&rid=ensmat-001:1970:16::59&subp=hires

Hao, Jing. (2018). Reconsidering 'cause inside the clause' in scientific discourse – from a discourse semantic perspective in systemic functional linguistics. *Text & Talk - An Interdisciplinary Journal of Language Discourse Communication Studies*. 38. 10.1515/text-2018-0013.

Hersh, R. (1997). *What is Mathematics, Really?* Oxford University Press.

Hersh, R.& K. Umland (2006). Mathematical discourse: the link from pre-mathematical to fully mathematical thinking. *Philosophy of Mathematics Education Journal*, 19. https://philpapers.org/rec/HERMDT.

Halliday, M. A.K. & C. Matthiessen (2014). *An introduction to functional grammar*. Fourth edition. Routledge.

Halliday, M.A.K. & Martin, J.R. (1993). *Writing science: literacy and discursive power*. University of Pittsburgh Press.

Halliday, M. A.K. & Matthiessen, C. (1999). *Construing experience through meaning: a language-based approach to cognition*. Cassell.

Hasan, R. (2009). The place of context in a Systemic Functional model. In Halliday, M. A.K. & Webster, J. (Eds.) *Continuum Companion to Systemic Functional Linguistics*. Continuum International Publishing Group.

Herbel-Eisenmann, B. & D. Wagner (2005, July 10-15). *In the Middle of Nowhere: How a Textbook Can Position the Mathematics Learner.* Conference of the International Group for the Psychology of Mathematics Education. Melbourne. http://www.emis.de/proceedings/PME29/PME29RRPapers/PME29Vol3HerbelEisenmannEtAl.pdf

Herbel-Eisenmann, B. & D. Wagner (2010). Appraising lexical bundles in mathematics classroom discourse: obligation and choice. *Educational Studies in Mathematics*, 75. 43-63. https:// 10.1007/s10649-010-9240-y

Higham, N. (1998). *Handbook of Writing for the Mathematical Sciences*. Society for Industrial and Applied Mathematics.

Hinkel, E. (2002). *Second Language Writers Text*. Lawrence Erlbaum Associates, Inc. Publishers.

Hood, S. (2004). Managing attitude in undergraduate academic writing: a focus on the introductions to research reports. In L. J. Ravelli, & Ellis, R. A. (Eds.). *Analysing academic writing: Contextualized frameworks* (pp. 24–44). Continuum.

Hood, S. (2010). *Appraising Research: Evaluation in Academic Writing*. Palgrave.

Hood, S. (2019). Appraisal. In Thompson, G., Bowcher, W., Fontaine, L., & Schönthal, D. (Eds.) *The Cambridge Handbook of Systemic Functional*. (pp. 382-409). Cambridge University Press. https://10.1017/9781316337936.017

Hunston, S. (1989). *Evaluation in Experimental Research Articles*. [Unpublished doctoral dissertation]. University of Birmingham.

Hunston, S. & J. Sinclair (2000). A local grammar of evaluation. In Hunston, S. & Thompson, G. (Eds) *Evaluation in Text: Authorial Stance and the Construction of Discourse*. OUP.

Hunston, S. & Thompson, G. (2000). *Evaluation in Text: Authorial Stance and the Construction of Discourse*. OUP

Hunston, S. (2011). *Corpus Approaches to Evaluation: Phraseology and Evaluative Language*. Routledge.

Hyland, K. (2002). Genre: Language, context and literacy. *Annual Review of Applied Linguistics*, 22. 113 - 135. 10.1017/S0267190502000065.

Hyland, K. (2004). *Disciplinary discourses. Social interactions in academic writing*. The University of Michigan Press.

Jakobson, R. (1960). Linguistics and Poetics. In T. Sebeok (Ed.). *Style in Language.*, pp. 350-377, Massachusetts Institute of Technology Press.

Kaplan, N. (2004). Nuevos desarrollos en el estudio de la evaluación en el lenguaje: La Teoría de la Valoración. *Boletín de Lingüística*. https://www.researchgate.net/publication/26460353_Nuevos_desarrollos_en_el_estudio_de_la_evaluacion_en_el_lenguaje_La_Teoria_de_la_Valoracion

Kachru, B. (1990). World Englishes and applied linguistics. *World Englishes*, 9 (I) 3-20. 10.1111/j.1467-971X.1990.tb00683.x

Knuth, D., Larrabee T., & Roberts, P. (1988). *Mathematical Writing*. The Mathematical Association of America.

Kress, G., & T. van Leeuwen (1996). *Reading images – The Grammar of visual design*. Routledge.

Kress, G. (1996). Representational resources and the production of subjectivity: Questions for the theoretical development of critical discourse analysis in a multicultural society. In Caldas-Coulthard, C. R. & Coulthard, M. (Eds.) *Texts and practices* (pp. 15-31). Routledge.

Lakic, I. (1997). Genre analysis of article introductions in economics. *Asp,* 15-18. https://10.4000/asp.3239

Laviosa, S. (2002). *Corpus-based Translation Studies: Theory, Findings, Applications*. Rodopi.

Leech, G. (2013) *A Communicative Grammar of English*. Taylor and Francis.

Lemke, J. (2002). Mathematics in the Middle: Measure, Picture, Gesture, Sign, and Word. In Anderson, M., Saenz-Ludlow, A., Zellweger, S. & Cifarelli, V., (Eds.). *Educational Perspectives on Mathematics as Semiosis: From Thinking to Interpreting to Knowing.* (pp. 215-234). Legas Publishing.

Lövstedt, A-C & Rose, D. (2015). *Reading to Learn Maths: A teacher professional development project in Stockholm*. https://www.readingtolearn.com.au/wp-content/uploads/2016/01/Reading-to-Learn-Maths-A-teacher-professional-development-project-in-Stockholm.pdf.

Lucero Arrúa, G. (2013, June 12-14). *El discurso de los artículos de investigación en matemática: análisis léxico-gramatical de elementos valorativos* [Complete article]. VI Coloquio de Investigadores en Estudios del Discurso y III Jornadas Internacionales sobre Discurso e Interdisciplina. Quilmes, Argentina. http://www.comunidadaledar.org/wp-content/uploads/2016/05/Ponencias-ALED-Univ-Quilmes.pdf

Lucero Arrúa, G. (2013, October 8-11). *La estructura genérica y las marcas de evaluación en el discurso de la matemática* [PPT Presentation]. IX Congreso Latinoamericano de Lingüística Sistémico Funcional (ALSFAL). Gramática, Discurso y Alfabetización: Desafíos para la educación en Latinoamérica. Santiago de Chile, Chile. https://alsfal2013.weebly.com/

Lucero Arrúa, G. & L. Laurenti (2015, November 3-6). *Espejismo o realidad: la construcción de la prosodia evaluativa en la sección discusión de artículos de investigación en matemática aplicada* [PPT Presentation]. XI Congreso Internacional de la Asociación Latinoamericana de Estudios del Discurso. Buenos Aires, Argentina. http://www.buenosairesaled.filo.uba.ar/

Lucero Arrúa, G. (2018, October 4-5) *Estrategias discursivas de posicionamiento y discusión en la Matemática: análisis comparado de discursos orales y escritos* [PPT Presentation]. IX Coloquio de ALEDAr (Asociación Latinoamericana de Estudios del Discurso - Capítulo Argentina) Estudios del discurso: Desafíos multidisciplinares y multimodales. La Pampa, Argentina. http://www.humanas.unlpam.edu.ar/wordpress/aled/wp-content/uploads/sites/16/2017/12/LIBRO-DE-RES%C3%9AMENES.pdf

Martin, J. R. (1992). *English Text*. System and Structure. John Benjamins Publishing Company.

Martin, J. R. (2000). Beyond exchange: appraisal systems in English. In Hunston, S. & Thompson, G. (Eds.) *Evaluation in Text: Authorial Stance and the Construction of Discourse* (pp. 142-175). OUP.

Martin, J.R. (2004). Multimodal Semiotics: Theoretical Challenges. In Dreyfus, S., Hood, S. & Stenglin, M. (Eds.). *Semiotic Margins: Meaning in Multimodalities*. Continuum.

Martin, J. R. (2019). Discourse Semantics. In Thompson, G., Bowcher, W., Fontaine, L., & Schönthal, D. (Eds.) *The Cambridge Handbook of Systemic Functional Linguistics* (Cambridge Handbooks in Language and Linguistics, pp. 358-381). Cambridge University Press. 10.1017/9781316337936.016

Martin, J.R. & White, P.R.R. (2005). *The language of evaluation: Appraisal in English*. Pallgrave Macmillan.

Martin, J.R. & Rose, D. (2007). *Working with discourse. Meaning beyond the clause*. Continuum.

Martínez, I. (2011). Capitalizing on the advantages of the Latin American EAP situation: Using authentic and specific materials in EAP writing instruction. *Ibérica, 21,* 31-48.

Maton, K. (2013). Making semantic waves: A key to cumulative knowledge-building. *Linguistics and Education*, 24, 8–22.

Matsuda, P., & De Pew, K. (2002). Early second language writing: An introduction. *Journal of Second Language Writing*, 11, 261–268. 10.1016/S1060-3743(02)00087-5

Matsuda, A., & Matsuda, P. (2010). World Englishes and the Teaching of Writing. *TESOL Quarterly*, 44, 369-374. 10.2307/27896730.

McGrath, L. & Kuteeva, M. (2011). Stance and engagement in pure mathematics research articles: Linking discourse features to disciplinary practices. *English for Specific Purposes*, 31, 161-173. http://dx.doi.org/10.1016/j.esp.2011.11.002.

Mercado, F. (2010). The Genre of Research Article: A Comparative Analysis of Language Use in Writing Research in Linguistics, Mathematics and Science. *Official Research Journal of St. Michael's College*. http://research.smciligan.edu.ph/?p=35.

Meyer, C.F. (2002). *English Corpus Linguistics: An Introduction*. CUP.

Mirallas, C. (2017). *La escritura de abstracts y AIC de alumnos de ILE: análisis de escritos y evaluación de la efectividad de una pedagogía de género* [Master's Dissertation]. Universidad Nacional de Córdoba, Córdoba, Argentina.

Mirallas, C. & G. Lucero Arrúa (2016). *Gabinete de Asesoramiento en Escritura Científica en Inglés (GAECI): una muestra de trabajo colaborativo entre el inglés como lengua extranjera y las disciplinas en el ámbito de la Universidad Nacional de San Luis*. [Complete Paper]. Encuentro regional de investigación. Secretaría de Ciencia y Técnica. Facultad de Ciencias Humanas. Universidad Nacional de San Luis. http://humanas.unsl.edu.ar/archivospdf/i_encuentro_cyt.pdf

Morgan, C. (1998). *Writing Mathematically: the Discourse of Investigation*. Taylor & Francis.

Morgan, C. (2005). Word, Definitions and Concepts in Discourses of Mathematics, Teaching and Learning. *Language and Education*, 19:2, 102-116, 10.1080/09500780508668666.

Nwogu, K.N. (1997). The medical research paper: structures and functions. *English for Specific Purposes*, 16(2), 119-137.

O'Donnell, M. (2013). Exploring Identity through Appraisal Theory: a Corpus Annotation Methodology. *Linguistics and the Human Sciences*, 9 (1). https://doi.org/10.1558/lhs.v9i1.95

O'Donnell, M. (2007). UAM CorpusTool (Version 3.3) [Computer software]. http://www.corpustool.com/download.html

O'Donnell, M. (2019). Continuing Issues in SFL. In Thompson, G., Bowcher, W., Fontaine, L., & Schönthal, D. (Eds.) *The Cambridge Handbook of Systemic Functional Linguistics* (Cambridge Handbooks in Language and Linguistics, pp. 204-229). Cambridge University Press. doi:10.1017/9781316337936.010

O'Halloran, K. (1999). Towards a systemic functional analysis of multisemiotic mathematics texts. *Semiotica*, 124 (1-2), 1–30. https://10.1515/semi.1999.124.1-2.1

O'Halloran, K. (2006). *Mathematical discourse. Language, symbolism and visual images*. Continuum.

Oliveira, L. & Cheng, D. (2011). Language and the multisemiotic nature of mathematics. *The Reading Matrix*, 11 (3). http://www.readingmatrix.com/articles/september_2011/oliveira_cheng.pdf

Osbourne, M., & Rubinstein, A. (1994). *A Course in Game Theory*. [E-Reader version]. http://ebour.com.ar/pdfs/A%20Course%20in%20Game%20Theory.pdf

Parodi, G. (2008). Lingüística de Corpus: Una introducción al ámbito. *RLA, Revista de Lingüística Teórica y Aplicada*, 46 (1), 93-119. http://dx.doi.org/10.4067/S0718-48832008000100006

Partington A. & Marchi, A. (2015). Using corpora in discourse analysis. In Biber, D. and Reppen, R. (Eds.) *The Cambridge Handbook of English Corpus Linguistics*. UK: Cambridge University Press.

Pascual, M. (2015). La evaluación en el discurso de la ciencia: Entre las restricciones del género y la actitud crítica. In Pascual M. (Comp.) *La Evaluación en el Discurso Científico: Aportes a la Comprensión del Diálogo de Pares (pp. 11-28)*. Nueva Editorial Universitaria. ISBN 978-987-733-007-6. http://www.neu.unsl.edu.ar/wp-content/uploads/2018/03/La-EvaluacioO%CC%82n-en-el-Discurso...pdf

Pennycook, A. (2016). Politics, power relations and ELT. In G. Hall. (Ed.) *The Routledge Handbook of English Language Teaching* (pp. 26–37). Routledge.

Pérez-Llantada, C. (2012). *Scientific discourse and the rhetoric of globalization: the impact of culture and language*. Continuum.

Pimm, D. (1987). *Speaking Mathematically: Communication in Mathematics Classrooms*. Routledge Kegan and Paul.

Quiroz, B. (2013). The interpersonal and experiential grammar of Chilean Spanish: Towards a principled systemic-functional description based on axial argumentation. [Doctoral thesis] University of Sydney. Sydney, Australia.

Rose, D. (2014). Reading to learn: selecting and analysing texts. Teacher resource booklet 2. www.readingtolearn.com.au

Salager-Meyer, F. (2013). Writing and publishing in peripheral scholarly journals: How to enhance the global influence of multilingual scholars? *Journal of English for Academic Purposes,* 13 (2014) 78–82. https://doi.org/10.1016/j.jeap.2013.11.003

Sampieri Hernández, R., Collado Fernández, C. & Lucio Baptista, P. (2003). *Metodología de la Investigación*. McGraw-Hill Interamericana.

Scott, M. & Tribble, C. (2006). *Textual Patterns. Key Words and Corpus Analysis in Language Education*. John Benjamins.

Sinclair, J., & Ball, C. (1996) Preliminary Recommendations on Text Typology. EAGLES Documents EAG-TCWG-TTYP/P, June 1996. http://www.ilc.cnr.it/EAGLES96/texttyp/texttyp.html

Sinclair, J. (2004) (Ed). *How to Use Corpora in Language Teaching*. Benjamins.

Schleppegrell, M. (2007) The Linguistic Challenges of Mathematics Teaching and Learning: A Research Review. *Reading and Writing Quarterly: Overcoming Learning Difficulties*, 23 (2) 139-159, https:// 10.1080/10573560601158461

Shiro, M. (2003). Genre and evaluation in narrative development. *Journal of Child Language*, 30, 165-195.

Sobel J. (2009) *Signaling Games*. In: Meyers R. (eds.) Encyclopedia of Complexity and Systems Science. Springer, New York, NY. https://doi.org/10.1007/978-0-387-30440-3_481

Strauss, P. (2017). "It's Not the Way We Use English"—Can We Resist the Native Speaker Stranglehold on Academic Publications? *Publications* 5(4), 27. https://doi.org/10.3390/publications5040027

Thetela, P. (1997). Evaluated Entities and Parameters of Value in Academic Research Articles. English for Specific Purposes, 16 (2), pp. 101-118.

Thomson, W. (2011). *A guide for the young economist*. (2nd Ed). The MIT Press.

Tognini-Bonelli, E. (2001). *Corpus Linguistics at Work*. Benjamins.

Solow, D. (2014). *How to Read and Do Proofs an Introduction to Mathematical Thought Processes*. Wiley & Sons.

Stubbs, M. (1996). *Text and corpus analysis: Computer-assisted studies of language and culture*. Blackwell

Swales, J. (1990). *Genre analysis: English in academic and research settings*. University Press.

Umland, K., & Hersh, R. (2006). Mathematical discourse: the link from pre-mathematical to fully mathematical thinking. *Philosophy of Mathematics Education Journal*, 19. https://philpapers.org/rec/HERMDT

Veel, R. (1999). Language, Knowledge and Authority in School Mathematics. In Christie, F. (Ed.). *Pedagogy and the Shaping of Consciousness Linguistic and Social Processes*. Continuum.

Villayandre Llamazares, M. (2018). "HARTA" de noveles: un corpus de español académico. CHIMERA: *Revista de Corpus de Lenguas Romances y Estudios Lingüísticos*, 5. 131. 10.15366/chimera2018.5.1.011.

Waicekawsky, L. (2015). La sección Discusión de artículos de investigación de Fonoaudiología y psicología desde la Teoría de la Valoración [Masters' Thesis]. Universidad Nacional de Córdoba, Córdoba, Argentina.

White, P.R.R. (2000). Introductory Guide to Appraisal. http://www.grammatics.com/appraisal/AppraisalGuide/Stage3-Attitude-Appreciation.doc

APPENDICES

Appendix 1: Corpora

Main Corpus[5]

[AEJ_Txt1]

This paper studies various stability notions in a one-to-one matching problem with transfers. In this problem, the market has two sides, say, firms and workers. An outcome in this setup not only specifies which firm is going to hire which worker, but also the monetary payments. One intuitive solution concept for such a market is ex post stability: Ex post stability requires that each agent end up with nonnegative utility and that no firmworker pair would prefer to match with each other at some wage rather than the current matching. Ex post stability was introduced by Gale and Shapley (1962) for a matching market without transfers and was extended to a market with transfers by Shapley and Shubik (1971). Both of these papers show that ex post stable matchings exist when agents' preferences are known. However, most markets operate under asymmetric information, i.e., there is no common knowledge of preferences. Therefore, it is important to have an incentive-compatible mechanism, allowing agents to reveal their preferences truthfully to implement an ex post stable matching. Unfortunately, this is impossible: Roth (1982) shows in a matching problem without transfers that there is no incentive-compatible mechanism that produces stable matchings. Alcalde and Barberà (1994) strengthen the impossibility result of Roth (1982) by showing that there is no incentive-compatible, individually rational, efficient mechanism. Our first result complements these findings by showing that there is no incentive-compatible and ex post budget-feasible mechanism that produces ex post stable matchings in a market with transfers (Theorem 1). Ex post budget feasibility requires that the mechanism does not need an outside subsidy to run.1 Although desirable, ex post stability may not be necessary for the success of some matching markets, especially when the outcome of the mechanism can be enforced. For example, in the National Resident Matching Program, which assigns medical school graduates to residency programs, the matching outcome is binding. For similar markets in which agents cannot form blocking pairs after learning the outcome of the mechanism, we introduce two alternative stability notions. The first is ex ante stability. Even though agents can still unilaterally opt out of the mechanism ex post, they can only form blocking pairs at the ex ante stage before they learn their values. In other words, a matching mechanism is ex ante stable if agents get nonnegative utilities for all outcomes

[5]PDF versions available at
https://drive.google.com/drive/folders/1AfN7jGuYVIcl6OhWGFfwH1HFbSRwx3Bs?usp=sharing

and there exists no firm-worker pair who could match with each other at a particular wage and both get higher expected utilities at the ex ante stage. We show that incentive compatibility and ex ante stability can be satisfied in conjunction with efficiency and ex ante budget balance, under the assumption that either firms and workers are ex ante symmetric or that firms and workers are equal in number (Theorem 3). Efficiency requires that we implement the matching that maximizes the sum of match utilities, and ex ante budget balance requires that the mechanism does not run an expected deficit or create an expected surplus. The second stability notion is interim stability. We still allow agents to opt out unilaterally ex post. However, agents can now form blocking pairs at the interim stage when they already know their values. The interim no-blocking notion presents unique challenges. First, agents have asymmetric information at the interim stage. Therefore, if a firm and a worker agree to form a blocking pair, this provides information about the firm's valuation to the worker and, similarly, information about the worker's valuation to the firm. With this new information, they presumably update their estimates of expected utilities from participating in the mechanism. Therefore, it may no longer be beneficial for them to block the mechanism.2 In our definition of interim no blocking, each agent has a belief that her partner's private information lies in a subset. A pair of agents can form a blocking pair only if their beliefs are correct, in other words, when they update their beliefs, they get the same subsets back. The second challenge is a bargaining issue, since agents' willingness to pay depends on their private information. We resolve the bargaining problem as follows: in a one firm-one worker case, if each agent's value for her potential partner can be zero, then there is a unique, incentive-compatible, individually rational, efficient, and ex post budget-feasible mechanism. In this mechanism, the firm and the worker match without making any transfers. Therefore, we assume that this is the only deal they make. Even if agents' values are bounded away from zero, this matching becomes focal.

Finally, we show that interim stability can be satisfied together with incentive compatibility, efficiency, and ex ante budget balance when there is only one agent on the short side of the market (Proposition 3). This assumption is not innocuous, but it is satisfied by some important markets such as auctions and monopolies. In Table 1, we provide a summary of the main results. The first column shows whether the result is negative or positive. The rest of the columns refer to a property (EFF for (ex post) efficiency, IC for (ex post) incentive compatibility, IR for (ex post) individual rationality, BF for (ex post) budget feasibility, and BB for budget balance). Each row represents a result stating whether a mechanism with the required properties exists. Two strands of literature are related to the current work. The first strand considers weaker notions of incentive compatibility for matching markets without transfers to overcome the impossibility result of Roth (1982) that incentive-compatible and ex post stable mechanisms do not exist (Roth 1989; Majumdar 2003; Ehlers and Massó 2007). These papers either have nonexistence results or make harsh assumptions to get existence. In contrast, we consider a matching market with transfers and, while retaining the incentive compatibility constraint, consider weaker stability notions. The second strand

in the literature considers incentive-compatible core concepts for markets with transfers, following the seminal work of Wilson (1978). Although none of the solution concepts considered in this literature are directly related to ours, the ex ante incentive-compatible core is worth mentioning.3 An interim incentive compatible mechanism satisfies this property if there exists no coalition of agents that can get a higher ex ante payoff by an interim incentive-compatible mechanism. Forges (2004) establishes the nonexistence of the ex ante incentive-compatible core for a one-to-one matching model with discrete types. By contrast, our main result with ex ante stability establishes the existence of a (dominant strategy) incentivecompatible mechanism that is immune to blocking by pairs of agents at the ex ante stage who cannot sign a contract contingent on their type realizations. In addition, we also require ex post individual rationality, which is stronger than the ex ante individual rationality implied by the conditions in Forges (2004).

In a recent paper, Chakraborty, Citanna, and Ostrovsky (2010) takes a different approach than these two strands of literature. They study a two-sided matching problem of schools to students in which students have a common ranking of schools and schools have interdependent values over students. In this setting, they introduce a stability condition that depends on how much information the mechanism reveals and show that a stable mechanism exists if schools only observe their own matches. In contrast to their stability notion, our stability notions remain the same regardless of the mechanism considered. Finally, Yenmez (2009) studies incentive-compatible mechanisms for the allocation of discrete resources in general markets. In particular, he provides necessary and sufficient conditions for the existence of an incentive-compatible mechanism with other desirable properties such as individual rationality and ex post budget balance. Our benchmark results (Theorems 1 and 2), which do not impose any stability conditions, are applications of his general results. Whereas the current work's main focus is stability, Yenmez (2009) does not study any such notion. The rest of the paper is organized as follows. We introduce the formal model and establish two benchmark results that do not impose any stability conditions in Section II. In Section III, we study ex post stability. We then consider two novel stability notions: ex ante stability in Section IV and interim stability in Section V. Finally, we conclude the paper in Section VI.

Footnotes

Carnegie Mellon University, Tepper School of Business, 5000 Forbes Avenue, Pittsburgh, PA 15213 (e-mail: byenmez@andrew.cmu.edu). This paper is a part of my dissertation submitted to Stanford University, and it has been circulated as a part of "Incentive Compatible Market Design with an Application to Matching with Wages." I am grateful to my advisors, Michael Ostrovsky, Andrzej Skrzypacz, and Robert Wilson, for continuous guidance and support. I thank seminar participants at various institutions for helpful comments. The author acknowledges the financial support of the SIEPR Leonard W. Ely & Shirley R. Ely Graduate Student Fellowship.

Go to http://dx.doi.org/10.1257/mic.5.4.120 to visit the article page for additional materials and author disclosure statement(s) or to comment.

1.One needs to consider a budget feasibility constraint when agents can make endogenous transfers. Without such a requirement, ex post stability and incentive compatibility may be consolidated by subsidizing the system. Of course, no such condition is needed for markets without transfers.

2.Similar difficulties arise in auctions with collusion. See, for example, Laffont and Martimort (1997, 2000) and Pavlov (2008).

3. See Glycopantis and Yannelis (2005), for different solution concepts and their applications. Recently, Myerson (2007) provided a different approach and proved the existence of an interim core notion for games with a balanced structure.

[AEJ_Txt 2]

The theoretical literature on two-sided matching began with the simple one-to-one (marriage) model of Gale and Shapley (1962), in which agents on opposite sides of a market (men and women) seek to match into pairs. The central solution concept in this literature is stability, the requirement that, if two agents are not matched to each other, at least one of them prefers his or her assigned partner to the other agent. Gale and Shapley (1962) showed that stable one-to-one matches exist in general, and obtained conditions underwhichthisexistenceresultispreservedevenifagentsononesideofthemarketare allowed to match to multiple partners, that is, when the matching is many-to-one (as in college admissions and doctor-hospital matching). Following high-profile applications of matching in labor markets and school choice programs,1 the foundational work on matching has been extensively generalized.2 Recently, Ostrovsky (2008) illustrated that matching markets need not be two-sided—they may instead consist of a market of firms organized into supply chains. Earlier matching models easily embed into the Ostrovsky (2008) supply chain framework: for example, the many-to-one matching market between doctors and hospitals may be thought of as a "one-step supply-chain" in which doctors sell their services to hospitals. Although the expanding work on matching has eliminated nearly all the theoretical restrictions imposed in the early literature, two assumptions have been maintained throughout, either implicitly or explicitly:

• acyclicity – no agent may both buy from and sell to another agent, even through intermediaries, and

• full substitutability – upon being endowed with an additional item, an agent's demand for other items is lower, both in the sense of a reduced desire to buy additional items and an increased desire to sell items he currently owns.3

The acyclicity condition is implicit in all studies of two-sided matching, as the "two sides" may be identifed as the set of buyers and sellers, and so each agent acts only as a buyer or only as a seller.

Furthermore, acyclicity corresponds to the supply chain structure imposed by Ostrovsky (2008). Full substitutability generalizes a heavily-studied notion of preference (gross) substitutability first introduced by Kelso and Crawford (1982). Substitutability, in turn, generalizes the responsive preference condition introduced by Roth (1985). These successively more-general substitutability conditions have been shown to be essential for the existence of stable allocations in a variety of matching contexts.4 Moreover,bothacyclicityandfullsubstitutabilityarenaturalinmostpreviously-studied matching settings, such as the matching of residents to hospitals (Roth and Peranson (1999)), the assignment of students to schools (Abdulkadiroˇglu et al. (2005); Abdulkadiroˇglu, Pathak and Roth (2005, 2009)), and the supply-chain production of steel (Ostrovsky (2008)). Acyclicity may not hold, however, in electricity markets, where an individual firm may buy power from a neighboring firm in one region and sell power to that same firm in another region. Full substitutability is unlikely to apply in settings such as the matching of auto-parts suppliers and assemblers, where different parts are complementary in the production of the final good.5

In this paper, we introduce a matching model in which firms trade via bilateral contracts which specify a buyer, a seller, and the terms of the exchange. This model subsumes all classical matching models, and its generality allows us to make two novel theoretical contributions. First, we show that both acyclicity and full substitutability are necessary for classical matching theory. If either condition is violated, then stable allocations cannot be guaranteed.6 Intuitively, if a contracting relationship contains a cycle, and if a firm in the cycle has an outside option which the firm prefers to one contract in the cycle, then both the outside option and the complete trading cycle are unstable; the necessity of acyclicity follows. The necessity of full substitutability is more technical to illustrate, but follows closely upon prior results of Hatfield and Kominers (2010). Second, in the presence of acyclicity and fully substitutable preferences, we fully generalize the key results of classical matching theory. We prove that, in the presence of acyclicity and full substitutability, stable allocations correspond bijectively to fixed points of an isotone operator; Tarski's fixed point theorem then guarantees the existence of a lattice of stable allocations. We also prove a generalization of the classical rural hospitals theorem of Roth (1986) and the strategy-proofness results of Hatfield and Milgrom (2005) and Hatfield and Kojima (2009). These latter results display a surprising structure which can only be elicited within a framework as general as ours: in particular, we show that for each agent the difference between the numbers of buy- and sell-contracts held by that agent, rather than the absolute number of contracts held, is invariant across table allocations.7 In light of our necessity results, our work establishes a frontier of matching theory. Without acyclicity and fully substitutable preferences, stable allocations are not guaranteed to exist in general, and hence the results of classical matching theory fail. Up to the failure of these conditions, however, all of the results of classical matching theory hold. Thus, our work in some sense characterizes the set of applications to which classical matching theory applies without imposition of additional structure: Settings where both acyclicity and full substitutability hold—such as labor markets,

school choice, and steel production—are directly approachable via classical matching theory; settings where either acyclicity or full substitutability fails—such as electricity and auto-parts markets— are not. The remainder of this paper is organized as follows. We formalize our model, restrictions on preferences, and solution concepts in Section I. In Section II, we prove the sufficiency and necessity of fully substitutable preferences for the existence of stable contract allocations. In Section III, we discuss the structure of the set of stable allocations, proving our rural hospitals and strategy-proofness results. We conclude in Section IV. All proofs are deferred to the Appendix.

Footnotes

1.Roth and Sotomayor (1990) provide a survey of the pre-1990 theory of matching. Roth (2008) gives an updated account, as well as references for historical and recent applications of matching. For examples of specific applications, see the work of Roth and Peranson (1999) (National Resident Matching Program) and Abdulkadiroglu et al. (2005); Abdulkadiroglu, Pathak and Roth (2005, 2009) (school choice).

2.Kelso and Crawford (1982) extended many-to-one matching to a setting in which matches are supplemented by wage negotiations; Hatfield and Milgrom (2005) generalized this framework still further, by allowing agents to negotiate contracts which fully specify both a matching and the conditions of the match; the possibility of such a generalization was first noted by remarks of Crawford and Knoer (1981) and Kelso and Crawford (1982). Meanwhile, a host of work has studied the existence of stable matchings in many-to-many matching settings, two-sided markets in which all agents may match to multiple partners (as in the matching of consultants to firms). Many-to-many matching has been studied, for example, in the work of Sotomayor (1999, 2004), Echenique and Oviedo (2006), and Konishi and ¨Unver (2006). Recently, Klaus and Walzl (2009) and Hatfield and Kominers (2010) merged this line of research with that of Hatfield and Milgrom (2005), introducing a theory of many-to-many matching with contracts.

3.Full substitutability is a condition on firms' preferences familiar from auction theory. Indeed, full substitutability is an ordinal analogue of the conventional notion of substitutability from auction theory (see, for instance, Milgrom (2004)) and we prove more formally that it is equivalent to quasi submodularity of the associated indirect utility function.

4.The sufficiency and necessity of substititutability for the guaranteed existence of stable allocations holds in the settings of many-to-one matching (Roth (1984) proves sufficiency; Hatfield and Kojima (2008) prove necessity), many to-many matching (Roth (1984) and Echenique and Oviedo (2006) prove sufficiency; necessity follows from the results of Hatfield and Kojima (2008)), many-to- many matching with contracts (Klaus and Walzl (2009) and Hatfield and Kominers (2010) prove sufficiency; Hatfield and Kominers (2010) prove necessity). Substitutable preferences are sufficient for the existence of a stable

outcome in the setting of many-to-one matching with contracts (Hatfield and Milgrom (2005)) but are not necessary (Hatfield and Kojima (2008, 2010)).

5.In an econometric study of many-to-many matching with transferable utility, Fox (2010) identifies substantial complementarity between inputs into automobile production.

6.Note that we use a notion of stability which is distinct from the concept of chain stability introduced by Ostrovsky (2008). Our stability concept is more stringent than chain stability, although these two notions coincide on acyclic contract domains over which firm preferences are fully substitutable (Theorem7). As we detail in Section I.B, for domains where these conditions do not hold, chain stability has some unappealing properties.

7.The Roth (1986) rural hospitals theorem and its subsequent generalizations by Hatfield and Milgrom (2005) and Hatfield and Kominers (2010) all show that, under certain conditions, the number of contracts signed by each agent is invariant across stable allocations. The natural conjecture that this exact result would extend to our setting is false, as we demonstrate in Section III.A. We instead find that the proper invariant for each firm in our framework is the difference between the numbers of buy- and sell-contracts held by that firm. This result implies the previous rural hospitals results because, in a two-sided market, no firm can be both a buyer and a seller.

[ET_txt1]

Introduction

Unraveling—contracting long before the job begins and before all the relevant information is available—has been one of the most prevalent phenomena in entry-level labor markets. For example, in the market for hospital-interns before 1945, appointments were even made as early as two years prior to the students' graduation and the actual employment (Roth and Xiaolin 1994).1 A similar situation still exists in the market for federal court clerks (Haruvy et al. 2006; Avery et al. 2007).

Most existing studies on unraveling offer a number of driving factors behind it on multiperiod models with uncertainty as to relevant information.2 In such models, it has been shown that unraveling is undesirable, as it causes Pareto-inefficient outcomes to arise. For instance, in a two-period model with an initial uncertainty about the rankings of firms over workers, Halaburda (2010) demonstrates that, for an anonymous mechanism to be Pareto-optimal, it has to prevent unraveling. In a similar multiperiod model with uncertainty regarding the qualifications of workers, Roth and Xiaolin (1994) show that unraveling may be ex-ante as well as ex-post inefficient. Moreover, in similar models, as well as the efficiency loss results, Li and Rosen (1998) and Li and Suen (2000) also characterize the agents who would be better (worse) off in the presence of early contracting in their respective settings.

In contrast to the above-cited works, Sonmez (1999) approaches the unraveling phenomenon from the manipulation point of view in centralized matching markets without uncertainty. Namely, he addresses the question of whether a two-agent coalition can circumvent stable mechanisms via pre-arranging, whereby both of them are at least weakly better off while at least one of them is strictly better off. Then, unfortunately, he shows that no stable mechanism is immune to pre-arrangements. Moreover, he identifies two types of successful pre-arrangements: Type-1 and Type-2. In the former type, the pre-arranging intern would have already been assigned to the same hospital in the absence of the early contract.3 On the other hand, in the latter type, he accepts an early offer from a hospital that he would otherwise not have been matched with.4

Given the prevalence of unraveling, much attention has been given to the welfare effects of it. While studies analyzing the welfare effects of unraveling in different models do exist (we already cited some of them), to the best of our knowledge, there is no such in the setting of Sonmez (1999). This paper attempts to fill this gap in the literature. In the paradigm of the hospital-intern matching market, we investigate the welfare effects of each type of pre-arrangements (Type-1&2) under the hospital-optimal and intern-optimal stable mechanisms. Namely, we address the questions of (i) whether pre-arrangements result in inefficient outcomes and (ii) how each side of the market is affected in terms of welfare.

We start with the intern-optimal stable mechanism. As it is non-manipulable via Type-1 pre-arrangements (Kojima and Pathak 2009), we address the welfare questions for Type-2 pre-arrangements (we know it is manipulable via Type-2 due to Sonmez 1999). We show that Type-2 pre-arrangements might cause inefficient matchings to arise, and the welfare effects on each side are ambiguous in the sense that, apart from pre-arranging agents, there might be agents from each side of the market being strictly better and worse off.

For the hospital-optimal stable mechanism, while the results are the same with those under the intern-optimal stable mechanism for the Type-2 case, they change drastically for the Type-1 case. First, the hospital-optimal stable mechanism is manipulable via Type-1 pre-arrangements. Then, in contrast to the results in the existing literature on unraveling and the ones in this paper, we prove that this type of pre-arrangements does not result in Pareto-inefficient outcomes. Furthermore, the welfare effects on each side are unambiguous: All hospitals (interns) are at least weakly better (worse) off, and at least one hospital (intern), apart from the pre-arranging one, is strictly better (worse) off.

This paper is broadly related to the line of research on unraveling. As most of the existing studies in this literature (we already cited some of them) investigate unraveling in models with uncertainty as to relevant information, a direct comparison between the current paper and any one of them is subtle due to modeling differences. Nonetheless, our study complements this line of research by investigating the manipulability of the two widely adopted stable mechanisms via different types of pre-arrangements and their welfare effects. One sharp contrast between the current paper and the existing literature is that while

the lack of information at the pre-arrangement stage is the reason for ex-post inefficient outcomes in the literature, it is not the case in this paper, as all the relevant information is common knowledge at every stage. Changes in the mechanics of the intern-optimal and hospital-optimal stable mechanisms after a pre-arrangement result in inefficient outcomes in our study.

Apart from the unraveling literature, two important related papers are by Konishi and Unver (2006) and Kojima (2006). They both investigate the welfare effects of capacity underreportings under the intern-optimal and the hospital-optimal stable mechanisms. The former shows that hospitals anonymously prefer any pure Nash equilibrium outcome (if it exists) to the outcome of any larger capacity profile. Then, Kojima (2006) extends this result to mixed Nash equilibrium outcomes. Moreover, the current paper is also related to the extensive incentive theory literature. Due to Roth (1982), we know that no stable mechanism is strategy-proof.5 As well as preference manipulations, Sonmez (1997, 1999) show that every stable mechanism is vulnerable to capacity and pre-arrangement manipulations, respectively. Two other closely related papers are by Kesten (2011) and Kojima and Pathak (2009). While Kesten (2011) demonstrates that no mechanism is immune to pre-arrangement manipulations, the current paper investigates the manipulability of the two central stable mechanisms via different types of pre-arrangements along with their welfare effects separately. Despite the manipulability results in these papers, Kojima and Pathak (2009) reveal that the scope of profitable pre-arrangements diminishes under the intern-optimal stable mechanism in large markets.

Is there any real-life evidence on the type of unraveling we consider in the current study? In the National Resident Matching Program (NRMP), Roth (1990) points out that some hospitals ask interns to rank them first in return for placing them in their first choice group of interns. As NRMP has been using the intern-optimal stable mechanism, it collapses to pre-arrangements that we focus on in the paper. Moreover, for suggestive facts regarding the specific types of pre-arrangements (Type-1&2) separately, we can look at early admissions at colleges in USA (though it is not a perfect fit to our paper, as the admission system is not centralized). Christopher and Levin (2010) report that most of the top colleges offer early admissions, and many schools fill sizeable fractions of their quotas with early applicants.6 One of the stylized facts they derive from the data is that while, at the very top schools, early applicants have higher test scores on average than regular applicants, the converse is true at schools just below the very top ones. On the basis of this fact, one can argue that early admissions at the very top schools are examples of Type-1 pre-arrangements,7 whereas those at schools just below the top ones are examples of Type-2 pre-arrangements.8 These facts suggest, in the hope of getting better students in the regular admission round, the very top schools might match earlier with students who would already be admitted in the absence of early contracts (Type-1), while schools just below than the very top ones might contract with their dispreferred students to ones with whom they would be matched in the regular round in the absence of early contracting (Type-2).

Why do we focus on the intern-optimal and the hospital-optimal stable mechanisms? There are both theoretically and practically important reasons behind it. First, they are the central mechanisms in theory in part due to their appealing properties.9 On the other hand, from the practical point of view, they are widely used in many real-life matching problems. For instance, NRMP and the two largest school districts, New York City and Boston, have been using the intern-optimal stable mechanism (see Roth and Peranson 1999; Abdulkadiroglu et al. 2005; Roth et al. 2005).10 On the other hand, apart from the reported British markets in Roth (1991) where the hospital-optimal stable mechanism is in use, The Veterinary Internship and Residency Matching Program (VIRMP), which is a centralized placement mechanism matching veterinarians and hospitals in the United States, has also been using the hospital-optimal stable mechanism.11

Our paper is important not only for theoretical purposes, but also for practical issues. Even though unraveling and its efficiency-related consequences have received much attention in the literature, there is no study analyzing pre-arrangements and their welfare effects in the setting of Sonmez (1999). Hence, from the pure theoretical point of view, this paper fills this important gap in the literature. On the other hand, real-life evidences and suggestive facts on pre-arrangements (we already provided some) make it also practically important. In some markets, unstable solutions have been replaced with stable ones in the hope of overcoming experienced market failures including unraveling.12 Yet, due to Sonmez (1999), we know that agents might still have incentive to pre-arrange even under stable mechanisms. This paper, unfortunately, also shows that these pre-arrangements might result in Pareto-inefficient outcomes. Therefore, the current study along with Sonmez (1999) demonstrates that stable solutions are not enough by themselves to avoid welfare detrimental pre-arrangements and call for a further policy action to overcome the problem.

.

Footnotes

I am grateful to Muriel Niederle, Ahmet Alkan and especially Fuhito Kojima, the Associate Editor and two anonymous referees for their insightful comments and suggestions. I thank Tim Bresnahan, Bertan Turhan, and Irem Ozsara9oglu.

Roth and Xiaolin (1994) give other market examples which experienced unraveling.

Some of them include Halaburda (2010), Roth and Xiaolin (1994), Li and Rosen (1998), Li and Suen (2000), and Suen (2000).

This implies that only the pre-arranging hospital strictly benefits.

In this case, both agents in the pre-arranging pair are strictly better off.

A mechanism is strategy-proof if no agent ever has incentive to misreport his preference. This property is widely studied in the literature. Some of the recent papers related to it include Kojima and Manea (2010), Kesten and Yazici (2010), Svensson (2009) and Sanver (2009).

Christopher and Levin (2010) also mention that the admission rate among early applicants is higher than that among regular applicants. For instance, in 2009-2010, the former is nearly as twice as the latter.

Presumably, top schools are the preferred ones by students, and students with higher test scores are the favorite ones of these schools. Therefore, at least some of the early admitted students at the top schools would have already been matched with the same schools in the regular admission process.

Given that these schools are just below than the very top ones, one can suspect that some of the early admitted students could not have gotten admission in the regular round.

For an excellent source about the mechanisms, one could refer to Roth and Sotomayor (1990).

In the paradigm of school choice, the intern-optimal stable mechanism is called the student-optimal stable mechanism.

For further information about VIRMP, the reader could refer to http://www.virmp.org.

Roth (1991) gives market examples from U.K. illustrating this point.

[ET_Txt 2]

Introduction

In many matching markets, using monetary transfers to allocate partners would appear uncouth, if not morally repugnant. Similarly off-putting are the advances of potential partners who are too eager to enter into a commitment, thereby revealing their lack of quality or sophistication. Thus, holding back from entering a match can act as a signal of quality, helping others to deduce one's type in the absence of publicly observable expenditures of money or effort. Unlike an environment where independent contests for partners can be held on both sides of the market to provide credible signals, however, competition in this environment requires agents on both sides to agree on the time at which a match is consummated. Otherwise, agents will be tempted to exit early in favor of a lower quality partner but a shorter wait, and separating equilibria unravel. This paper provides sufficient conditions for efficient matching to be implementable when information about partner quality is private information on one or both sides and shows that when simple random matching can turn out to be the only implementable allocation.

Time is continuous and agents are impatient. Agents are divided into two disjoint sides of the market, and each agent seeks to match to someone from the opposite side. Each agent's suitability as a partner plays a role in determining the value of the match to the two parties. While unmatched, agents receive a flow payoff of zero, but upon matching, they leave the market and receive a match value that is

increasing and either strictly log-supermodular or strictly log-submodular in the quality of the two partners. Agents with private information in this paper reveal their types by delaying the point at which they "dropout" and announce they are ready to commit to a match. In this sense, the game resembles a one- or two-sided war of attrition. Unlike standard wars of attrition, signaling is not modeled as mounting pecuniary or effort costs, but instead as the opportunity cost of remaining unmatched. The main objective of the paper is to investigate when positive or negative assortative matching are implementable in the absence of costly signaling, as in Heidrun and () or J(). When competition for partners cannot be conducted independently—so that types on either side who are slated to be matched must agree on the time at which to match—either strong sufficient conditions are required to ensure that efficient matching can occur or the patterns of matching that are implementable are very restricted. The reason is that whichever side has the larger gains from getting a higher quality partner will need to delay longer to provide credible signals. In the meantime, the other side will be willing to sacrifice partner quality for a shorter wait, and separating equilibria unravel.

Section considers the case where only one side of the market has private infor- mation. The informed side issues proposals over time, where higher-quality agents wait longer before dropping out. When match surplus is strictly log-supermodular or log-submodular, positive or negative assortative matching is the only implementable allocation, and the market clears from the top or bottom, respectively. I extend the mechanism design analysis so that the strategy space of the agents is a dropout time rather than a type report, and proposing agents can approach any potential partner, rather than just the one prescribed by the mechanism. Such a deviation might lure a higher-quality agent on the uninformed side into leaving the market early with lower- quality partners in order to match sooner. A simple sufficient condition to ensure that matching is synchronized across the two sides and the positive assortative outcome prevails is available: swapping partners among any two pairs matched under positive assortative matching always produces a larger log gain to the side that is waiting for proposals than the side that is making them. This ensures that in equilibrium, the side waiting for proposals has nothing to gain by accepting an early proposal by an agent of lower quality than the one they expect to match with in equilibrium. Negative assortative matching, however, is not implementable since the highest type partners are available at the earliest dates, and all agents on the signaling side can profitably deviate by approaching these high-quality partners immediately, unraveling the market.

When information is incomplete on both sides of the market, however, the results are more negative. I first consider complete separation, where agents on both sides are continuously matching and exiting. This requires that agents on both sides have the same marginal return to waiting an additional instant, which requires their utility functions to grow as a function of partner quality at the same rate. Except for particular situations where the payoffs are determined by a constant split of a shared surplus, this kind of alignment of the two sides' preferences seems unlikely, and assortative matching fails to arise purely from using delays

as a signal of type. If complete separation fails to be implementable, then any equilibrium must then involve some kind of random matching. I consider the case of "coarse matching" ; Damiano and), where the market is split into a finite number of sub-markets which meet at discrete dates. Within these sub-markets, agents are matched randomly, but later sub-markets reward patience by providing a more attractive distribution of partner types on each side of the market, creating some stochastic assortativity in the matching. Implementability of a given coarse matching then requires that the lowest and highest types in a given sub-market are indifferent between deviating and joining the previous or next sub-market, respectively, and the measures of agents on both sides are equal. While a given coarse matching then only requires a finite number of indifference conditions hold, I show that if one side always experiences a larger log gain from receiving a higher-quality partner at the same type quantile, then only simple random matching is implementable. This implies that when the environment does not allow costly effort signaling and transferable utility, assortative matching will be difficult to implement even under standard supermodularity assumptions. The final result of the paper then provides a condition under which non-trivial coarse matching is possible, which takes the form of a crossing condition in the log gains to the two sides: for example, the gain to workers from matching to a mediocre firm rather than the worst firms is larger than the gain to matching to the best firms instead of a mediocre firm, while the gain to firms from matching to the best workers rather than mediocre workers is larger than the gain from matching to mediocre workers rather than the worst workers.

While () showed that stable matching is generally impossible in discrete time with incomplete information and in the absence of transfers, positive results in the literature with transfers (Damiano and ; Heidrun and ; J; Pavan and ; Utgof; Liu et) suggested that the assumption of supermodularity might generally be enough to implement the efficient allocation when agents were allowed to signal or otherwise "bid" in a market for partners. The intuition for these positive results is that if surpluses are supermodular in types, higher-quality agents can expend part of the returns to getting a better partner on costly signaling and keep the rest, leading to a separating equilibrium. This, however, exploits the fact that contests can be held instantly and independently on both sides of the market. When transferable utility or costly effort signaling are eliminated, however, rationing partners on the basis of time is an obvious substitute: low-type agents will agree to a match quickly since they have little to gain by waiting, allowing higher-type agents to compete and "wait one another out" for the most desirable partners. While this intuition is correct on each side of the market considered in isolation, the two schedules must synchronize in equilibrium, and heterogeneity in payoffs and type distributions between the two sides will lead to different schedules.

This paper considers a dynamic matching model following Beck () and Beck(), rather than a search-and-matching following Shimer and (). The market here is non-stationary, since the expected quality of the remaining agents will be monotonically increasing or decreasing in time, and the market clears in a finite amount of time. In the infinite horizon search literature, market churning is achieved by assuming match

quality varies idiosyncratically over time or matches end according to some exogenous process, making them unsuitable for studying markets where fixed sets of agents that meet once and clear quickly, like the primary market for labor in professions where hiring occurs once a year or marriage markets where transfers are not possible and people typically marry within narrowly defined age and socioeconomic groups.

The "large market" approach used in this paper is similar to Olszewski and Sie (), who consider profit-maximizing contests with large numbers of players for heterogeneous prizes, but with quasi-linear utility and "prizes" rather than partners that are indifferent to their winner. This is similar to the first part of Sect. , but in subsequent analysis the "prizes" in my paper have preferences over their partners, leading to very different results. The coarse matching analysis in Sect. uses the general idea of taking a matching market and breaking it into discrete sub-markets within which the agents match randomly. I focus on the incentives of the "boundary" types and use a similar approach to Damiano and () to reduce the problem to analysis of local constraints. While Damiano and () show that a profit- maximizing matchmaker might desire to use coarse matching to improve profits, I consider it as an alternative when full separation is not implementable. Even when the designer's goal is to implement the most efficient allocation of partners possible, however, coarse matching turns out to be extremely restricted.

[GEB_txt 1]

Introduction

The fields of bounded rationality and mechanism design are increasingly attracting the attention of economists. This paper contributes to solving the classical implementation problem when players are boundedly rational. Implementation theory considers the cases in which there are different states of the world but in each state what is optimal for a society (summarized by social choice rule (SCR)) is known. Even though the SCR is fixed, there are usually states in which the social optima contradict the individual optima for some agents. Therefore, it is unreasonable to expect society to make a choice consistent with the SCR after a state is realised. However, a benevolent third party who is not aware of the realised state may be able to guarantee socially optimal outcomes in each state by designing a mechanism (a set of rules that result in an outcome based on the information sent by this society's members) that is played by the members once the uncertainty is resolved. Implementation theory investigates whether any mechanism can deliver the socially optimal outcomes in each state.

Since Hurwicz's seminal works inand the implementation problem has been studied from many different perspectives.However, a majority of papers assumes that players are fully rational. But what if the players are not fully rational? This is the main concern of this paper.

In this paper, we model irrationality as simple mistakes that occur when the players evaluate their best responses: the players try to be rational, but because of their imperfect calculating ability, they may

play non-optimal strategies. If some probabilistic structure is imposed on the mistakes, then the players have probabilistic responses. Assuming that the players are aware that the others are mistake-prone, one can define equilibrium as a fixed point of the players' responses. This equilibrium is the well-known Quantal Response Equilibrium (QRE) from

Logit QRE (LQRE) is a QRE if mistakes are distributed iid with an extreme value distribution parameterized by X e R+ which we interpret here as the sophistication level. Due to this specification of the mistakes, the logit quantal response function has two desirable features. First, the players are more likely to make a smaller mistake than a bigger one. Second, as the sophistication level approaches infinity, the probability of a player playing a strategy not in the true best response monotonically decreases to 0. Therefore, the higher the X, the more rational players are. In addition, if X = ro, then the players are fully rational, hence, any limit LQRE (LLQRE) is a Nash equilibrium. Moreover, if the players are close enough to being fully rational, then any resulting LQRE is very close to one of the LLQREs.

In addition to its desirable theoretical features, LQRE seems to explain the experimental results better than the Nash equilibrium does. The original paper ofdemonstrates the predictive ability of LQRE on several well-known experiments whose results systematically deviated from the ones Nash equilibrium predicts. Since then, LQRE has been used to explain many experimental results including all-pay auctions , first price auctions , coordination games (, and information cascades .

Given the theoretical and empirical plausibility of LQRE, we assume that games result in LQREs. This paper studies the implementation problem when the equilibrium concept is LLQRE (not LQRE) because the LQREs can be proxied by the corresponding LLQREs when the players are highly sophisticated. Therefore, any mechanism which implements an SCR in LLQREs will implement the SCR in LQREs with high probability.

We characterize the sufficient conditions for LLQRE implementation in environments with at least three players and in which at least one player has a state-independent worst alternative. In such environments, any SCR satisfying quasimonotonicity (QMON) - a small variation of Maskin monotonicity - and no worst alternative (NWA) is LLQRE implementable. Quasimonotonic SCRs satisfy the following condition: if the strict lower contour set of an SCR alternative weakly expands for every agent when going from one state to another, then the alternative is also in the SCR of the second state. On the other hand, an SCR satisfies NWA if it does not prescribe any player's worst alternative in any state.

In the proof of the sufficiency result, we construct a mechanism that delivers each SCR alternative in each state through some strict LLQRE of the corresponding state. We should remark that this does not mean that all the LLQREs have to be strict. There can be some non-strict LLQREs in any state as long as each one of them yields an alternative prescribed by the SCR in the corresponding state. We use the above mentioned restriction because non-strict LLQREs are sometimes not preserveunder monotonic

transformations (including affine) of the utility functions of the players. This is a highly undesirable problem for the planner who only has information about the players' preference relations because in this case she needs to ensure that LLQRE implementation is robust to monotonic transformations of the utility functions of the players. Otherwise, some SCR alternatives might not be implemented for certain utility representations which must be avoided. In this paper, we show that strict LLQREs do not depend on the utility representations of the players' preferences. In addition, using several examples, we illustrate the complexities of determining the conditions under which non-strict LLQREs are robust to monotonic transformations of the utility functions of the players. Consequently, for our sufficiency result, we look for a mechanism that delivers each SCR alternative in each state through some strict LLQRE of the corresponding state. If one concentrates on the LLQRE implementation in which each SCR alternative in each state is delivered by some strict LLQRE, then we also show that QMON and NWA are necessary conditions. In this sense, the current paper (almost) fully characterizes LLQRE implementation under the restriction that each SCR alternative in each state is delivered by some strict LLQRE.

The paper most closely related to ours is the one by: They study implementation in strict Nash equilibria and find that QMON and NWA are necessary and (almost) sufficient for implementation in strict Nash equilibria. The authors furthermore conjecture that these conditions are important for implementation in environments with boundedly rational players because many equilibrium concepts in such environments are closely related to strict Nash equilibria. Indeed their conjecture holds in our setting. In this sense, this paper andcomplement each other and seem to be consistent with a "bigger picture" of implementation when the players are boundedly rational. However, the proof of our sufficiency result uses a mechanism much different than the one used in because not all strict Nash implementing mechanisms implement in LLQREs.Furthermore, our mechanism implements any given SCR in both LLQREs and strict Nash equilibria. In this sense, our mechanism is perhaps more relevant for implementation when the players are boundedly rational.

There are a handful of papers which consider the irrationality of players in implementation theory.and consider implementation in existing mechanisms under learning dynamics. investigate the case in which the players adjust their strategies in the direction of better responses. Interestingly, QMON is again key for implementation in recurrent strategies of better response dynamics. These papers require dynamic settings, while the setting used for this paper is static.studies simple pricing schemes used in implementing efficient SCRs in evolutionary setting. The idea that some players are completely unpredictable has been studied by Even though in LQRE, players play in this fashion when the sophistication level approaches 0, there is a substantive difference between our paper and that of Eliaz. In his setup only some of the players make mistakes while the others are rational. In contrast, in this paper every player makes small mistakes.

This paper is organized as follows: Sectioncontains preliminaries. Sectiondefines LLQRE and restricted LLQRE implementations and discusses their sufficient and necessary conditions.

Footnotes

See Example

See Example

[GEB_Txt 2]

Introduction

Variants of the Gale-Shapley deferred acceptance (DA) algorithm (Gale and Shapley, 1962) and what has come to be known as the Boston mechanism (which we will often refer to simply as 'Boston') are widely used by school districts throughout the United States to assign K-12 students to schools. Understanding the advantages and disadvantages of these mechanisms is a matter of great practical importance that has been the focus of extensive research, both theoretical and empirical.1 The two mechanisms have been widely studied, and choosing one over the other involves trade-offs between incentive and welfare properties. While earlier work promoted DA over Boston (e.g., Abdulkadiroglu and Sonmez, 2003 and Ergin and Sonmez, 2006), several recent papers have re-examined the Boston mechanism in settings in which participants have limited information about the preferences of the other students and the (post-lottery) priority structures at the schools and have found advantages for the Boston mechanism. In particular, in incomplete information environments with common ordinal preferences for the students and no school priorities,2 Abdulkadiroglu et al. (2011) (henceforth, ACY) and Miralles (2008) show that the (symmetric) equilibrium outcomes of the Boston mechanism actually interim Pareto dominate3 that of deferred acceptance.4

The contribution of the current work is two-fold. First, we study the robustness of the interim Pareto dominance results mentioned above to the introduction of more realistic, nontrivial priority structures. We give two examples with weak priorities in which some students are strictly (interim) better off under DA, and show in a general model that the same will be true for any priority structure that satisfies a mild condition that is satisfied by many real-world priority structures. These results are similar in flavor to many "impossibility theorems" in the matching literature, which construct preferences to show that certain properties of matching mechanisms will not hold in general (see Roth and Sotomayor, 1990, Chapter 4).

Second, because neither mechanism will interim Pareto dominate the other once we allow for priorities, we must search for alternative criteria by which to compare school choice mechanisms. We thus introduce a new criterion, ex-ante Pareto dominance, to the school choice literature. This criterion examines welfare before students know their own types (cardinal utilities and priorities). From this perspective, we

show that we can once again rank mechanisms, and in particular, that Boston ex-ante Pareto dominates any strategyproof (and anonymous) mechanism (including DA and the top trading cycles algorithm), even allowing for arbitrary priority structures. Thus, there is an explicit welfare cost associated with the use of strategyproof mechanisms.

Our ex-ante Pareto dominance concept is similar to the ex-ante viewpoint often used in the mechanism design literature, and one of our main contributions is to show how it can be useful in school choice settings. Since it seems reasonable to assume that students do in fact know their own preferences and priorities at the time they submit their rankings, our ex-ante viewpoint is more usefully interpreted as a normative concept, rather than a positive one, and can be given several normative justifications. First, we can argue from behind a Rawlsian "veil of ignorance" and say that a student who was asked to choose between mechanisms before she knew her place in society (i.e., her preferences and priorities) would pick the ex-ante Pareto dominant mechanism. Additionally, from a policy perspective, we argue that this criterion is especially relevant for school districts, as they can be thought of as social planners who do not know the realised preferences of any individual and whose goal is to maximize overall ex-ante welfare. Finally, since policymakers must decide on mechanisms that will be used for several years, they may be more concerned with how the mechanism performs relative to the underlying distribution of preferences and priorities rather than optimizing performance for one specific realization.

Thus, while introducing nontrivial priorities causes difficulties in ranking mechanisms from an interim perspective, our results provide some justification for the use of Boston mechanism from an ex-ante perspective, even when schools have priorities. This is not to say that DA (or strategyproof mechanisms in general) should be rejected in favor of the Boston mechanism; strategyproofness certainly has its own advantages. However, we are able to generalize the results of Abdulka- diroglu et al. (2011) and Miralles (2008) to situations with arbitrary priority structures and point out clear welfare losses due to strategyproofness. Which of these issues is more important empirically, or whether there exist mechanisms that will perform better than both Boston and DA, is an important open question.

Related literature

This paper is related to the large number of works that have aided in the design of real-world institutions by examining the incentive and welfare properties of centralized matching mechanisms in general, and school choice mechanisms in particular. On the incentives side, Roth (1982) and Dubins and Freedman (1981) show that deferred acceptance is strategyproof, while Abdulkadiroglu and Sonmez (2003) point out that the Boston mechanism requires students (or parents) to play a complicated strategic game and may harm naive students who fail to strategize. In fact, it is this feature that was important in the city of Boston's decision to abandon its namesake mechanism for a deferred acceptance procedure. On the efficiency side, Ergin (2002) and Ergin and Sonmez (2006) were the first to discuss possible ex-post Pareto

inefficiencies of the two mechanisms in a school choice context. However, here we will be concerned with interim and ex-ante efficiency losses as a result of the tie-breaking necessary to construct schools' strict priority orderings over students, issues first raised by Abdulkadiroglu et al. (2008), Erdil and Ergin (2008), and Abdulkadiroglu et al. (2009).6

The papers most closely related to this one are Abdulkadiroglu et al. (2011) and Miralles (2008), both of which investigate interim efficiency and show that Boston may actually interim Pareto dominate DA in situations with common ordinal preferences and no school priorities. The intuition for these results is that Boston allows students to indicate a relatively high cardinal utility for a school by promoting it above its true ordinal rank. However, the assumption of no school priorities may not apply in many contexts. Many cities classify students into several priority levels at each school,7 with a student in a higher priority level being admitted before a student in a lower priority level under Boston, if they rank the school the same. To highlight the role of priorities, we keep the common ordinal preferences assumptions found in the prior work for most of the analysis, but allow for arbitrary priority structures. As we show, when this is done, the interim welfare comparison between the two mechanisms is no longer clear cut. However, the ex-ante criterion we propose allows us to rank mechanisms in a wider range of scenarios, providing guidance in mechanism selection for school districts that may have complicated priority structures.

The remainder of the paper is organized as follows. In Section 2, we give two examples of school choice problems with nontrivial priority structures in which the Boston mechanism no longer interim Pareto dominates DA. Section 3 extends these examples to a general model, and identifies a sufficient condition on the priority structure under which Boston and DA cannot be interim Pareto ranked. Section 4 examines welfare from an ex-ante perspective, showing that from this viewpoint, Boston Pareto dominates any strategyproof and anonymous mechanism, even with priorities. Section 5 concludes. The proofs of the main propositions are relegated to Appendix A.

Footnotes

By interim utility, we mean a situation in which students know their own types but only the distribution of the types of other students. Much of the previous work on this topic calls this "ex-ante" utility, but, in this paper we will also examine welfare from the perspective before students know even their own types, and we reserve the term "ex-ante" for this situation.

See also Featherstone and Niederle (2008) who find gains to the Boston mechanism over DA in an experimental setting. Pais and Pinter (2008) is another experimental study that examines the top trading cycles (TTC) algorithm in addition to Boston and DA, finding that limited information may actually improve efficiency. Ozek (2008) provides simple examples of problems in which the Boston mechanism may Pareto dominate DA.

A similar argument is made by Featherstone and Niederle (2008).

While most of the above works focus on specific aspects of the mechanisms, Kojima and Unver (2010) take a general axiomatic approach to understanding the Boston mechanism, while Kojima and Manea (2007) do the same for deferred acceptance.

[JET:Txt 1]

Introduction

In a school choice problem, students have preferences for schools and in turn, schools ranklist students by their priorities. An allocation mechanism matches students with seats at schools. The Gale and Shapleystudent-proposing deferred acceptance algorithm (henceforth, DA) always selects the optimal stable matching for students. Nevertheless, it is well known that the DA matching is not necessarily Pareto efficient. Abdulkadiroglu et al. using NYC high school match data, show that in practice, such inefficiency is empirically significant. Kesten [12] proposes the school choice problem with consent, which seeks to improve the efficiency of the DA algorithm by obtaining students' consent to give up their priorities. We revisit this problem and offer a new perspective.

As is well known in the literature (see e.g.,, the inefficiency of DA may arise when certain cycles exist in schools' priority lists. Consequently, it may happen that during the DA algorithm procedure, some student i applies to school s and is tentatively accepted, but her tentative acceptance at s initiates a chain of rejections that eventually lead s to reject student i herself. By applying to school s, student i gains nothing, but potentially blocks trading among other students. In Kesten [12], i is called an "interrupter" at s.

In a school choice problem with consent, some or all students consent to give up their priorities at schools that are better than their assignments. To improve the efficiency of the DA algorithm, Kesten designs the efficiency-adjusted DA mechanism (henceforth, EADAM), which iteratively reruns DA after removing the last interruptions caused by consenting interrupters in the DA procedure. He then shows that no student has incentive to not consent under EADAM, and when all students consent, EADAM is Pareto efficient.

We take a new perspective on school choice with consent by directly examining consenting incentives. We observe that to make sure that students do not have incentives to not consent, we should use their consent only when they are (Pareto) unimprovable, so that their consent won't hurt their opportunities of being improved to better schools. This perspective brings us transparency in consenting incentives and makes the algorithms designed or interpreted based on this perspective more accessible to practitioners. For a given matching, to identify unimprovable students in a convenient way, we define underdemanded schools. We say that a school is underdemanded at a matching if no student prefers it to her assignment. Since Pareto improvements of any non-wasteful matching must take the form of trading cycles, students

matched at underdemanded schools at the DA matching are all unimprovable. Moreover, a school is underdemanded at the DA matching if and only if it never rejects any student during the DA procedure.

By focusing on unimprovable students, we propose a new algorithm - the simplified EADAM - for school choice problems with consent. If all students consent, this algorithm starts by running DA, and then iteratively reruns DA after removing students matched with underdemanded schools together with their assignments. If not all students consent, whenever we remove a non-consenting student, for each remaining school that she desires, we also make sure that the remaining students who have lower priorities than her are unacceptable to this school. We show that in each round, there exists at least one underdemanded school; therefore, at least one school will be removed. As a result, this algorithm stops within m + 2 rounds if there are m schools. We also show that the simplified EADAM is Pareto efficient when all students consent and is constrained efficient otherwise. A matching is constrained efficient if it does not violate the priorities of non-consenting students, but any matching that Pareto dominates it does.

Although the simplified EADAM and Kesten's EADAM differ in several ways, they share the same iterative structure and, more fundamentally, they can be unified under the perspective of focusing on unimprovable students. To show the latter, we prove in a lemma that the lastly rejected interrupters of the DA procedure are all matched with essentially underdemanded schools and hence are unimprovable at the DA matching. Therefore, under both mechanisms, even if a student consents, her consent will be used only after her assignment becomes unimprovable. Consequently, her consent decision can only affect other students' assignments, but not her own assignment. This argument makes the mechanisms' consenting incentives transparent and renders them more accessible to practitioners. We then show that the two mechanisms are outcome equivalent and that this equivalence holds more generally among mechanisms designed by focusing on unimprovable students.

We also apply the simplified EADAM to school choice with weak priorities, following the works of Erdil and Ergin [8] and Kesten [12]. The goal is to recover the efficiency loss of DA caused by fixed tie-breaking. We begin by transforming the problem by assuming that no student consents to give up priorities, except at tied priorities. In the adaptation of the simplified EADAM, we iteratively rerun DA after removing students matched at underdemanded schools and making them yield tied priorities to the remaining students. This adapted algorithm can be viewed as a stable improvement cycles mechanism proposed by Erdil and Ergin, but with endogenous cycle selection. Since this algorithm recovers all efficiency loss caused by the inappropriate tie-breaking of each round's removed students at once - and because at least one school is removed in each round - it stops very quickly.

Our contribution is to propose a perspective on designing for school choice with consent, and to design new mechanisms and interpret existing mechanisms based on that perspective. Bando shows that the EADAM outcome can be supported by a strictly strong Nash equilibrium of the preference revelation game

under DA, and along the way independently proposes another simplification of Kesten's EADAM. Bando's algorithm focuses on the removal of the last-step proposers of the DA algorithm, instead of the lastly rejected interrupters. Since last-step proposers are unimprovable, his approach can also be unified under our perspective of focusing on unimprovable students. Kesten and Kurino [13] define underdemanded schools in the same way as we do and they are the first to introduce this concept; they also study some general properties of this concept. However, they have a different motivation in mind - by restricting the preference domain, they try to resolve the trade-off between strategy-proofness and Pareto efficiency of DA. Our paper also relates to other literature that studies the inefficiency of DA; for example, Kesten Abdulkadiroglu et al.and Erdil [7], among others. Kesten and Kurino offer a detailed review of the literature on the trade-off between strategy-proofness and efficiency in improving on DA.

This paper is organized as follows. We introduce the basic model and Kesten's EADAM in Sectionand define underdemanded schools and the simplified EADAM in Section In Section we present our main results and the application of the simplified EADAM on weak priorities. Sectionconcludes. All proofs are relegated to

[JET_Txt 2]

The matching with contracts framework is a key model in recent market design research. It has been successfully applied to the matching of cadets to branches in the United States Military Academy and the Reserve Officer Training Corps as well as to the design of affirmative action matching mechanisms in school choice . Despite this practical success, some theoretical questions about the model have not been satisfyingly answered. In particular, it is not clear to what extent the model is more general than the job matching model of

In the discussion of their job matching model with salaries, already discuss the possibility to extend their model to allow for arbitrary "endogenous job char acteristics", i.e. multidimensional contracts. Moreover, they give an informal discussion of how to reduce this seemingly more general model to their original framework. As long as agents' preferences allow for "a well-behaved utility possibility frontier" and contracts are "negotiated efficiently" a parameterization of the utility possibility frontiers for matched firms and workers can be interpreted as salaries and the original results extend to the more general framework. Recently, it was shown that this informal argument can be made precise and that the assumption that contracts are substitutes for firms is important for the argument . Under this assumption, there exists an embedding that assigns to each market with contracts a corresponding market with salaries such that the set of stable allocations of the market is invariant under the embedding. Moreover, the gross substitutability condition that is the key assumption of the analysis ofis satisfied in the market with salaries. This result can be extended to many-to-many models of matching with contracts .

The embedding results show that, under the assumption of substitutability, the matching with contracts model is essentially not more general than the matching model with salaries. Nevertheless, substitutability is not the most general condition for which the key results of the theory of many-to-one matching with contracts hold. The main results of the theory can be proved under the weaker condition of unilateral substitutability, respectively under the even weaker condition of bilateral substitutability .Furthermore, these weaker substitutability conditions play a central role in recent market design applications .

We extend the result ofand show that a market where contracts are unilateral substitutes for firms can be embedded into a market with salaries where workers are gross substitutes for firms. In particular, the result applies to the cadet-to-branch matching problem studied by We show that under a weaker notion of embeddability, an embedding is possible even when contracts are bilateral substitutes for firms. For this purpose, we introduce a natural condition for the demand in market with salaries, net substitutability, that guarantees the convergence of a descending auction to a stable allocation. We then show that a market where contracts are bilateral substitutes for firms can be embedded into a market with salaries where workers are net substitutes for firms. Both results are the most general that we can hope for. We show that, for the embedding method proposed in this paper, unilateral substitutability is necessary for an embedding into a market with gross substitutability and that bilateral substitutability is necessary for an embedding into a market with net substitutability. In this sense, the results of this paper clarify to what extent the model with contracts is more general than the model with salaries.

The main technical contribution of this paper is a new embedding technique that does not rely on separabilitof firm preferences, i.e. the property that a firm's ranking of contracts with a given worker is independent of the contracts it signs with other workers. Separability in many-to-one markets with contracts is implied by substitutability but not by unilateral substitutability . Thus, under unilateral substitutability we cannot use the embedding construction ofand The underlying observation behind our embedding is that firm preferences can be modified such that they become essen tially separable while the essential properties of the market (in particular, the set of efficient and individually rational allocations and preferences over those allocations) remain unchanged.

Independently of establishing an equivalence between matching with contracts and with salaries, the result also gives new insight into the matching model with contracts itself. For the case where contracts are unilateral substitutes for firms, we introduce a new class of mechanisms (corresponding to the salary-adjustment process of that can be interpreted as versions of the firm-proposing mechanism of

In addition, we also provide new insights in the structure of the set of stable allocations when contracts are unilateral substitutes for firms. Under this condition, the set of stable allocations forms an upper semi-lattice with respect to worker preferences . We show that the semi-lattice can be completed to a

lattice by altering firm preferences in a very particular way. It turns out that there are modified preferences under which contracts are substitutes for firms and that differ from the original preferences only in so far as they alter the ranking of allocations that were previously Pareto dominated through a change of contract terms. Every stable allocation under the original preferences is still stable under the modified preferences. Modifying the preferences in this way may extend the set of stable allocations. Yet, only stable allocations are added that match the same agents as a previously stable allocation under different contracts. This provides some intuition why an embedding is still possible under unilateral substitutability.

Footnotes

Unilateral substitutability is sufficient for the existence of a worker-optimal stable allocation. Unilateral substitutability and the law of aggregate demand are sufficient for the rural hospitals theorem, group-strategy-proofness and weak Pareto optimality of the cumulative offer mechanism for the workers. Bilateral substitutability guarantees the existence of stable allocations.

Other authors starting withuse the term “Pareto separability”.

The approach taken here resembles the approach ofwho also study preference changes in many-to-one matching markets that leave the set of stable allocations invariant. One difference is that consider classical matching markets without contracts.

[JME_Txt 1]

Introduction

We study two-sided matching problems. ‘Stability’ of outcomes in these problems is considered to be the main property that accounts for the success of matching rules. We identify a large and maximal preference domain for which ‘underdemanded’ institutions (or agents) have the same partners at each stable outcome. Consequently, no stable rule can implement possibly desirable changes in the set of partners assigned to such institutions.

Our study is motivated by issues raised in certain centralized labor markets. As an illustration, many countries employ each year a centralized mechanism to assign newly graduated medical students to positions in residency programs. Hospitals in rural areas are typically less preferred than those in urban areas by medical graduates, i.e., they are ranked below urban hospitals in a typical student’s preference list. Also, graduates from relatively successful programs are more popular among hospitals, i.e., they are ranked above other students in a typical hospital’s preference list. Rural hospitals complain that their positions may not be filled by the stable matching rule in use and that they may not be assigned popular graduates. The ‘rural hospital theorem’ established in several matching models states that the number of medical graduates as-

signed to a hospital and the set of graduates assigned to a hospital in a rural area do not vary across stable outcomes. Even though the theorem's name is a useful reminder of its content and origin, the 'rural hospital theorem' equally applies to other labor markets with similar concerns about the numerical distribution of workers or the composition of the workforce of firms.

We study the 'rural hospital theorem' in the context of many-to- many labor markets, i.e., markets where each agent can engage in multiple partnerships. There are several reasons to focus on many- to-many markets instead of many-to-one markets (where each worker can be employed by at most one firm). First, a well-known many-to-many market is the medical labor market in the UK. More specifically, each medical graduate in the UK has to seek two positions (a medical position and a surgical position) to be able to register as a medical doctor. () mentioned concerns of doctor shortages in rural areas in the UK. Second, as pointed out by (), even if in a labor market most workers are employed by one firm, the presence of a few workers with multiple employers can make a crucial difference. Precisely, (Example 2.2) showed that the presence of only one worker with two part-time jobs can already change the stable outcome for all other agents. Thus, the functioning of even 'almost many-to-one' labor markets can only be understood through the study of many-to-many matching models. Third, the literature on many-to-many matching markets has grown in the last couple of years, but there is still a wide gap with respect to many-to-one markets. Fourth, there are important structural differences between many-to-one and many-to-many matching markets, even if all agents have so-called 'responsive' preferences. For instance, () showed that unlike many-to-one markets, in many-to-many markets the set of stable outcomes need not coincide with the core. Finally, our results are not only novel to the many-to-many framework. Indeed, the restriction of all our results to the many-to-one framework yields new results and subsumes existing results for that framework as well.

Next, we describe in more detail the model we study, the existing literature, and our contribution. In a two-sided many-to-many matching problem there are two disjoint sets of agents, which we call 'firms' and 'workers'. Each firm (worker) can only form partnerships with workers (firms). Each agent has a preference order over the set of all subsets of partnerships, i.e., subsets of agents in the other set. For each agent, there is a maximal number ('quota') of partnerships the agent can or is willing to be involved in. An out- come of the problem is a 'match' which consists of a collection of partnerships.

A match is 'stable' if no agent prefers to be matched to a proper subset of its current partners, and no set of firms and workers prefers to establish new partnerships only among themselves and possibly break up some existing partnerships. This definition is more stringent than so-called pairwise stability which is another standard solution concept but that only eliminates blocking by firm–worker pairs. Stability proved to be a crucial property in many entry-level labor markets where workers are matched to firms through a clearinghouse. It has been observed that clearing- houses that use stable rules often perform better than those

that use rules that do not necessarily produce stable matches. Accord- ing to (p. 422) even the weaker stability concept, pair- wise stability, can be of primary importance for many-to-many markets as well.

There are many-to-many problems for which no stable match

exists. Certain assumptions on preferences have been identified to guarantee that they do. A firm's preferences are 'substitutable' if whenever a worker is chosen from a group of workers by this firm, she is also chosen from any of the group's subsets to which she belongs. Substitutability of workers' preferences is defined similarly. Substitutability is a standard assumption in the literature and it guarantees the existence of a pairwise stable match. () showed that for substitutable preferences, stability and pairwise stability are equivalent. Thus, when preferences are substitutable, the set of stable matches is non-empty and coincides with the set of pairwise stable matches. With the important exception of , we assume substitutability throughout.

Taking the requirement of stability as granted, an important question is whether the choice of a particular stable rule affects the numerical distribution of workers; and if not, whether different matches assign different sets of workers to a firm that does not fill all its positions. For instance, in the context of the assignment of medical graduates in the US, the National Resident Matching Program (NRMP) failed to fill the posts of many hospitals in (typically less preferred) rural areas (Sudarshan). However, provided that the preferences satisfy certain conditions, the problem of the rural hospitals cannot be attributed to the particular stable rule used by the NRMP. Indeed, the results obtained by () and , suggest that any other stable rule would yield (R1) the same numerical distribution of medical graduates and would assign (R2) the same medical graduates to each rural hospital that does not fill all its posts. The two results are known as weak and strong versions of the rural hospital theorem.

Both versions of the rural hospital theorem play a functional

role in proving many appealing results. For instance, R1 is used to show the lattice structure of the set of stable matches (Martínez) and the group strategy-proofness (for the workers' side) of the worker-optimal stable rule (Martínez) in a many-to-one model. () studied refinements of Nash equilibrium based on 'truncations at the match point' for the preference revelation game induced by any stable rule. He used R2 to prove that each equilibrium outcome is stable for the true preferences. () studied ordinal Nash equilibria of the preference revelation game induced by any probabilistic stable rule. She used R2 to show that any equilibrium induces a match that is individually rational for the true preferences. () also employed R2 to extend the latter result to many-to-many matching with a more general preference domain. These results show that the relevance of the rural hospital theorem goes beyond its direct interpretation: it is a powerful tool in establishing structural results and analyzing strategic matching games.

The first papers on the rural hospital theorem (e.g., ; , ,) studied many-to-one match- ing problems and assumed firms' preferences to be 'responsive'. A firm's preferences over groups of workers are

responsive to its preferences over individual workers if (i) for two groups that only differ in one worker, the firm prefers the one with the pre- ferred worker, and (ii) adding an acceptable (unacceptable) worker to a group that does not fill its quota makes the group better (worse). Responsiveness implies substitutability. Several papers have shown R1 and R2 for preference domains that are strictly larger than the domain of responsive preferences. A firm's preferences are 'separable' if condition (ii) above holds. R1 and R2 hold for substitutable and separable preferences (Martínez , Proposition 2). Since responsiveness implies separability, () result subsumes the previous rural hospital theorem results.

Concerning the many-to-many framework, (Propo- sition 6) showed that R1 holds for substitutable and 'cardinally monotonic' preferences. A firm's preferences over groups of workers are cardinally monotonic if whenever the group of workers available to the firm expands, it will not employ fewer workers.

Since separability implies cardinal monotonicity, (, Proposition 6) many-to-many result subsumes (, Proposition 2a) many-to-one result on R1. On the other hand, R2 has only been shown to hold for responsive preferences (Alkan, , Proposition 2i) and for so-called 'quota-filling' prefer- ences that satisfy separability (Alkan, , Corollary 1). The latter two results on R2 do not subsume (, Proposi- tion 2b) many-to-one result on R2.

The contribution of our paper is twofold. We first introduce a new preference domain called weak separability by relaxing separability. We prove that the strong rural hospital theorem, i.e., R2, holds on the domain of substitutable and weakly separable preferences (). Thus, our result generalizes the results of (Proposition 2b) for many-to-one matching and (, Proposition 2i and , Corollary 1) for many- to-many matching. Our short proof is based on a strong structural result regarding the set of stable matches due to ().

Our second contribution shows that the two largest domains for R1 and R2 discussed above are in fact maximal (in a sense made precise below). First, we provide a maximal domain result that complements 's (, Proposition 6) result regarding R1 and cardinal monotonicity. Precisely, if some agent's preferences do not satisfy cardinal monotonicity then we construct substitutable and cardinally monotonic preferences for the other agents such that R1 fails (). Second, we provide a maximal domain result that complements our regarding R2 and weak separability. Precisely, if some agent has substitutable preferences that are not weakly separable then we construct substitutable and weakly separable preferences for the other agents such that R2 fails (). In fact, our two maximality results are stronger in two ways: (1) the constructed preferences are responsive and (2) the proofs are also effective for the many-to-one framework (and yield novel results in that framework as well).

Concerning many-to-one matching with contracts, (Theorems 8 and 9) proved R1 for substitutable and cardinally monotonic preferences and established a maximality result. () introduced a weaker condition than substitutability called bilateral substitutability. (Theorem 6) extended R1 to bilaterally substitutable and cardinally monotonic preferences for many-to-one matching with contracts. () studied many- to-many

matching with contracts where multiple contracts can be signed between any firm–worker pair. (Section 4.2) also obtained R1 for substitutable and cardinally monotonic preferences. In matching without contracts, substitutability and bilateral substitutability are equivalent. Thus, in that framework, (Theorem 9), (Theorem 6), and (2012a, Section 4.2) boil down to the earlier mentioned result of Alkan (2002, Proposition 6) for many-to-one and many-to-many

matching. In Remark 8 we show that in the framework without contracts, Hatfield and Milgrom (2005, Theorem 9) does not imply nor is implied by our Proposition 1.

Hatfield and Kominers (2012b) studied matching in networks with bilateral contracts: agents trade goods via contracts and each agent may be both a seller and a buyer of a good. Their Theorem 8 shows that a 'generalized version of R1' holds if preferences sat- isfy 'same-side and cross-side substitutability' and two laws of aggregate demand and supply. In terms of two-sided many-to-many matching without contracts, their result boils down to Alkan (2002, Proposition 6). (Theorem 9) also proved a maximality result similar to . More precisely, if some agent's preferences violate the law of aggregate demand or supply but do satisfy same-side and cross-side substitutability, then there are same-side and cross-side substitutable preferences for the other agents satisfying the laws of aggregate demand and supply such that the generalized version of R1 fails. In we show that in terms of two-sided many-to-many matching without contracts (Theorem 9) does not im- ply our .

In Section , we present the model. In Section , we formally introduce and relate the aforementioned preference domains. In Section , we state and prove our results on the rural hospital theorem.

Footnotes

Recent papers on many-to-many matching include, among others, , (), (), (), (), (), (), Sotomayor (), and ().

See the discussion that precedes and and .

This is an adaptation of the stability definition in ().

See, for instance, ().

Substitutability is an adaptation of the gross substitutability property (

) by () and () to matching problems without monetary transfers.

The existence of a pairwise stable match can be shown via an algorithm for strict

preferences ( ) and via a non-constructive proof for non-necessarily strict preferences ( ). See also () for the computation of the full set of pairwise stable matches.

We are thankful to a referee for pointing this out.

Since R2 implies R1, R1 (R2) is often referred to as the weak (strong) rural hospital theorem.

For the reader's convenience, we refer to the Venn diagram of (in

Section) which depicts the inclusion relations among the preference domains we discuss.

() also introduced the domain of separable preferences with so-

called affirmative action constraints. This domain is a strict superset of the domain of separable preferences but a strict subset of the domain of cardinally monotonic preferences. () showed that on his domain an appropriately adjusted version of R2 holds.

Cardinal monotonicity is called size monotonicity and law of aggregate demand

in () and (), respectively

[JME_Txt_2]

Introduction

Two-sided, many-to-one matching models study assignment problems where a finite set of agents can be divided into two disjoint subsets: the set of institutions (called firms) and the set of individuals (called workers). Each firm has a preference relation on all subsets of workers and each worker has a preference relation on the set of firms plus the prospect of remaining unmatched. A preference profile is a list of preference relations, one for each agent. A matching assigns each firm with a subset of workers (possibly empty) in such a way that each worker can work for at most one firm. Given a preference profile a matching is called stable if all agents have acceptable partners (individual rationality) and there is no unmatched worker- firm pair who both would prefer to be matched to each other rather than staying with their current partners (pair-wise blocking).

The "college admissions model with substitutable preferences" is the name given byo the most general many-to-one model with ordinal preferences in which stable matchings exist. Each firm is restricted to have a substitutable preference relation on all subsets of workers; namely, each firm continues to want to employ a worker even if other workers become unavailable (ere the first to use this property in a more general model with money). For each substitutable preference profile the deferred-acceptance algorithms produce either the firms-optimal stable matching or the workers-optimal stable matching, depending on whether the firms or the workers make the offers. The firms (workers)-optimal stable matching is unanimously considered by all firms (respectively, workers) to be the best matching among all stable matchings.

A more specific many-to-one model, called the "college admissions problem" byupposes that firms have a maximum number of positions to be filled (their quota), and that each firm, given its ranking of individual workers, orders subsets of workers in a responsive manner; namely, for any two subsets that differ in only one worker a firm prefers the subset containing the most preferred worker.n this model the Blocking Lemma says the following. Fix a responsive preference profile. Suppose that the set of workers that strictly prefer

an individually rational matching to the workers-optimal stable matching is nonempty. Then, we can always find a firm and a worker with the following properties: (a) the firm and the worker block the individually rational matching, (b) the firm was hiring another worker who strictly prefers the individually rational matching to the workers-optimal stable matching, and (c) the worker (member of the blocking pair) considers the workers-optimal stable matching to be at least as good as the individually rational matching. The interest of the Blocking Lemma lies in the fact that it is an instrumental result to prove key results on matching. For instance, the fact that in the college admissions problem the workers-optimal stable mechanism is group strategy-proof for the workers (Dubins and Freedman, 1981)nd the strong stability theorem in the marriage model (Demange et al., 1987) follow directly from the Blocking Lemma. The first result says that if in centralized markets (like entry-level professional labor markets or the admission of students to colleges) a mechanism selects for each preference profile its corresponding workers-optimal stable matching then, no group of workers can never benefit by reporting untruthfully their preference relations. This is an important property and it becomes critical if the market has to be redesigned, in which case the declared preference profile conveys very valuable information. The second result says that every non-stable matching is either nonindividually rational or we can identify a blocking pair (a firm and a worker) and another stable matching such that both members of the blocking pair weakly prefer to the original one.

It is known that the Blocking Lemma does not hold for the many-to-one matching model with substitutable preference profiles. The purpose of this paper is two-fold. First, we consider a weaker condition than responsiveness, called quota q-separability, that together with substitutability implies that the Blocking Lemma holds for all these preference profiles (firm is said to have a separable preference relation over all subsets of workers if its partition between acceptable and unacceptable workers has the property that only adding acceptable workers makes any given subset of workers a better one. However, in many applications such as entry-level professional labor markets, separability alone does not seem very reasonable because firms usually have fewer openings (their quota) than the number of “good” workers looking for a job. In these cases it seems reasonable to restrict the preference relations of firms in such a way that the separability condition operates only up to their quota, considering unacceptable all subsets with higher cardinality. Moreover, while responsiveness seems the relevant property for extending an ordered list of individual students to a preference relation on all subsets of students, it is too restrictive, though, to capture some degree of complementarity among workers, which can be very natural in other settings. The quota q-separability condition permits greater flexibility in going from orders on individuals to orders on subsets. For instance, candidates for a job can be grouped together by areas of specialization. A firm with quota two may consider as the best subset of workers not the set consisting of the first two candidates on the individual

ranking (which may have both the same specialization) but rather the subset composed of the first and fourth candidates in the individual ranking (i.e., the first in each area of specialization).

Second, we show (in that the Blocking Lemma holds on a subset of substitutable preference profiles (not necessarily quota q-separable) if and only if the workers-optimal stable mechanism is group strategy-proof for the workers on this subset of profiles. This means that, in contrast with what the literature has considered so far, the Blocking Lemma is more fundamental than just a key step to prove general results like group strategy-proofness of the workers-optimal stable mechanism for the workers. Observe that our former result (Martínez et al., 2004) showing that the workers-optimal stable mechanism is group strategy-proof for the workers on the set of substitutable and quota q-separable preference profiles was proved assuming that the Blocking Lemma was true on the set of all these profiles. Hence, nd our former result does not imply that the Blocking Lemma holds on the set of all substitutable and quota q-separable preference profiles. tates that this is indeed the case.

The paper is organized as follows. In Section 2, which closely followse present the preliminary notation and definitions. In Section 3 we present the Blocking Lemma and state, in hat it holds on the set of all substitutable and q-separable preference profiles. In Section 4 we state and prove the equivalence, on any subset of substitutable preference profiles (not necessarily quota q-separable), between the Blocking Lemma and group strategy-proofness

of the workers-optimal stable mechanism for the workers. In Section 5 we conclude with an example of a substitutable and quota q-separable preference profile for which the symmetric Blocking Lemma for the firms does not hold. We collect all proofs in two Appendices at the end of the paper.

Footnotes

1. Observe that the marriage model (i.e., the one-to-one matching model) is a particular instance of the "college admissions problem" when all firms have quota one.

2. To be precise, they show it forthe marriage model, but their result can be extended to the college admissions problem. Some results concerning stability in the college admissions problem are immediate consequences of the fact that they hold forthe marriage model. Each college is split intoas many pieces as positions it has, so transforming the original many-to-one model into a one-to-one model. Responsiveness allows then the translation of stability from one model to another. Seer a complete description of this procedure as well as for its applications. Observe that this reduction is possible only if preferences are responsive.

3. We have already showed that if firms have substitutable and quota q-separable preference profiles then, (a) the set of unmatched agents is the same in all stable matchings (Martínez et al., 2000), (b) the set of stable matchings has a lattice structure with two natural binary operations (Martínez et al., 2001), (c) the

workers-optimal stable matching is weakly Pareto optimal for the workers, relative to the set of individually rational matchings (Martínez et al., and (d) the workers-optimal stable mechanism is group strategy-proof for the workers (Martínez et al., 2004). This last result is proven assuming that the Blocking Lemma holds for all substitutable and quota q-separable preference profiles; here, we are providing a proof that this is indeed the case.

[IJGT_Txt 1]

Introduction, main concepts and results

In a decentralized setting in which players can interact with each other and get together in groups, the game theoretic predictions are that a matching that can be upset by a coalition will not occur. The outcome of such coalitional interactions should then be a stable matching, if it exists. However, such predictions should be revised in the cases in which preferences are not necessarily strict. In such cases, it is justifiable that recontracts between pairs of agents already allocated according to a stable matching, leading to a weak Pareto improvement of the original matching, should be allowed. In this context, it makes sense to predict that only Pareto-stable matchings, i.e. stable matchings that are Pareto optimal, will occur.

This paper takes up this approach and proves some characteristic properties of the Pareto-stable matchings. It concentrates on the well-known Roommate and Marriage models, both introduced by Gale and Shapley in their famous paper of 1962. We follow the notations and concepts presented in (). The Roommate model is described as the pair (N, P), where N 1, 2,.. ., n is the set of players and P is the set P(1),.. ., P(n) , where P(j) is an ordered list of preferences (strict or non-strict) for player j . The Marriage model is regarded as a sub-model of the Roommate model in which N M W , M is a set of men and W is a set of women. For the sake of exposition the main concepts will be introduced along this section, as well as the main results of this paper, which will be presented, motivated, discussed and illustrated with examples. We will not always provide a formal statement. The intuitive proofs will be provided here and the technical proofs will be presented in the next section.

To figure out the kind of coalitional interaction taking place among agents allocated according to a stable matching that is not Pareto-optimal, see Example below.

Example 1 (Pareto-stability is a natural solution concept for the roommate model) Consider a decentralized setting where a set of eight boys, 1, 2,.. ., 8, wish divide up into pairs of roommates. The boys' preferences over acceptable partners are rep- resented by the following ordered lists, where P(j) denotes boy j 's list for all j

1,..., 8:

P(1) = 8, 2, 1 P(5) = 8, 6, 5,

P(2) = [3, 1], 2 P(6) = [3, 5], 6

P(3) = 2, 6, 4, 3 P(7) = 4, 8, 7

P(4) = [3, 7], 4 P(8) = [1, 5, 7], 8

The brackets in the preference lists of boys 2, 4, 6 and 8 mean that these agents are indifferent among the boys inside the brackets. The matching z, where z(1)

2, z(3) 4, z(5) 6, z(7) 8, doesn't have any blocking pair, so it is stable. This means that no two boys can be both better off by becoming roommates.

However, we cannot expect to observe this matching as the final outcome. In fact, boy 3 prefers boy 6 to his partner, boy 4; in his turn boy 6 is indifferent between boy 3 and his partner, boy 5; boy 5 prefers boy 8 to his partner, boy 6; boy 8 is indifferent between boy 5 and his partner, boy 7; on the other hand boy 7 prefers boy 4 to his partner, boy 8 and boy 4 is indifferent between boy 7 and his partner boy 3. Thus, boys 3, 5 and 7 can act together and be better off by exchanging their partners 6, 8 and 4 among them. It is natural to expect that this exchange will be accepted by boys 6, 8 and 4, since these boys are indifferent between their current partners under z and the new proposed mates. It is then reasonable to expect that these boys will form a new set of partnerships, 3, 6 , 5, 8 and 7, 4 , and that matching w, such that w(1) 2, w(3) 6, w(5) 8 and w(7) 4, will be the resulting matching of this coalitional interaction. Matching w is a weak Pareto improvementof matching z via coalition 3, 4, 5, 6, 7, 8 , which weakly blocks matching z. Since a weak Pareto improvement of a matching does not create any blocking pair, and z is stable, then matching w is also stable.

Considering that an exchange of partners is acceptable if it does not hurt anybody, it is then evident that an exchange of partners is acceptable only if (1) the agents involved are either all indifferent between their current partners and the new ones or they form a weak blocking coalition and (2) by matching the agents of the weak blocking coalition among them in an appropriate way, a weak Pareto improvement of the current matching is obtained.

Having this in mind observe that once matching w is reached no more acceptable

exchange of partners is possible. In fact, boys 7 and 5 are assigned to their first choice, so there is no acceptable exchange involving these boys and their partners. On the other hand, any exchange involving some of the remaining boys will necessarily involve boy 8, partner of boy 5, who will not accept such pairwise interaction. Hence, although z and w are stable, only w can be expected to occur.

The pairs {3,2} and {1,8} are the only weak blocking pairs of matching w but the coalition {3,2,1,8} does not produce any weak Pareto-improvement of w. Matching z is also weakly blocked but only w is Pareto-stable.

Observe that in this example coalition {1,2,3,4,7,8} also weakly blocks matching z, and yields a weak Pareto-improvement given by the matching wj, which assigns 1–8, 2–3, 4–7 and 5–6. This new matching is also Pareto-stable. No more acceptable exchange of partners will occur. hh

The Pareto-stability concept can be viewed as an intermediate concept between the stability concept and the strong-stability concept. In fact, the set of strongly stable matchings is contained in the set of Pareto-stable matchings, since if a stable matching is not Pareto-optimal then it has a weak Pareto-improvement via some weak-blocking coalition. When preferences are strict, these two sets coincide with the set of stable matchings, because there is no weak blocking coalitions. With indifferences, the pre- vious example illustrates that the set of strongly stable matchings may be a proper subset of the set of Pareto-stable matchings, which may be a proper subset of the set of stable matchings. In that example the set of strongly stable matchings is empty.

It is immediate that Pareto-stable matchings exist if and only if the set of stable matchings is non-empty. In fact, starting at any stable matching that is not Pareto opti- mal, a finite sequence of weak Pareto-improvements leads to a Pareto-stable matching. This is due to the fact that any weak Pareto improvement of a stable matching is still stable and the set of stable matchings is non-empty by assumption, it is finite and preferences are transitive. Consequently, a Pareto-stable matching always exists for the Marriage model.

Assuming we have a stable matching, a natural question is how to test it for Pareto optimality. Clearly, if x is a stable matching then matching z is a weak Pareto improve- ment of x if: (i) the set S j N z(j) x(j) is a weak blocking coalition of x ;

(ii) x(S) z(S) S; (iii) if j, k S and z(j) k then (j, k) is a weak blocking pair of x or both agents are indifferent between each other and their mates under x and (iv) if j S and j is unmatched under z then j must be indifferent between being unmatched and being matched to x(j). Equivalently, given a stable matching x , we can say that x is Pareto optimal if none of the following requirements occurs:

There are sequences (j1, j2,..., jq) and (k1, k2,..., kq) with x(j1) kq , x(jt) kt 1 for all t 2,..., q, and such that either (jt , kt) is a weak blocking pair of x or both agents are indifferent between each other and their mates under x , for all t 1,..., q. Moreover, (jt , kt) is a weak blocking pair of x for some t 1,..., q.

There are sequences (j1, j2,..., jq) and (k0, k1,..., kq) where kq is unmatched under x, x(jt) kt−1 for all t 1,..., q, k0 is indifferent between being unmatched at x and being matched to j1 x(k0), and either (jt , kt) is a weak

blocking pair of x or both agents are indifferent between each other and their mates under x , for all t = 1,..., q. Moreover, (jt , kt) is a weak blocking pair of x for some t = 1,..., q.

There are sequences (j1, j2,..., jq+1) and (k1,..., kq) where j1 is unmatched under x, x(jt) kt−1 for all t 2,..., q 1, jq+1 is indifferent between being unmatched at x and being matched to kq x(jq 1), and either (jt , kt) is a weak

blocking pair of x or both agents are indifferent between each other and their mates under x , for all t = 1,..., q. Moreover, (jt , kt) is a weak blocking pair of x for some t = 1,..., q.

In fact, if (1) occurs a weak Pareto improvement of x is obtained by matching jt to kt , for all t 1,..., q and keeping the other matches. If (2) occurs then a weak Pareto improvement of x is obtained by matching jt to kt , for all t 1,..., q, leav- ing k0 unmatched and keeping the other matches. If (3) occurs then a weak Pareto improvement of x is obtained by matching jt to kt , for all t 1,..., q, leaving jq 1 unmatched and keeping the other matches.

The remaining part of this paper is devoted to finding the main properties that characterize the Pareto-stable matchings for the Roommate and Marriage models. Our main finding concerns the role played by the simple matchings and Pareto-simple matchings in the characterization of such outcomes.Simple matchings can be defined as follows:

Definition 1 Matching x is simple (respectively, strongly simple) if it is individually rational and no matched agent is part of a blocking pair (respectively, weak blocking pair).

Simple matchings exist even when stable matchings do not, since the matching where everyone is unmatched is simple. Clearly, every stable matching is simple.

The concept of Pareto- simple matching is the following:

Definition 2 An individually rational matching z extends the individually rational matching x if z is a weak Pareto improvement of x . If z and x are simple we say that z is a simple extension of x . A matching x is Pareto-simple if it is simple and does not have any simple extension.

That is, matching x is Pareto-simple if it is simple and it is not weakly-dominated by any other simple matching. Pareto-simple matchings always exist since the set of simple matchings is non-empty, finite and preferences are transitive. Correspondingly, matching x is called Pareto-strongly simple if it is strongly simple and it is not weakly dominated by any strongly simple matching.

The following example, due to (), shows that the set of Pareto-simple matchings may be disjoint from the set of Pareto-optimal matchings, as well as from the set of Pareto-stable matchings.

Example 2 (The set of Pareto-simple matchings, the set of Pareto-optimal matchings and the set of Pareto-stable matchings are disjoint) Consider the Roommate model where the set of boys is N 1, 2, 3, 4 . The boys' preferences over acceptable partners are given by:

P(1) = 2, 3, 4, 1 P(3) = 1, 2, 4, 3

P(2) = 3, 1, 4, 2 P(4) = arbitrary

The set of Pareto-stable matchings is empty because the set of stable matchings is empty. There is no Pareto-simple matching that is Pareto-optimal. In fact, matching x where every agent is unmatched is the only

simple matching because any other match- ing has a blocking pair where at least one boy is matched. Then it is Pareto-simple. However, it is not Pareto-optimal since it is weakly dominated by, for example, matching x 1, which matches 1–2 and 3–4. Matching x 1 is Pareto-optimal but it is not simple. The set of Pareto-optimal matchings also includes x 2, which matches 1–3 and 2–4 and x 3 which matches 1–4 and 3–2. hh

The set of Pareto-stable matchings may be a non-empty proper subset of the set of Pareto-simple matchings and of the set of Pareto-optimal matchings, as illustrated in the example below.

Example 3 (Pareto-stable matchings is a non-empty proper subset of the set of Pareto- simple matchings and of the set of Pareto-optimal matchings.) Consider the Room- mate model where the set of boys is N 1, 2,.. ., 6 . The boys' preferences over acceptable partners are given by:

P(1) = 2, 3, 1 P(4) = [5, 6], 4

P(2) = 3, 1, 2 P(5) = 4, 3, 5

P(3) = 1, 5, 2, 3 P(6) = 4, 6

The set of stable matchings is non-empty since matching y, such that y(1)

2, y(3) 5 and y(4) 6, is stable. This is the only stable matching for this market. Since any weak Pareto-improvement of y must be stable then y is Pareto-optimal, so it is Pareto-stable and Pareto-simple. The pair {5,4} weakly blocks y, so the set of strongly stable matchings is empty. Now, let yj be the matching that assigns 5–4 and leaves unmatched the other boys. It is easy to see that yj is simple and unstable. On the other hand, there is no way to extend yj to a simple matching. In fact, boy 5 is matched to his first choice. Consequently, any weak-Pareto-improvement of yj will only involve the unmatched boys. However, any arrangement with these boys will have a blocking pair where at least one boy is matched. Then, any weak-Pareto-improvement of yj is not simple, so yj is a Pareto-simple matching. Since it is not stable then it is not Pareto-stable. Matching yj is not Pareto-optimal, since matching zj that assigns 5–4, 1–2 and leaves unmatched the other agents, for example, is a weak-Pareto improve- ment of yj. However, matching zj is not simple since the pair {2,3} blocks it and boy

2 is matched. Then, zj is Pareto-optimal but it is not Pareto-stable. hh

As these examples suggest, the set of Pareto-stable matchings is the intersection of two non-empty Pareto sets:

Theorem 1 The set of Pareto-stable matchings equals the intersection of the set of Pareto-simple matchings with the set of Pareto-optimal matchings.

The proof of this result is straightforward. If a matching is Pareto-stable then it is simple and it is not weakly dominated by any individually rational matching, in particular it is not weakly dominated by any simple matching, so it is a Pareto-simple matching. Conversely, if a matching is simple and Pareto-optimal then it

must be stable, since otherwise it would have a blocking pair formed with unmatched agents and so, by matching these agents with each other, we would get a weak-Pareto-improvement of the given matching, which would contradict its Pareto-optimality.

Thus, by Theorem , in order to show that Pareto-stable matchings exist it is suffi- cient to find just one Pareto-simple matching that is Pareto-optimal. It turns out that under strict preferences, if Pareto-stable matchings exist then every Pareto-simple matching must be Pareto-optimal, so every Pareto-simple matching must be stable. In fact, Theorem provides a characterization of the set of Pareto-stable matchings as the set of Pareto-simple matchings. For the Roommate model it is required strictness of the preferences and non-emptiness of the set of stable matchings. For the Marriage model it is not imposed any restriction.

Theorem 2 (a) Consider the Roommate model with strict preferences and suppose the set of stable matchings is non-empty. Then the set of Pareto-stable matchings equals the set of Pareto-simple matchings.

(b) Consider the Marriage model. Then the set of Pareto-stable matchings equals the set of Pareto-simple matchings.

The idea of the proof of this result is to show that every Pareto-simple matching is stable. If this is established then every Pareto-simple matching is Pareto optimal, since otherwise there would be a weak Pareto improvement of it, which would still be stable, so it would be simple, which is a contradiction. This is equivalent to show that every unstable and simple matching has a simple extension:

Proposition 1 (a) Consider the Roommate model with strict preferences. If the set of stable matchings is non-empty then every unstable and simple matching has a simple extension.

(b) Every unstable and simple matching for the Marriage model has a simple extension.

The proof of this proposition is given in the next section. Unlike the other results of this paper it is not straightforward. It is easy to obtain an extension B of an unstable and simple matching A for the Roommate model. It is enough to keep the partner- ships formed under A, if any, and to add some new partnerships. Of course, these new partnerships are formed with blocking pairs of A. What is not clear is that if the set of stable matchings is non-empty and preferences are strict, then matching B can be constructed so that it is still simple. Without these requirements such construction of B is not always possible. Indeed, to match the correct blocking pairs of A is the inventive part of the proof. (Remember that matching x of Example and matching yj of Example are simple and unstable matchings but they cannot be extended to a simple matching. In the first case there is no stable matchings in the market and in the second case the preference of player 4 is not strict).

The proof of Proposition -(a) uses a key lemma. This is a technical result, which is a one-sided version of the Decomposition lemma for the Marriage model from Gale and Sotomayor (1985). For part (b), the proof strongly uses the fact that the Marriage model has two sides.

By Proposition , in order to conclude that the set of Pareto-stable matchings for the Roommate model with strict preferences is empty, it is enough to find just one Pareto-simple and unstable matching. See the example below.

Example 4 (An application of Proposition -(a)) Consider the Roommate model where the set of boys is N={1,2,…,7}. The boys' preferences over acceptable partners are given by:

P(1) = 5, 6, 1 P(4) = 6, 5, 4 P(7) = 2, 1, 3, 7

P(2) = 3, 7, 2 P(5) = 4, 1, 6, 5

P(3) = 7, 2, 3 P(6) = 1, 4, 6

The matching that assigns 4–5, 1–6 and leaves the other agents unmatched is simple and unstable. Any extension of this matching will match a pair of agents in {2,3,7}. However, one of the agents in the pair will form a blocking pair with the agent left unmatched. Hence, the original matching does not have a simple extension. Since the preferences are strict, we need not check that every Pareto-simple matching is unstable. (Observe that the matching that assigns 4–6, 1–5 and leaves the other agents unmatched is also Pareto-simple and unstable). Proposition implies that the set of

stable matchings is empty, so the set of Pareto-stable matchings is also empty. hh

The following corollary is then immediate:

Corollary 1 (a) Suppose the preferences in the Roommate model are strict. The set of stable matchings is non-empty if and only if every unstable and simple matching has a simple extension.

(b) The set of stable matchings for the Marriage model is always non-empty.

The fact that the condition in (a) is necessary is immediate from Proposition . It is sufficient since, if every unstable and simple matching has a simple extension then the Pareto-simple matchings must be stable. The conclusion follows since Pareto-simple matchings always exist. The proof of part (b) is immediate from Proposition a), since a Pareto-simple matching always exists and cannot have a simple extension.

It is easy to construct examples for the Marriage model where, as in the Room- mate model, the set of strongly stable matchings is empty. However, it is well known that the existence of two sides in the Marriage market causes fundamental differences between the two models. There are properties of the Marriage model which depend on the two-sidedness of the market, as the non-emptiness of the set of stable matchings under any kind of preferences and the lattice property of the set of stable matchings when preferences are strict. This last property guarantees the existence of the optimal stable matchings for each side of the market. Moreover, it implies that if the two opti- mal stable matchings coincide then the set of stable matchings is a

singleton. When preferences need not be strict, the lattice property may fail to hold even when the man-optimal and the woman-optimal stable matchings exist. Moreover, the man-optimal stable matching may coincide with the woman-optimal stable matching when the set of stable matchings is not a singleton. See the example below.

Example 5 (The woman-optimal and the man-optimal stable matchings coincide but the set of stable matchings is not a singleton) Consider the Marriage model where the set of agents are M m1, m2 and W w1, w2 . Agent m1 is indifferent between w1 and w2 m2 prefers w1 to w2 w1 is indifferent between m1 and m2 and w2 prefers m1 to m2. Both matchings under which no agent is unmatched are stable and are the only stable matchings. The matching y1 where y1(m1) w1 and y1(m2) w2 is not Pareto-optimal and is not strongly stable. It is weakly Pareto improved by matching y2 where y2(m1) = w2 and y2(m2) = w1. Matching y2 is strongly stable. Matching

y2 is clearly optimal for the men and for the women but matching y1 is also stable. hh

The key lemma mentioned above is also used in this paper to extend, to the Room- mate model with strict preferences, two well-known properties for the Marriage model with strict preferences. The first result reflects an opposition of interests between the two players involved in a partnership regarding two Pareto-stable matchings. It asserts that if x and y are Pareto-stable matchings and j prefers x to y then j is matched under both matchings and both of his mates prefer y to x . The second result implies that the set of trading agents at a simple matching can be regarded as a sort of stable coalition in the sense that such agents always make their transactions under a stable matching within the same pool. In particular, the set of matched agents under a Pareto-stable matching is the same under any Pareto-stable matching. The proof of both results will be given in the next section.

The present work also addresses the case of non-necessarily strict preferences. Sim- ilar results to those stated under the assumption of strict preferences, by focusing on strongly stable matchings and strongly simple matchings, are obtained and presented in Sect. . The proofs of these results follow the lines of the proofs of the correspond- ing results under strict preferences and are left to the reader. Some final conclusions and related work are presented in the last section.

Footnotes

Matching w is a weak Pareto improvement of matching z if everyone weakly prefers w to z and at least one player strictly prefers w to z.

We say that a matching is strongly stable if it is not weakly blocked by any coalition.

The idea of focusing on simple matchings has already been used in the literature for the proof of existence theorems in several matching models. (See the last section of this paper).

[IJGT_Txt 2]

Introduction

The canonical evolutionary game theory model of Maynard Smith and () plays an important role in biology, economics, political science, and other fields. Its equilibrium concept, an evolutionarily stable strategy (ESS) describes evolutionary outcomes in environments where populations are large and matching is uniformly random. Since an ESS is a refinement of Nash equilibrium, it obviously cannot explain any behavioral departure from purely self-serving behavior in the one-shot Nash sense. In particular it cannot account for cooperative behavior in say, a prisoners' dilemma, or shed light on altruism more generally, nor can it account for any other non-Nash behaviors such as spite ; Alger and Weib) or costly punishment (Fehr and).

In order to explain such deviations from Nash behavior, evolutionary game theory turned to models with a finite number of agents hence departing from the first of the mentioned conditions of Maynard Smith and (). Thus in Schaf(), the finite set of individuals have "market power" and can influence average fitness while making simultaneous decisions (playing the field). In the model preferred by Maynard S()—namely repeated games—a few agents, usually just two, can perfectly monitor and record each others' past actions and condition their strategies hereupon (in evolutionary theory, the repeated games approach is usually referred to as direct reciprocity). Both of these frameworks have led to a large body of research in economics and game theory (see e.g. Alós-Ferrer and ; L; ; Veg, and references therein).

Others, beginning with (,) and his F -statistic, focused on studying populations where individuals do not get matched in a uniformly random manner. When matching is non-uniformly random the fitness of an individual will depend on the group of individuals he is matched with, and groups with different compositions will on average meet with varying reproductive success (Kerr and Godfre); see also Ber(). Take the prisoners' dilemma. If cooperators have a higher chance compared to defectors to be matched with cooperators, matching is non-uniformly random, and specifically it is in this case assortative. If the matching is assortative enough, cooperators will end up receiving a higher average fitness than defectors and thus positive levels of cooperation can become evolutionarily stable. Assortative matching has also been shown to lead to more cooperative outcomes in Moran processes (Cooney et a).

Non-uniformly random matching is a realistic description in situations where a large group of individuals cannot perfectly monitor each others' past behaviors but receive some signals about opponents' types and exert some influence on the matching process (Wilson and Duga; Ber). It can also result due to prolonged interaction of individuals in separated groups (Maynard S), if individuals are matched according to a "meritocratic matching" process in the sense of Nax et (), if matching depends on the geographical location

of individuals (Eshel et ; Nowak and ; Skyrm), or if (genetically) similar individuals match assortatively as in models of kin selection (; Grafe; Hines and Maynard S ; Alger and Weib ; O). Several other processes are listed in Ber() who also shows that the index of assortativity of Berg() and Wright's F -statistic are formally equivalent. In general, the above conditions lead to what biologists refer to as structured populations.

Now, the existing literature on non-uniformly random matching usually deals with special cases—the typical being the two-player, two-strategy case where matching is assortative. Exceptions to this include Kerr and Godfre() who study many-player games with two strategies, van V() who uses a setting similar to ours and discusses inclusive fitness, and Alger and Weib() who develop a general model to investigate the evolutionary stability of preferences. Here we consider the general case and define Nash equilibrium and ESS within the resulting population game , pp. 22–23). The fitness function of the population game is derived from two primitives: a (symmetric, normal-form game) payoff matrix and a function that assigns particular population compositions to group compositions (called a matching rule). Given this structure of fitness functions, we then show how several results known from population games carry through to our setting. In particular, any Nash equilibrium is a steady state for the replicator dynamic, any (Lyapunov) stable steady state for the replicator dynamic is a Nash equilibrium, and any ESS is an asymptotically stable state of the replicator dynamic.

More substantially, we push the literature forward by deriving results on the effi- ciency of the Nash equilibria of population games. A key point—well known from the prisoners' dilemma—is that under uniformly random matching, Nash equilibrium may be inefficient in the sense that the average fitness of the population is not max- imized. Since ESS and Nash equilibrium coincide in evolutionary models based on uniformly random matching, it follows that uniformly random matching generally fails to produce outcomes that are efficient. When matching is non-uniformly random, this raises the following question: If we keep the payoff structure fixed and vary the matching rule, will some matching rule lead to efficiency? Our main result in this regard (Proposition) tells us that any efficient outcome will in fact be a Nash equi- librium under some matching rule. Such efficient outcomes could, for example, be reached endogenously by populations who can influence the matching process.

The structure of the paper is as follows. Section describes the general setup, introduces matching rules, and defines Nash equilibrium and ESS. Section contains our main theoretical results. Section provides a number of applications in two-player, two-strategy normal-form games. Finally, Sect. concludes.

Footnotes

Intuitively, uniformly random matching means that an individuals' type has no influence on what type of individual he is likely to be matched to.

For an overview see (). For a survey of more recent advances in the social sciences, see Newton (, Sect. 3). An interesting study is that of van Veelen et () who use a model where interactions are repeated and the population is also structured. They find that an assortative population structure significantly increases cooperation levels.

Nax and () show that while this is true for certain classes of games, it is not always the case. In a similar setting, W() studies coordination games in a stochastic setting and shows that the Pareto dominant outcome is always stochastically stable. Studying the evolution of (other-regarding) preferences, Ne() shows that if assortativity itself is subject to evolution, Pareto inefficient behavior can be evolutionarily stable.

[SCW_Txt 1]

1 Introduction

We consider the following situations. There are positions in a queue for a facil- ity (for example, a supercomputer or an expensive software equipped for research in a university) and an infinitely divisible good, referred to as money, to allocate. Then, an allocation is a list of bundles to each agent such that each agent receives:

(a) a position, with no two agents having equal positions, and (b) a positive or negative amount of money, with the agents' amounts summing up to at most zero. The agents' using times of the facility are the same, and are normalized to unity. Each agent has a constant unit waiting cost, which may differ among agents, and his waiting cost is the product of his waiting time and unit waiting cost. Each agent has a linear preference, and his utility is the sum of the waiting cost and monetary transfer. The queueing problems are concerned with what queues and monetary transfers we select for each unit waiting cost profile. A rule is formulated as a "function" assigning a queue and monetary transfers to each unit waiting cost profile.

There are three important properties for rules: incentive compatibility, equity, and
efficiency.

Strategy-proofness is an incentive compatibility condition. It says that it is a domi- nant strategy for each agent to report his true unit waiting cost. Without strategy-proof- ness, agents may have incentives to misrepresent their unit waiting costs to manipulate the outcome (that is, a queue and monetary transfers) for their own benefits. These manipulated outcomes may not constitute socially desirable ones relative to agents' true unit waiting costs. Strategy-proof rules are immune to such strategic misrepre- sentation.

Anonymity in welfare is an equity condition. It requires that when the unit waiting costs of two agents are switched, their welfares under the rule also be switched. That is, it says that the names of the agents do not matter from the viewpoint of the welfare level.

Pareto-efficiency is an efficiency condition. An allocation is Pareto-efficient if there is no other allocation which makes each agent at least as well off and at least one agent better off. A rule is Pareto-efficient if it assigns, for each unit waiting cost profile, a Pareto-efficient allocation. In our setting, Pareto-efficiency is decomposable into two conditions of efficiency: queue-efficiency (minimization of the total waiting cost among agents) and budget-balance (zero-sum transfers).

Dolan () has provided a rule that satisfies strategy-proofness and queue-effi- ciency. The class of equally distributed pairwise pivotal rules (rules assigning an efficient queue and transfers considering each pair of agents in turn, making each agent in the pair pay the cost he imposes on the pair, and distributing the sum of these two payments equally among the others) proposed by () satisfies not only strategy-proofness and queue-efficiency but also budget-balance (that is, Pareto-effi- ciency). Moreover, this class is the only one of rules that satisfy strategy-proofness, equal treatment of equals in welfare, and Pareto-efficiency (Kayı and Ramaekers). Equal treatment of equals in welfare requires that the rule assign an allocation for which the welfare levels of agents are equal whenever their unit waiting costs are the same.

Pareto-efficiency is a desirable but demanding condition for not only queueing prob- lems but also allocation problems of indivisible objects and money [see ()]. Thus, it is interesting to search rules without Pareto-efficiency.

We analyze the rules satisfying strategy-proofness and anonymity in welfare and show that under strategy-proofness, anonymity in welfare implies queue-effi- ciency.As equally distributed pairwise pivotal rules satisfy anonymity in welfare, by combining the result of () with ours, we also give another characterization of the class of equally distributed pairwise pivotal rules as the only one of rules that satisfy strategy-proofness, anonymity in welfare, and budget-balance. In Sect. , we set up the model and state the results. In Sect. , we provide proofs.

Finally, in Sect. , we make some concluding remarks.

Footnotes

Several authors such as () and () have defined a rule as a correspondence. Kayı and Ramaekers () have studied the queueing problems from the viewpoints of both a function and a correspondence.

This name was given by ().

() have characterized the class of rules that satisfy strategy-proofness and Pareto-efficiency.

() has shown that envy-freeness implies queue-efficiency. Envy-freeness requires that no agent should end up with a higher utility by consuming what any other agent consumes. However, since

Footnote 4 continued

anonymity in welfare and envy-freeness are independent, it is not obvious what rules are strategy-proof and anonymous in welfare. () and () have analyzed envy-free rules.

[SCW_Txt 2]

Introduction

We study the possibility of designing strategy-proof rules that yield satisfactory solutions to matching problems. By matching problems, we refer to the several important allocation problems in two-sided matching markets where agents, from the start, belong to one of two disjoint sets: for example, workers and firms, students and col-leges, and athletes and teams. Allocations in these markets are matchings, assigning each agent on one side of the market to the agent(s) on the other side.

A matching rule chooses a matching for each preference profile. A matching rule is efficient if it always chooses a matching such that no other matching exists that would make all agents weakly better off and at least one agent strictly better off. A matching rule is individually rational if an agent is never assigned to a partner to whom the agent prefers being unmatched. Individual rationality is necessary for agents to participate voluntarily in matchings. A matching is blocked by a pair of agents if each agent in the pair prefers the other in it to the assigned partner. A matching rule is stable if a match¬ing rule is individually rational, and for any preference profile, the chosen matching is not blocked by any pair. Stability guarantees the rights of individual agents and pairs.

Because the agents' preferences are not known to others, there may be incentives for agents to misrepresent their preferences in order to manipulate the final outcome in their favor unless they are given incentives to represent their true preferences. As the result of misrepresentation, the chosen matching may not be socially desirable relative to the agents' true preferences. Therefore, matching rules need to be immune to such strategic misrepresentation to choose desirable matchings based on agents' true preferences. A matching rule is strategy-proof if it is a dominant strategy for each agent to announce its true preference.

The possibility of matching rules satisfying desirable properties has been explored by many authors. Gale and Shapley (1962) prove that a stable rule, called the "deferred acceptance algorithm", exists. Roth (1982, 1985) shows that all stable rules are not strategy-proof. Alcalde and Barbera (1994) and Sonmez (1994) pursue the possibility of a strategy-proof rule by relaxing stability to efficiency and individual rationality, and show the impossibility of designing rules satisfying efficiency, individually rationality, and strategy-proofness.

In the present article, we pursue the possibility of a strategy-proof rule by relaxing efficiency or employing an alternative concept. A preference profile is unanimous if, unless agents prefers to being unmatched, they have reciprocal top preferences, i.e., if any agent x most prefers x', then x' also prefers x. A matching rule respects unanimity if for each unanimous preference profile, each agent is matched to the partner the agent

most prefers. First, we establish that there is a strategy-proof rule that is individually rational and respects unanimity. However, this rule is unreasonable in the sense that mutually best pairs are matched on only rare occasions.

In order to explore the possibility of better matching rules, we introduce a natural condition, which we call "respect for 2-unanimity". A matching rule respects 2-una¬nimity if a mutually best pair is matched, and an agent wishing to being unmatched is unmatched.1 Compared with stability, respect for 2-unanimity "weakly" guarantees the rights of individual agents and pairs. Secondly, we establish that no strategy-proof rule respects 2-unanimity. Since stability implies respect for 2-unanimity, this Sectionintroduces the matching model. Sectionpresents our results. Section concludes the article by addressing a research agenda and relating our results to the literature on matching problems.

Footnotes

) introduces a similar axiom, mutually best: a mutually best pair should be matched. However, respect for 2-unanimity is slightly stronger than mutually best.

Contrast Corpus[6]

[Txt 1]

The National Organ Transplant Act of 1984 decreed that it is "unlawful for any person to knowingly acquire, receive, or otherwise transfer any human organ for valuable consideration for use in human transplantation." In the absence of a pricing mechanism for this scarce resource, vast organ shortages have developed, with roughly 122,000 persons awaiting organ transplants in the U.S.1 This number grows dramatically every year, in spite of numerous efforts to increase the supply of transplantable organs, including educational campaigns (Siminoff et al., 2009; Rodriguez et al., 2007), social media outreach (Cameron et al., 2013), and coordination of paired kidney exchanges (Roth et al., 2004, 2005; Ausabel and Morrill, 2014). Additional reform proposals include moving to a system of presumed consent for donors (Abadie and Gay, 2006; Bilgel, 2012), allowing financial exchanges for organs (Becker and Elias, 2007; Lacetera et al., 2014; Wellington and Sayre, 2011) and altering the organ allocation rules to induce more donations (Kessler and Roth, 2012; Li et al., 2012). The evidence on the success of these efforts to increase the supply of organs is limited, and we know very little about how a shift in supply of organs may affect transplant candidates' behavior and outcomes.2

[6] PDF versions available at https://drive.google.com/drive/folders/10MmmEw002amF-QMoTofUaNHj6JkHPc2Z?usp=sharing

Without a pricing mechanism in place, the effect of an increase in the supply of organs will depend on the nature of the alternative system for allocating the scarce resource. The United States government oversees a system for allocating organs that attempts to address a balance of equity and efficiency - as the nationwide Organ Procurement and Transplantation Network (OPTN) defines it, a balance of "justice (fair consideration of candidates' circumstances and medical needs), and medical utility (trying to increase the number of transplants performed and
the length of time patients and organs survive)."3 The system is complex and varies by organ, but it generally begins by generating a waitlist of medically compatible transplant recipients in a geographic area. Geographic proximity plays a central role because organs have a limited time when they are viable between procurement and transplantation.4 As a result, shocks to the local supply of organs will likely affect the outcomes and composition of local transplant waitlists.

We use data on organ donors and transplant recipients from the Scientific Registry of Transplant Recipients (SRTR) to consider whether shifts in the supply of transplantable organs affect the behavior and outcomes of transplant candidates and their physicians. We focus on shocks to organ supplies generated by variation in state-level motorcycle helmet laws; all else equal, these shocks might be expected to affect organ shortages and the resulting waiting time for individuals on transplant waitlists. However, without a price mechanism in place, expected waiting time serves as a signal of the scarcity of the organs. If supply shocks change expected waiting times, it is possible that the demand for organs will respond and mitigate some or all of the effects of changes in supply.

We estimate whether the demand for organs in response to a higher supply of organs manifests itself in increased inflows onto waitlists following statewide helmet law repeals. In addition, we consider whether transplant candidates with the option of receiving an organ from a living donor and more likely to exercise this option when the supply of organs is higher. Finally, we consider the overall effect of helmet laws on exits from transplant waiting lists, including both the means of and the timing of exits.

We have four main substantive findings. First, repeals of motorcycle helmet laws substantially increase the supply of transplantable organs. This finding is closely related to Dickert-Conlin, Elder and Moore (2011; DCEM hereafter), who find that motorcycle helmet laws generate (presumably unintentional) shocks to the supply of organ donors. Because each donor can potentially contribute multiple organs to persons on multiple waitlists, we extend DCEM's analysis by quantifying how helmet laws affect the supply of individual organs. We estimate that repeals of statewide helmet laws increase the local supply of transplantable organs from donors killed in motor vehicle accidents by nearly 20 percent. These shocks are particularly large for lungs, kidneys, and livers.

Second, we find that transplant candidates respond strongly to local supply shocks, with inflows to local transplant waitlists increasing by roughly 12 percent in following helmet law repeals. These inflows are

largely driven by those who live outside the local area, rather than by more candidates signing up for their "home" waitlists. The implication is that transplant candidates' decisions of which waitlists to enter are driven, at least in part, by variation in expected waiting time across the waitlists. Moreover, we find that candidates who are listed on multiple waitlists have by far the largest response to helmet law repeals, inflows onto waitlists increasing by over 40 percent relative to baseline. Taken together, these results suggest that in the absence of a formal pricing mechanism, waiting times for organs are the relevant "price" determining where candidates choose to list.

Third, we find that donations from living donors decline when the supply of organs from deceased donors increases due to helmet law repeals. As the relative price - again, as measured by expected waiting time - of a transplant from a decreased donor declines, some candidates are induced to opt for a transplant from a deceased donor rather than a living one. These effects are

most pronounced for potential transplants from living donors who are not blood relatives or spouses of the candidate, suggesting that these are disproportionately the "marginal" cases where the relative costs of living and deceased donors are most influential. Increases in the supply of deceased donors also decrease living donations from parents, children, spouses, and siblings, but by smaller magnitudes. These findings are consistent with those of Fernandez, Howard and Stohr (2013), who estimate that an increase in the supply of deceased kidney donors nearly completely crowds out kidney donations among non-biologically-related living donors.

Our findings on both waitlist inflows and living donors are suggestive that increases in the supply of transplantable organs generates behavior that at least partially offsets the direct effects of reduced waiting time. In order to estimate the overall effect on candidate outcomes, we estimate how increases in organ supply effects health outcomes for transplant candidates.

We focus on time-to-transplant, the probability of exiting the waitlist through various means (including successful transplant or death prior to receiving a transplant), and, conditional on a transplant occurring, the probability that it is successful (known as "graft survival") for one, two, and five years post-transplant. We find little evidence that an increase in the supply of organs increases graft survival time conditional on transplant, but we do find evidence for a decline in the likelihood of dying while waiting for an organ. It is likely that behavioral responses offset at least some of the beneficial effects of an increase in supply of organs on the outcomes of transplant candidates, but the offset is not complete.

Finally, our findings raise questions about the balance of justice and medical utility in the current allocation mechanism, which relies heavily on geographic boundaries. Those transplant candidates who have informational or financial advantages might be most likely to be able to capitalize on violations of the law of one price, which in this setting implies that expected

waiting times for organs should not vary across location. For example, several articles in the popular press alluded to the lack of "fairness" in the organ allocation mechanism in 2009 when Steve Jobs, who lived in California at the time, obtained a liver transplant in Memphis, Tennessee, which had a median wait time roughly 85 shorter than the national average.5

In the following section we explain the setting in which organ donation exists and describe our data sources on organ donations and transplants. Section III estimates a causal relationship between helmet laws and organ donations. In Section IV we estimate transplant candidates responses to the supply shocks estimated in Section III and Section V considers how the supply and demand for organs combine to affect transplant candidates outcomes. Section VI concludes.

Footnotes

2.Fernandez. Howard and Stohr (2013) are an exception. in that they consider the effect of an increase in deceased kidney donors on living kidney donations.

4. OPTN reports the maximum preservation times for hearts and lungs at 4 to 6 hours; liver at 8 to 12 hours; pancreas at 12 to 18 hours and kidney at 24 to 36 hours (from).

5. A substantial part of the criticism was based on the argument that Jobs used his significant financial means to obtain an organ that might be "better served" by being transplanted to a candidate without metastatic pancreatic cancer, which eventually led to Jobs' death in 2011. See

for an example of this sort of response to Jobs' situation.

[Txt 2]

Introduction

The most interesting cooperative game questions can be summarized as follows: (I) which coalitions are formed?, and (II) how are their values distributed between their members? The fundamental concept of a cooperative equilibrium is the core which always assumes that the grand coalition forms. However, the power of the core concept is limited by the fact that the non-emptiness of the core may be assured only in certain ideal environments where the grand coalition formation is reasonable. A natural non-empty extension of the core is the aspiration core introduced by Cross (1967) (see also Albers (1979), Bennett (1983), Bejan and Gómez (2012) and Cesco (2012)). The idea behind the aspiration core is to search those outcomes generated by non-trivial families of coalitions called balanced families that no coalition can improve. This solution takes on the two problems simultaneously, stressing the evident relations between questions (I) and (II).

In the context of games with transferable utility (TU games), the aspiration core has been recently characterized by Bejan and Gómez (2012) and Cesco (2012) who presented axiomatizations of the aspiration

core on the entire class of TU games extending the core axiomatization given by Peleg (1986).1 The contribution of our paper is to offer an axiomatization of the aspiration core on games without transferable utility (NTU games) extending the core axiomatizations given by Peleg (1985). We give an axiomatization of the aspiration core on the domain of all NTU games as the only solution that satisfies non-emptiness, individual rationality, some appropriately- modified version of consistency (reduced game property) and independence of individual irrelevant alternatives. Quoting Peleg (1985), "...we may consider a solution to be 'acceptable' if its axiomatization is very similar to that of the core", then our aspiration core axiomatization posits the aspiration core as an acceptable non-empty solution for NTU games. When we consider solutions supported only by the grand coalition, our axioms also characterize the classical core on an appropriate subdomain. Furthermore, if we consider solutions supported by partitions, our axioms also characterize the c-core ((Guesnerie and Oddou (1979); Sun et al.(2008) and Koczy and Lauwers (2004). Many core axiomatizations (see, for example, Peleg (1985)) work on the class of games with non-empty core, so there is some circularity when they use the core to define their domain of games. It is important to highlight that our aspiration core axiomatization works on the entire class of NTU games, then such circularity does not occur in our axiomatization.2

The traditional consistency axiom and the corresponding reduced game (Davis and Maschler (1965), Peleg (1985)) are defined in a framework in which it is assumed that the grand coalition forms. We use a modified reduced game and its corresponding consistency axiom introduced by Moldovanu and Winter (1994) (see also, Hokari and Kibris (2003), Bejan and Gomez (2012)) for solutions supported by non-trivial families of coalitions. The difference between the traditional reduced game and the modified reduced game arises in the way that the coalition of all the remaining players has to cooperate with the departing players. In the traditional reduced game, the remaining coalition has to get together with all the departing players while in the modified reduced game, the remaining coalition can do it with any subgroup of the departing players that it wishes.

Axioms of independence of irrelevant alternatives have been studied by several authors, for example Aumann (1985), Peleg, Sudholter and Zarzuelo (2012), among others. In general, if an alternative is prescribed as a solution to a problem, and this remains as a feasible outcome in a game where some feasible payments of some coalitions are removed, independence of irrelevant alternatives requires that such alternative be in the solution of the problem in which the feasible payments were removed. In this work, we use a version of this axiom that only considers the case in which some feasible payments of individual coalitions are removed. Therefore, the axiom is called independence of individual irrelevant alternatives.

This paper has the following organization. In Section 2, we give basic definitions and notations. In Section 3, the axioms are presented. Section 4 includes our main axiomatization results. Section 5 shows the independence of the axioms. The Appendix contains two omitted proofs.

Footnotes

1.Cesco (2012) works with a solution concept called M-core which is equivalent to the aspiration core.

2.An alternative axiomatization of the core on the entire class of NTU games is presented by Hwang and Suldholter (2001), but their axioms characterize the empty solution outside the domain of games with non-empty core.

[Txt 3]

Introduction

The partitioning games have been introduced by Kaneko and Wooders (1982), and recently studied by Solymosi (2008), and Auriol and Marchi (2009), among others. These games are useful in modeling situations with restricted cooperative possibilities between the players, and therefore, only some coalitions may be formed. Certainly, the number of coalitions is exponentially large, and it may not be feasible in practice to consider all of them. It may be the case that some of the players in a coalition may not get to meet or communicate with each other, so that actually only some coalitions may be formed. In other contexts, it could be very hard to form a large coalition and then, only small coalitions may play essential roles. But even if all coalitions are allowed, it may still happen that only small coalitions play essential roles, because the game has some special structure, as in the bridge game of Shubik (1971), and the assignment games of Shapley and Shubik (1972).

Partitioning games are represented by a finite set N of players, an a priori set T of coalitions of N (subsets of N) and a payoff function v on T. Only coalitions in T play an essential role and players have to be organized through partitions taken from The fundamental concept of a cooperative equilibrium is the core, which always assumes that the grand coalition forms. However, the power of the core concept is limited by the fact that the non-emptiness of the core may be assured only in certain ideal environments. Kaneko and Wooders (1982) give necessary and sufficient condi¬tions on T which guarantee that every partitioning game, associated to (N, T), has non-empty core. These conditions are considered by the authors “extremely restric¬tive and, without some very special structure on the collection of basic coalitions, we would not expect these conditions to be met”. A large and current literature has studied these conditions to provide a graph-theoretical characterization of these families; see, for instance, Aguilera and Escalante (2010).

In this paper, we study and compare two non-empty extensions of the core that give alternative solutions to the restrictive condition established by Kaneko and 1 Wooders (1982). One of the solutions is the approximate core which proposes the replication of games to obtain non-empty e—cores if the number of replications is sufficiently large. This idea has been introduced by Wooders (1981-1983)2 and stud¬ied in Kaneko and Wooders (1982), Kovalenkov and Wooders (2003) and Wooders (2008), among others. In this

approach, the existence results are based on the fact that, with a finite number of types of players and bounded basic group sizes, large games have non-empty approximate cores.

The other solution concept is the aspiration core which proposes that the coop¬eration (or negotiation) of the players can be supported by overlapping structures of coalitions (not just the grand coalition) called balanced families. The aspiration core has been introduced by Bennett (1983) (see also, Cross (1967), Albers (1979)) and recently, studied by Bejan and Gomez (2012), Cesco (2012) and Arribillaga (2013), among others.

Although the approximate core and the aspiration core are two solutions that have the same motivation -to give an answer to (partitioning) games with an empty core- they have not yet been compared and linked in the literature. The main contribution of this paper is to show different relations between the approximate core and the aspiration core in partitioning games. First, we show that the cores of the replicated games, in a subsequence of the replica games, are equal to the aspiration core of the (original) game. Second, we prove that the collection of e—approximate cores converges to the aspiration core when e tends to zero. All the obtained results are completely independent of the set T of feasible coalitions and the payoff functions.

The paper is organized as follows. In the next section, preliminary definitions and notation are introduced. In section 3, approximate core and aspiration core definitions are presented. In section 4, we present the main results.

Footnotes

1.(N, n, v) is called a game with restricted cooperation in Pulido and Sanchez-Soriano (2006). In that paper the grand coalition is always feasible (N 2 n) and the players are not reorganized in partitions taken from n.

2.The 1981 version of the paper is the Cowles Foundation Discussion Paper No. 612 that was published in 1983.

3.As usual, 2n denotes the set of all the coalitions (subsets) of N.

[Txt 4]

Introduction

In coalitional game theory we have games with transferable utilities (TU games), and games without transferable utilities (NTU games). The latter can be seen as a generalization of the first. There exist many applications where a coalitional game is utilized to describe an economic model. Some examples include: TU- Market (studied in Shapley and Shubik (1969)) and NTU-Market (investigated by Scarf (1967)), NTU Game of Public (and private) Good Economy (presented in Moulin (1988)), Minimum Cost Spanning Tree Games (investigated by Gra- not and Huberman (1984)). In general researches into the NTU case have had a higher complexity than for the TU case. Though this paper could be applied to the TU, the primary focus is on NTU games, which are described as a pair (N, V) where N is the set of players and V is the

characteristic function that assigns to each coalition S C N a subset VS of RS, the set of feasible payments for S. There are many interesting coalitional game questions that have yet been fully addressed, but in general they could be summarized by two: (I) Which coalitions would form? And (II) how to distribute their values between their members?

There exists much literature answering the second question, proposing a set of payoffs or a single payoff as a solution (for example, the core and Shapley's value). The core is one of the solution concepts central to coalitional games, as introduced for NTU games by Aumann (1961). Scarf (1967) presented the most classic condition on the game, for non-emptiness of the core, which is the balanced condition. Peleg (1985) gives an axiomatic characterization of the core as the only solution that satisfies non-emptiness (NE), individual rationality (IR) and the reduced game property (RGP) on the class of non-leveled games with non-empty core.

In respect to the first question most prior researches have supposed that the coalitional structure is given exogenously. However, nowadays there is available an ongoing growing literature that is taking on the two problems simultaneously and stressing the evident relationship between questions (I) and (II),(A. Sen- gupta and K. Sengupta (1994), Koczy and Lauwers (2004), Cesco (2008), Zhao (2008) and others). This current paper builds upon that approach.

To answer the interrogations I) and II) made in the first paragraph we propose a solution which is always non-empty and has interesting links with the core by relationship of inclusion and in term of axioms. This solution emerges from considering: a) the balanced families (collection of coalitions) to organize the participation of the players in more than one coalition in an orderly way and b) the feasible payoffs for a balanced family. The balanced families suggest to us what coalition must be formed and " for how long". We term the new solution as the balanced core. As the balanced core needs to consider the distribution of payoff, x, and the balanced family, B, which guarantees the assignment x, the elements in the balanced core are pairs (x, B). Unlike the great majority of papers adopting a similar approach we do not impose that B is a partition of the player set (coalition structure). Instead we ask only that B is a minimal balanced family of coalitions. The balanced core is always non-empty and "contains" the core on the class of games that satisfies the non-leveled condition (which is necessary in Peleg's characterization of the core and is amply noted in the literature, and is many times a condition asked in the definition of NTU games). Moreover the two solutions "coincide" on the class of non-leveled and balanced games.

The idea embedded in the balanced core definition of some kind of resistance to a certain type of objections, locates this solution as a core-type concept that is able to be characterized by a set of axioms similar to those used by Peleg (1985) for the core of a NTU-games. Indeed, we show that the balanced core satisfies non-emptiness (NE), individual rationality (IR) and the reduced game property (or consistence) (RGP) on the class of quasi non leveled NTU games (which includes the non leveled ones). Accordingly we ought to use

appropriate reformulations of these axioms in order to take into account the new structure of the solution elements. To characterize completely the balanced core we utilize one additional axiom, the axiom of independence of irrelevant alternatives (IIA), which was used and discussed by Aumann (1985) and others. As a result, we show a axiomatic characterization of the balanced core as the only solution satisfying NE, IR, RGP, IIA on the class of quasi non leveled NTU games.

There exist two new solutions studied in Cesco (2008) and Zhao (2008)1, which are very near to the balanced core. Cesco studies a solution for TU games which is named the M — core. The balanced core and the M — core coincide on TU games. Considering this, the balanced core defined by us for NTU games can be regarded as an extension to NTU games, of the M-core. Zhao presents a solution for TU games and then extends this to NTU games. The solution is called new core. The balanced core and the new core "coincide" on the TU games. As such, the balanced core defined by this paper for NTU games can be viewed as another extension to the NTU games, of the new core defined for TU games. Furthermore for NTU games the payoffs in the balanced core are the payoffs in the new core that are not (weakly) dominated by any other payoff in the new core. With a simple equivalence the axiomatic characterization displayed for the balanced core can be adapted to a refinement (with the criterion of weak Pareto dominance) of the new core presented by Zhao for NTU games.

This paper has the following organization. In the next section are introduced some foundations of NTU games. In section 3 the balanced core is presented and it is proven that the balanced core is always non-empty and is related, in inclusive words, to the core. In section 4 an axiomatic characterization of the balanced core as the only solution satisfying NE, IR, RGP and IIA is presented. Also at the end of this section it is proven that the four axioms used in that characterization are logically independent and reveal further details regarding the domain restriction. Finally in section 5 we establish the connection of the balanced core, with both the M — core studied by Cesco (2008) and also the new core presented by Zhao (2008). In this current work are applied the new concepts to a NTU-market.

Footnotes

There exits an older version of this paper and a new (2010) version.

[Txt 5]

Introduction

The project concerns a linear programming (LP) characterization of the set of stable matchings in the context of many-to-one markets and matching with contracts, under substitutable preferences that satisfy the law of aggregate demand.Beyond enhancing the toolbox of the literature to this environment, a LP characterization would allow for an analysis of quantile- mechanisms in the context of many-to-one problems, as well as a deeper understanding of the connection between matching with contracts and matching with salaries. To

provide such a characterization we transform the problem of finding stable (many-to-one) matchings to finding stable* matchings in an associated one-to-one matching problem. The spirit of the idea is similar to the "related marriage market" that has been applied to the case of responsive preferences (see). In the case of substitutable preferences, an agent is "decomposed" into several copies which are not necessarily identical, but whose aggregate behavior mirrors the behavior of the agent in any choice situation. Stability* is the suitable adaptation of pairwise stability to the presence of these copies.

Footnotes

1In the future, we seek to extend the LP characterization result to more general settings that allow for externalities and complementarities.

[Txt 6]

Introduction

The marriage model describes a matching problem in which the agents can be divided into two disjoint subsets: the set of men and the set of women. The objective of this problem is to assign a woman to a man, allowing the possibility that men and/or women might stay single or without a partner. In this paper, we study the marriage model when agents in both sides of the market may be indifferent among agents of the other side.

Most works dealing with matching models assume that agents are not indifferent to the agents on the other side of the market. Many results for

the matching model when preferences are strict cannot be extended to the matching model when agents may have preferences with indifferences.1

In matchings models, stability is considered as the main property to be satisfied by any matching. A matching is called stable if all agents have acceptable partners, and no unmatched men-women pair would strictly prefer to be matched to each other rather than staying with their current partners. Gale and Shapley [7] show that at least one stable matching for the marriage model always exist even when agents may have indifferences in their preferences. They study the college admission problem. In this problem, colleges have preferences over students and students have preferences over colleges. Each college has a maximum number of positions to be filled, its quota. The college admission problem became very popular in recent decades in school districts around the world because it allowed parents to choose a school for their children. The matching model theory is used to design mechanisms for assigning students to schools. Usually, this theory assumes that preferences are strict. However, a college may rank students according to their test scores (A to F), in which case, the college will have indifferences over sets of dents with the same scores. For this

reason, the college admission problem with indifferences is one of the most studied models. (See [1]r44W5], [6]).

Matching problems are also study using linear programming. Rothblum [12] introduces a list of linear inequalities, which generate a convex polytope.2 He characterizes the stable matchings of the marriage model with strict preferences as extreme points of this convex polytope. Roth, Rothblum and Vande Vate [11] present a linear program and use linear programming theory to give alternative proves to already well-know results in the marriage model with strict preferences.

Design of mechanism for matching models with indifferences either force agents to reveal strict rankings, or break ties as part of the mechanism. Given a preference profile with indifferences, a tie-breaking is a preference profile in which each agent replaces indifferences by some strict order. In other words, a tie-breaking is a new strict preferences profile.

Usually the procedure to compute a stable matching is breaking ties and then applying Gale and Shapley's Deferred Acceptance Algorithm [7]. The way in which these indifferences may be ordered have both strategic and welfare consequences. (See Erdil and Ergin [6] and Abdulkadiroglu, Pathak and Roth [1]).

For every strict preference profile obtained after a tie-breaking we can calculate the system of linear inequalities. These linear inequalities generate the convex polytope defined by Rothblum [12]. Consider all systems of linear inequalities corresponding to all the ways of breakjn^indifferences^ -On might align these inequalities and generate a new system of inequalities. Then we have a new convex polytope. The problem here is that there are matchings that are stable in some tie-breaking and not stable in other ones. (See Example 2). In other words, there are stable matchings that satisfy some systems of linear inequalities corresponding to some tie-breaking, but do not satisfy other ones.

One of our contribution in this paper is to characterize the stable matchings of the marriage model with indifferences via linear inequalities. To do this, we exchange the stability inequalities from the strict model, for new specific stability inequalities in the model with indifferences. These new inequalities, together with others inequalities, generate a new convex polytope. This new polytope may have strictly fractional extreme points (see Example 2). We prove that the stable matchings are the integer extreme points of this convex polytope.

We say that a stable matching i is men-optimal at a preference profile if there is no other stable matching that Pareto dominates i according to men's opinions. In the marriage model with strict preferences a unique menoptimal stable matching always exist. However, when indifferences in preferences are allowed, the optimal stable matching may not be unique. That is to say there can appear more than one stable matching that is not Pareto dominated by another stable matching. (See Example 2).

A social planner may need to compute an optimal stable matching for one side of the market, for instance a men-optimal stable matching. One can be tempted to break ties and use the Deferred Acceptance Algorithm

(Gale and Shapley [7]) to compute the men-optimal stable matching at the strict preference profile associated. Nonetheless, the men optimal stable matching at this strict preference profile, may not be a men-optimal stable matching at the original preference profile with indifferences. (See example 1). Erdil and Ergin [5] establish an algorithm that computes optimal stable matchings in the college admission problem with indifferences. To this end, they break ties and apply a Pareto improvement cycles and Pareto improvement chains.

The Linear Program presented by Roth, Rothblum and Vade Vate [11] does not distinguish among any stable matchings. That is, all stable matchings give the same objective value. Another of our contributions is to present a linear program that computes a men-optimal (women-optimal) stable matching for the marriage model with strict preferences. This is present as a particular case of the marriage model with indifferences. To do this, we define a new objective function in the linear program that is correlated with the men (women) preferences. In this way the solution of the linear program is unique: the men-optimal (women-optimal) stable matching.

For the case of the marriage model with indifferences we present a linear program that computes one of the men-optimal (women-optimal) stable matchings without using any tie-breaking. Since the constraints for this linear program are the linear inequalities above mentioned, the optimal solution may be fractional. For this reason, we define an integer linear program, such that the optimal solution is one of the men-optimal (women-optimal) stable matchings.

The paper is organized as follows. In Section 2, we introduce the model, preliminary notations and definitions. In Section 3, we characterize the set of stable matchings as integer extreme points of a convex polytope. Finally, in Section 4, we present a integer linear program that computes a men-optimal stable matching.

Footnotes

1. See Roth and Sotomayor for a more detailed explanation.

2. Vande Vate [13](1989) characterized stable matchings as extreme points of a linear inequality system in a model when all agents are mutually acceptable.

[Txt 7]

Introduction

A large part of the matching literature assumes that agents are not indifferent between any two agents on the other side of the market. Many of the known results for the matching model when preferences are strict cannot be generalized to the matching model when agents may have preferences with indifferences (see Roth and Sotomayor 1990).

The college admission problem was introduced by Gale and Shapley (1962). In this problem, colleges have strict preferences over students, and students have strict preferences over colleges, and each college has a maximum number of positions to be filled, its quota. The college admission problem became very popular in recent decades in school districts around the world because it allowed parents to choose a school for their children. Since then, the matching model theory has been used to design mechanisms for assigning students to schools. Usually, this theory assumes that preferences are strict. However, a college may rank students according to their test scores (A to F). In this context, the college will have indifferences over sets of students with the same scores. For this reason, the college admission problem with indifferences is one of the most studied models. (See Ergin and Erdil (2006, 2008) Abdulkadiroglu, A., Pathak, A. and Roth, A. (2009))

The marriage problem is the case in which all colleges have a quota equal to one. This model describes a matching problem in which the agents can be divided into two disjoint subsets: the set of men and the set of women.

Unlike the marriage model with strict preferences, where there exists a unique concept of stability by pairs of agents, in the marriage model with indifferences there are several concepts of stability. A matching is stable if all agents have acceptable partners and there is no unmatched men-women pair in which both of them would strictly prefer to be matched rather than staying with their current partners. Irving (1994) formulates two other possible definitions of stability for the marriage model with indifferences. A matching is strongly stable if all agents have acceptable partners and there is no unmatched men-women pair in which one of them would weakly prefer to be matched, and the other one would strictly prefer to be matched rather than staying with their current partners. A matching is super stable if all agents have acceptable partners and there is no unmatched men-women pair in which both of them would weakly prefer to be matched rather than staying with their current partners.

In practical situations, stability is the most appropriate definition of stability, since there is no real incentive to form a blocking pair of a matching if one agent is indifferent between an other agent and his/her partner in the matching. In this paper, we focus in stable matchings.

One of the most significant results in the matching literature is the one establishing that the set of stable matchings has a lattice structure. A set has a lattice structure if we can define a partial ordering and two binary operations (the least upper bound and the greatest lower bound) on it. The structure is important for at least two reasons. First, it indicates that even if agents on one side of market compete for agents on the other side, this conflict is attenuated since, agents of the same side have a coincidence of interests on the set of stable matchings. Second, it has proved to be very useful: many algorithms that yield stable matchings (and are used in real centralized markets) are based on this lattice structure. The lattice structure of the stable matching set for the marriage model (with strict preferences) was first established by Knuth (1976), who attributed the result to Conway. Manlove (2002) studied the marriage model with indifferences. He shows

that the strongly stable matching set has a lattice structure when it is partitioned by a suitable equivalence relation.

In this paper, we give a sufficient condition, the closing property, under which we can generalize results from the marriage model with strict preferences to the marriage model with indifferences. We define an equivalence relation over the stable matching set. Since the relation defined over the stable matching set is an equivalence relation, it defines a partition (of stable matching set). If the set of stable matchings satisfies the closing property, we prove the lattice structure over equivalence classes of the stable matching set of the model with indifferences. Also, if the strongly stable matching set is not empty, we prove that the lattice over equivalence classes of strong matchings (Manlove 2002) is a sublattice of the lattice over equivalence classes of stable matchings. In the marriage model with strict preferences the stable matching set satisfies the closing property trivially, because each representative class only has one stable matching.

In the hospital resident problem with strict preferences, the Hospital Rural theorem (Roth 1984, 1986) is of great importance, since it produces dissatisfaction in stable allocation mechanisms. The Hospital Rural theorem states that if one hospital has a vacancy in some stable matching, then the set of residents assigned to it is the same (in particular, we have the same number of vacancies in any stable matching). In the context of the marriage model, the theorem states that the set of agents remaining single is the same for all stable matchings (McVitie and Wilson 1970). The Hospital Rural theorem is not valid when agents may have indifferences in preferences. Irving, Manlove and Scott (2003) studied the hospital resident problem with indifferences. They prove that if the set of super stable matchings is not empty, then the Hospital Rural theorem is valid over all stable matchings. In this paper, we generalize the Hospital Rural theorem over all stable matchings, under certain restrictions.

Sotomayor (2011) proposes the Pareto stability concept as a solution concept for matchings with indifferences. Sotomayor (2011) shows that the set of stable matchings coincide with the set of Pareto stable matchings. In this paper, we prove that, under indifferences, if the stable matching set satis fies the closing property, then the stable matching set and the Pareto stable matching set coincide.

This paper is organized as follows. In Section 2, we formally describe the model and present some preliminaries. In Section 3, we define the closing property. We proceed with the study of equivalence classes to matchings. In Section 4, we assume that the stable matching set satisfies the closing property. We show the lattice structure over equivalence classes of stable matchings. In Section 5, we extend the Hospital Rural theorem to the marriage model with indifferences. Finally, in Section 6, we establish the relation between stable matchings and Pareto stable matchings.

[Txt 7]

Introduction

A large part of the matching literature assumes that agents are not indifferent between any two agents on the other side of the market. Many of the known results for the matching model when preferences are strict cannot be generalized to the matching model when agents may have preferences with indifferences (see Roth and Sotomayor 1990).

The college admission problem was introduced by Gale and Shapley (1962). In this problem, colleges have strict preferences over students, and students have strict preferences over colleges, and each college has a maximum number of positions to be filled, its quota. The college admission problem became very popular in recent decades in school districts around the world because it allowed parents to choose a school for their children. Since then, the matching model theory has been used to design mechanisms for assigning students to schools. Usually, this theory assumes that preferences are strict. However, a college may rank students according to their test scores (A to F). In this context, the college will have indifferences over sets of students with the same scores. For this reason, the college admission problem with indifferences is one of the most studied models. (See Ergin and Erdil (2006, 2008) Abdulkadiroglu, A., Pathak, A. and Roth, A. (2009))

The marriage problem is the case in which all colleges have a quota equal to one. This model describes a matching problem in which the agents can be divided into two disjoint subsets: the set of men and the set of women.

Unlike the marriage model with strict preferences, where there exists a unique concept of stability by pairs of agents, in the marriage model with indifferences there are several concepts of stability. A matching is stable if all agents have acceptable partners and there is no unmatched men-women pair in which both of them would strictly prefer to be matched rather than staying with their current partners. Irving (1994) formulates two other possible definitions of stability for the marriage model with indifferences. A matching is strongly stable if all agents have acceptable partners and there is no unmatched men-women pair in which one of them would weakly prefer to be matched, and the other one would strictly prefer to be matched rather than staying with their current partners. A matching is super stable if all agents have acceptable partners and there is no unmatched men-women pair in which both of them would weakly prefer to be matched rather than staying with their current partners.

In practical situations, stability is the most appropriate definition of stability, since there is no real incentive to form a blocking pair of a matching if one agent is indifferent between an other agent and his/her partner in the matching. In this paper, we focus in stable matchings.

One of the most significant results in the matching literature is the one establishing that the set of stable matchings has a lattice structure. A set has a lattice structure if we can define a partial ordering and two binary operations (the least upper bound and the greatest lower bound) on it. The structure is important for at least two reasons. First, it indicates that even if agents on one side of market compete for agents on the other side, this conflict is attenuated since, agents of the same side have a coincidence of interests on the set

of stable matchings. Second, it has proved to be very useful: many algorithms that yield stable matchings (and are used in real centralized markets) are based on this lattice structure. The lattice structure of the stable matching set for the marriage model (with strict preferences) was first established by Knuth (1976), who attributed the result to Conway. Manlove (2002) studied the marriage model with indifferences. He shows that the strongly stable matching set has a lattice structure when it is partitioned by a suitable equivalence relation.

In this paper, we give a sufficient condition, the closing property, under which we can generalize results from the marriage model with strict preferences to the marriage model with indifferences. We define an equivalence relation over the stable matching set. Since the relation defined over the stable matching set is an equivalence relation, it defines a partition (of stable matching set). If the set of stable matchings satisfies the closing property, we prove the lattice structure over equivalence classes of the stable matching set of the model with indifferences. Also, if the strongly stable matching set is not empty, we prove that the lattice over equivalence classes of strong matchings (Manlove 2002) is a sublattice of the lattice over equivalence classes of stable matchings. In the marriage model with strict preferences the stable matching set satisfies the closing property trivially, because each representative class only has one stable matching.

In the hospital resident problem with strict preferences, the Hospital Rural theorem (Roth 1984, 1986) is of great importance, since it produces dissatisfaction in stable allocation mechanisms. The Hospital Rural theorem states that if one hospital has a vacancy in some stable matching, then the set of residents assigned to it is the same (in particular, we have the same number of vacancies in any stable matching). In the context of the marriage model, the theorem states that the set of agents remaining single is the same for all stable matchings (McVitie and Wilson 1970). The Hospital Rural theorem is not valid when agents may have indifferences in preferences. Irving, Manlove and Scott (2003) studied the hospital resident problem with indifferences. They prove that if the set of super stable matchings is not empty, then the Hospital Rural theorem is valid over all stable matchings. In this paper, we generalize the Hospital Rural theorem over all stable matchings, under certain restrictions.

Sotomayor (2011) proposes the Pareto stability concept as a solution concept for matchings with indifferences. Sotomayor (2011) shows that the set of stable matchings coincide with the set of Pareto stable matchings. In this paper, we prove that, under indifferences, if the stable matching set satis fies the closing property, then the stable matching set and the Pareto stable matching set coincide.

This paper is organized as follows. In Section 2, we formally describe the model and present some preliminaries. In Section 3, we define the closing property. We proceed with the study of equivalence classes to matchings. In Section 4, we assume that the stable matching set satisfies the closing property. We show the lattice structure over equivalence classes of stable matchings. In Section 5, we extend the Hospital Rural

theorem to the marriage model with indifferences. Finally, in Section 6, we establish the relation between stable matchings and Pareto stable matchings.

[Txt 8]

Introduction

Two-sided matching models are used to study assignment problems in which agents can be divided into two disjoint subsets from the very beginning. One of these subsets contains institutions like men, firms, hospitals, colleges, sororities, orchestras, schools, clubs, etc. The other one includes agents like women, workers, medical interns, students, musicians, children, sportsmen, etc. The fundamental question of these assignment problems is how to match each institution on one side with one or a group of agents on the other side. Stability has been considered as the main property to be satisfied by any sensible matching. A matching is called stable if all agents have acceptable partners and there is no unmatched institution-agent pair such that both would prefer to be matched to each other rather than staying with their current partners under the proposed matching. Giving all blocking power to an agent and institutionagent pairs seems a weak requirement. Moreover, in many cases it may be the right solution concept. This is due to the fact that to destroy an individually rational unstable matching, only a telephone call (or a couple of e-mails) is required. The "Marriage Model" is the name given by Roth and Sotomayor [8] to a one-to-one model with ordinal preferences. In this paper we focus on this model.

Gale and Shapley [3] show that deferred acceptance algorithms produce either the men-optimal stable matching (or the women-optimal stable matching), depending on whether men or women make the offers. The men (women)- optimal stable matching is unanimously considered by all men (respectively, women) to be the best among all stable matchings.

Irving and Leather [5] introduce the concept of cycles on preferences and cyclic matching and present an algorithm that finds all stable matchings for the marriage model. To seek for cycles on preferences, these authors first reduce the preference lists of all agents. This reduction of preferences allows us to find cycles. A cycle is a set of men that satisfies the condition that the second woman in the reduced list of a man in the cycle is the first woman in the reduced list of another man in the cycle. A cyclic matching is the stable matching in which all men in the corresponding cycle are matched to their second woman in the reduced preference lists, and in which the remaining men stay matched to their first woman in the reduced preference lists.1 Since the cycles of a reduced lists are disjoint, we extend the definition of cyclic matching to a set of cycles in the reduced preference lists. That is to say, for any set of cycles in the reduced preference lists, a cyclic matching is the stable matching in which all men in the set of cycles is matched to their second woman in the reduced preference lists, and in which the remaining men stay matched to their first woman in the reduced preferences lists.

Shapley and Shubik [11], study the assignment game in terms of its linear programming formulations. This game consists of two finite, disjoint sets of players M and W and an |M | x | W | matrix of non-negative numbers joj,j : (i, j) € M x W} . The interpretation is that any pair of players (i,j) G M x W is free to form a coalition whose worth is a,j which the two players may divide between themselves in any way. Any player is free to remain single and receive zero, and the worth of an arbitrary coalition equals the maximum worth that can be obtained by the sum of the worth of the elements of a partition of this coalition in pairwise coalition and singleton (with pairs consisting of one player from M and one from W). Each matching can be represented by an assignment matrix, called the incidence vector of the matching. In this way, the authors show that the matchings are exactly the integer solution of a system of linear inequalities. These integer solutions -incidence vectors- are exactly permutation matrices. They define a fractional matching to be a not necessarily integer solution of this system of linear inequalities. According to Birkhoff's Theorem [1], any fractional matching must be a convex combination of incidence vectors.2

Based on the similarity between the assignment game and the marriage model, Vande Vate [12] adds some inequalities to the Shapley and Shubik's system of linear inequalities, capturing the stability condition for the marriage model with these inequalities. He shows that the stable matchings for the marriage model correspond to the set of incidence vectors (integer solution for the linear inequalities). In other words, the stable matchings are exactly the extreme points of the polytope generated by the system of linear inequalities that he proposes. The model that he considers assumes that all pairs of agents are mutually acceptable, and that the two sides of the market have the same number of agents. Following Gale and Shapley's results [3], Vande Vate assures that the polytope generated by this linear system is non-empty.

Rothblum [10] extends this result by considering the possibility that agents stay unmatched, and that both sides of the market do not necessarily have the same number of agents. He also gives a proof for the characterization of extreme points of the polytope as stable matchings.

Roth et al. [9] introduce a linear program, where the objective function is the sum of all entries of the incidence vector and the constraints are Roth- blum's linear inequalities. They characterize all stable matchings as the integer solutions of this linear program. Using the Duality and the Complementary Slackness Theorems of linear programming, they show that the primal program and its dual program have a remarkable relationship. Each optimal solution of the primal program is contained in an optimal solution of the dual program.

Roth et al. [9] define a stable fractional matching to be a (not necessarily integer) solution of the linear program. These stable fractional matchings can be interpreted either as lotteries over possible matches, or as time-sharing arrangements. In other words, a stable fractional matching assigns a probability distribution to each agent over possible partners, or the probability that an agent is matched to another agent. In centralized markets, where agents submit their preference list on prospective partners to a clearing-house, a

matching is produced by processing these lists according to an algorithm. This may result in a random process depending on the structure of the algorithm. This random process may end on lotteries over stable matchings. Likewise, in discrete problems where agents have opposite interests, randomization is surely one of the most practical tools to achieve procedural fairness. Roth et al. [9] observe that stable fractional matchings can be blocked by a pair (m, w). We interpret an entry of the incidence vector of a stable fractional matching as the fraction of time that man m and woman w are assigned one another. This stable fractional matching may assign both m and w a portion of time to the agents they like less than each other. These two agents, m and w, have an incentive to increase the time they spend with each other at the expense of those they like less. If this happens, we say that the pair (m, w) fractionally blocks the stable fractional matching. The authors define a strongly stable fractional matching, whose set is denoted by SS(P), as a stable fractional matching that does not have a fractional blocking pair in the above sense. We will see that the convex combination may or may not be fractionally blocked, depending on which stable matchings are involved. That is to say, a stable fractional matching will or will not be strongly stable, depending on the stable matchings involved. The importance of these matchings is to avoid private time-sharing arrangements outside the matching. Roth et al. [9] assure that a stable fractional matching is strongly stable only if each agent may assign time only to two agents on the other side of the market.

The set SS(P) is a subset of the stable fractional matchings. Moreover, depending on the model, it may be a proper subset. We illustrate these facts with an example.

The main result of this paper is a characterization of a strongly stable fractional matching. We prove that a stable fractional matching is strongly stable if and only if all the stable matchings in its support belong to a connected set. A stable matching p belongs to a connected set if there is a stable matching such that p is one of its cyclic matchings. In this way, we also characterize the set SS(P) as the union of the convex hull of the connected sets. Using an adaptation of Irving and Leather's algorithm, we present an algorithm that computes the connected sets.

An important result about stable matchings is that the set of stable matchings has a lattice structure under the partial order of men's (women's) common preferences; these lattices are dual. This is a very significant result for at least two reasons. First, it indicates that even if agents of the same side of the market compete for agents on the other side, this conflict is attenuated. The reason for this is that agents of the same side of the market have a coincidence of interests on the set of stable matchings. Secondly, such lattice structure has proved to be very useful: many algorithms that yield stable matchings are based on such structure or some related properties. Roth et al. [9] show that the set of stable fractional matchings has a lattice structure. For this task they define an appropriate partial order over the set of stable fractional matchings -the stochastic dominance.

Using the same partial order over stable fractional matchings, we also show that the set SS(P) has a lattice structure. The binary operations associated with the lattice, x V y and x A y, defined by Roth et al. [9] for the set of stable fractional matchings, and the fact that the set of SS(P) is a subset of the stable fractional matchings set, tell us that SS(P) is a sub-lattice.

We proceed as follows. Section 2 presents the marriage model, the concept of cycle on preferences, properties of cycle and properties of cyclic matchings and the linear programming approach. In Section 3, we present a characterization of a strongly stable fractional matching, and an algorithm to calculate the stable matchings necessary to characterize the set of all strongly stable fractional matchings. In this section we also prove that this set has a lattice structure. Finally, we include two lemmas necessary for our characterization in section 4 (Appendix).

Footnotes

1. For more detail, see Roth and Sotomayor [8].

2. Any (non-integer) solution of the above mentioned system of linear inequalities must be a convex combination of permutation matrices.

[Txt 9]

Introduction

A large part of the matching literature, studies many-to-one matching markets. The agents in these markets are divided in two disjoints sets: The manyside of the market, namely resident doctors, students, workers, etc, and the one-side, namely hospitals, college, firms, etc. The main property studied in the matching literature is the notion of stability of a matching. A matching is called stable if all agents have acceptable partners and there is no unmatched institution-agent (hospital-doctor, college-student, firm-worker, etc.), that both agents would prefer to be matched to each other rather than staying with their current partners under the proposed matching. Each agent has a preference list, that gives an order over the agents or sets of agents on the other side of the market, with the possibility to keep unmatched.

Irving and Leather [6] introduce the concept of cycles on preferences and cyclic matching and present an algorithm that finds all stable matchings for the one-to-one matching market, also known as ”the marriage model”. Bansal et.al. [3] and Cheng et.al. [4] extended the concept of cycles and cyclic matchings for a many-to-many and many-to-one matching markets, respectively. To seek for cycles on preferences, these authors first reduce the preference lists of all agents. This reduction of preferences allows us to find cycles. Since the cycles of a reduced list are disjoint, we extend the definition of cyclic matching to a set of cycles in the reduced preference lists. The linear programming is a mathematical tool used in matching theory.

Each matching can be represented by an assignment matrix, called the incidence vector of the matching. First, Vande Vate [13] present a system of linear inequalities, that characterize the stable matching of the marriage model. He shows that the stable matchings for the marriage model correspond to the set of incidence vectors (integer solution for the linear inequalities). In other words, the stable matchings are exactly the extreme points of the polytope generated by the system of linear inequalities that he proposes. The model that he considers assumes that all pairs of agents are mutually acceptable, and that the two sides of the market have the same number of agents. Following Gale and Shapley's results [5], Vande Vate assures that the polytope generated by this linear system is non-empty. Rothblum [11] extends this result by considering the possibility that agents stay unmatched, and that both sides of the market do not necessarily have the same number of agents. He also gives a proof for the characterization of extreme points of the polytope as stable matchings. Roth et al. [9] introduce a linear program, where the objective function is the sum of all entries of the incidence vector and the constraints are Rothblum's linear inequalities. They characterize all stable matchings as the integer solutions of this linear program. Ba¨ıou and Balinski [2] present two generalizations of the convex polytope for the many-to-one matching market. Roth et al. [9] for the marriage model, as well as Ba¨ıou and Balinski [2] for the many-toone matching market, define stable fractional matchings as a not necessarily integer solution of the linear inequality system. These stable fractional matchings can be interpreted as the time that each agent spends with one agent of the other side of the market. Sethuraman et al. [12] explores the geometry of the stable fractional matchings in the many-to-one matching markets, and gives an alternative prove to one of the characterizations of Ba¨ıou and Balinski [2]. Roth et al. [9] observe that some stable fractional matchings can be blocked by a pair (m,w). This stable fractional matching may assign both m and w a portion of time to the agents they like less than each other. These two agents, m and w, have an incentive to increase the time they spend with each other at the expense of those they like less. If this happens, we say that the pair (m,w) fractionally blocks the stable fractional matching. The authors define a strongly stable fractional matching, whose set is denoted by SS(P), as a stable fractional matching that does not have a fractional blocking pair in the above sense.

In the school-choice market with priorities, Kesten and ¨Unver [7] study this stability condition for the fractional matchings, they called strongly ex ante stable random matchings. In the school choice market, they deal with indifferences in the preferences of the schools. Then a strongly ex ante random matching, is a random matching that fulfil two condition: ex ante discrimination, and ex ante justified envy. In our market, ex ante discrimination, is trivially fulfil since the firms preferences over workers are strict preferences. The ex ante justified envy condition is represented by our strong stability condition. They, also present a fractional deferred-acceptance algorithm that computes an unique for us a strongly stable fractional matching. Their work analyze strategy proofness and efficiency of this mechanism. Alkan and Gale [1] consider a deterministic two-sided schedule matching market. In their model a worker may be employed for

at most one hour, but he can share his time between different firms. A firm can hire fractions of workers until it fills its quota of hours. Both, firm and workers are equipped with complex preferences structure over these schedules. Our work goes in another direction, we analyze the the linear programming structure for the many-to-one matching market when the firms has a preferences that are q-responsive. We extend the definition from Roth et al. [9] for the strongly stable fractional matchings to a many-to-one matching market. The importance of these matchings is to avoid private time-sharing arrangements outside the matching. We focus on one of the characterization of Ba¨ıou and Balinski [2]. The convex polytope generated by the linear inequalities of this characterization, may have fractional extreme points. We prove that, these fractional extreme points, violates our strong stability condition. Using Bansal et.al. [3] and Cheng et.al. [4] extension of cyclic matching, we define a connected set generated by a stable matching μ, as the set of all cyclic matching of μ (including μ). Then, we characterize a strongly stable fractional matching, as a convex combination of stable matchings that belong to the same connected set. This way, we characterize the set of all strongly stable fractional matching as the union of the convex hull of these connected sets. The paper is organized as fallows. Section 2, formally introduces the model, preliminary results, and one of the Ba¨ıou and Balinski [2] characterizations of stable matchings. Section 3, discuses definition of strongly stable fractional matching, some properties of these matchings. Also we discus cycles and cyclic matchings properties. In section 4 is presented our characterization of a strongly stable fractional matching. Section 5, Appendices contains proofs of the lemmas needed on our characterization.

[Txt 10]

Introduction: A large part of the matching literature studies many-to-one matching markets. The agents in these markets are divided into two disjoint sets: The many-side of the market,

namely resident doctors, students, workers, etc, and the one-side, namely hospitals, college, firms, etc, The main property studied in the matching literature is the stability. A matching is called stable if all agents have acceptable partners and there is no unmatched pair (hospital-doctor, college-student, firm-worker, etc.), where both agents would prefer to be matched to each other rather than staying with their current partners under the proposed matching. Each agent has a preference list that determines an order over the agents or sets of agents on the other side of the market, with the possibility of staying unmatched. In this paper the firms have q-responsive and strict preferences.

Linear programming is a widely used mathematical tool in matching theory. Each matching can be represented by an assignment matrix called the incidence vector of the matching.

Vande Vate [19] and Rothblum [16] present a system of linear inequalities that characterizes the

stable matching of the marriage market for two different restrictions of the market. Both papers show that the stable matchings for the marriage market correspond to the set of incidence vectors (integer solutions for linear inequalities). In other words, the stable matchings are exactly the extreme points of the polytope generated by the system of linear inequalities. Roth et al. [15], for the marriage market, introduce a linear program that characterizes all stable matchings as the integer solutions.

Linear programming approach has been developed to the theory of stable matching markets also by Abeledo and Rothblum [4] [3], Abeledo and Blum [1], Abeledo et al. [2], Fleiner [9], [8], Sethuraman and Teo [18] and by others.

Ba ̈ıou and Balinski [5] present two characterizations of the convex polytope for the many-to-one matching market. We focus on one of these characterizations.

Lotteries over stable matchings has been study in many instance in the literature. Roth et al. [15] for the marriage market, studied lotteries over stable matching via linear programming. When the extreme points of the convex polytope generated by the constraints of a linear program are exactly the stable matchings of the market, for instance the marriage market, a random matching coincides with the concept of stable fractional matching. Roth et al. [15] defines stable fractional matching as a not necessarily integer solution of the linear program. When the extreme points are not all integer, these two concept are not the same, for instance a many-to-one matching market with q-responsive and strict preferences. That is to say, a random matching is always a stable fractional matching, but some stable fractional matching can not be written as a lottery over stable matchings. Example 1 expose a many-to-one matching market with an extreme point that is a stable fractional matching.

Each entry of an incidence vector of a stable fractional matching can be interpreted as the time that each agent spends with one agent on the other side of the market. For a stable fractional matching, it can happen that two agents, one of each side of the market, have an incentive to increase the time that they spend together at the expense of those matched agents that they like less than each other at a stable fractional matching. To study a ”good” fractional solution, the idea is to avoid this and prevent that agents have incentive ”block” the stable fractional matching in a fractional way. For a Marriage Market, Roth et al. [15] define a strongly stable fractional matching as a stable fractional matching that fulfill a non-linear equalities that represent this non-blocking condition mentioned above. In other words, a stable fractional matching that fulfill the non-linear equalities from Roth et al. [15], is a strongly stable fractional matching. Neme and Oviedo [13] give a characteriazation of the strongly stable fractional matching for the marriage market. Our work extends their result and provides a characterization for the many-to-one strongly stable fractional

matching set. We analyze the linear programming structure for the many-to-one matching market when the firms have q-responsive preferences. We extend the strong stability condition from Roth et al. [15] to a many-to-one matching market. We focus on one of the characterizations of Ba¨ıou and Balinski [5]. As we mentioned before, the convex polytope generated by the linear inequalities of this characterization may have fractional extreme points. We prove that these fractional extreme points violate our strong stability condition.

In the school choice set-up, strong stability for lotteries has been introduced by Kesten and Unver [¨11], they called ex-ante stability for lotteries. For this market, they deal with indifferences in the preferences of the schools. Kesten and Unver [¨11] also present a fractional deferred-acceptance algorithm that computes a unique strongly ex-ante random matching. Their paper analyses strategy proofness and efficiency of this mechanism. Our characterization goes in another direction, we study the relationship between the stable matchings that are involve in the lotteries.

Bansal et al. [6] and Cheng et al. [7] study the concept of cycles in preferences and cyclic matchings for many-to-many and many-to-one matching markets, respectively. These papers are an extension of Irving and Leather [10]. To seek for cycles in preferences, these authors first reduce the preference lists of all agents. We present the reduction procedure for our market in the Appendix. This reduction procedure allows us to find cycles in preferences. Since the cycles of a reduced list are disjoint, we extend the definition of cyclic matching to a set of cycles in the reduced preference profile.

Following the extension of cyclic matching used by Bansal et al. [6] and Cheng et al. [7], we define a connected set generated by a stable matching μ as the set of all cyclic matching of μ (including μ). Then, we characterize a strongly stable fractional matching as a lottery over stable matchings that belong to the same connected set. Moreover, we prove that these stable matching of the lottery, have a decreasing order on the eyes of all firms. In this way, we characterize the set of all strongly stable fractional matchings as the union of the convex hull of these connected sets.

Roth et al. [15], (in Corollary 21) proves that in a strongly stable fractional matching, each agent is matched with at most two agents of the other side of the market. Schlegel [17] generalizes this result for the school choice set-up with strict priorities. They show that an ex-ante stable lottery fulfills that each worker has a positive probability with at most two distinct firms, and for each firm, all but possibly one position are assigned deterministically. For the one position that is assigned by a lottery, two workers have a positive probability of been employed. Our characterization gives an alternative prove for this two result, for the school choice set-up due to Schlegel [17] is straigtforgard, and for the marriage marker due to Roth et al. [15], its necessary only to set all quotas of all firms equal to one.

This paper is organized as follows. Section 2 formally introduces the market, preliminary results, and one of Ba¨ıou and Balinski's characterizations of stable matchings [5]. Section 3 discusses the definition of strongly stable fractional matching and some properties of these fractional matchings. We also discuss cycles and cyclic matching properties. In section 4, we present our characterization of a strongly stable fractional matching. The Appendix contains the reduction procedure, lemmas and proofs of the lemmas needed for our characterization.

Appendix 2: Complete Software-Assisted Analysis, Manual Analysis Sample

This appendix includes:

1. Links for downloading the complete analysis:

https://www.dropbox.com/sh/66bgrw55y5fryz7/AABWz4G64b45BKQbpLa9jjUca?dl=0, or from https://drive.google.com/drive/folders/1-vFJN6msi2OW_m5rAF2F91mhK_GIgPu7?usp=sharing

2. A link for downloading the UAM Corpus Tools software, whose installation is necessary to visualize the analysis: https://www.dropbox.com/sh/ybmhw2okfghrs2p/AAAA7W3-OR6tNAzE35VMRi_Aa?dl=0

3. A sample of the manual analysis (Table 17)

Table 17: Table 17: Example of manual analysis (MC text)Clause	**Appraised Entity**	**STAGE ^Phase**	**Appraisal Type**	**Comments**
This paper studies various **stability notions in a one-to-one matching problem with transfers**	OS	Justification: Announcing topic	Eng: Mon	
In this problem, the market has two sides, say, firms and workers.		Justification: Providing explanations/definitions		
An outcome in this setup not only specifies which firm is going to hire which worker, but also the monetary payments		Offer: Introducing contribution		
One **intuitive** solution concept for such a market is ex post stability Ex post stability requires that each agent end up with nonnegative utility and that no firm worker pair would prefer to match with each other at some wage rather than the current matching	EC PR		ATT:App + Eng: Mon Eng: Mon	
Ex post stability was introduced by Gale and Shapley (1962) for a matching market without transfers and was extended to a		Proof:Reviewing PR (Indicating relevance of topic)	Eng:Exp Eng:Exp	This part is especially noteworthy, as it reveals the researchers' attitude towards the object of

market with transfers by Shapley and Shubik (1971).				study. On the one hand, they invite the reader to value the object of study as relevant by citing previous works dealing with it. However, they react to other authors' contributions by foregrounding a gap, which acts as the introduction of another contribution which reinforces the statement
Both of these papers show that ex post stable matchings exist when agents' preferences are known	PR		Eng:Con	
However, **most markets** operate under asymmetric information, i.e., there is no common knowledge of preferences.	EC	Proof: Providing explanations/definitions	Eng:Con Eng:Con	
Therefore, it is important **to have an incentive-compatible mechanism**, allowing agents to reveal their preferences truthfully to implement an ex post stable matching.	EC		Att:App	
Unfortunately, **this** is impossible:	EC	Proof:Reviewing PR (Identifying gap)	Att:Aff + App+ Eng:Mon	
Roth (1982) shows in a matching problem without transfers that there is no incentive-compatible mechanism that produces stable matchings	PR	Proof:Reviewing PR (Supporting the gap statement)	Eng:Contr	
Alcalde and Barberà (1994) strengthen the **impossibility result** of Roth (1982) by showing that there is no incentive-compatible, individually rational, efficient mechanism.	PR		Eng:Contr Att:App r Eng:Exp Eng:Contr	
Our first **result** complements these findings by showing that there is no incentive-compatible and ex post budget-feasible mechanism that produces ex post stable matchings in a market with transfers (Theorem 1).	CR	Comparing contributions between CR and PR	Eng:Exp Eng:Con	The term "theorem" within brackets has been marked as an instance of Engagement, as it reveals the researchers' intention to attribute the previously introduced statement to a section of the paper which is important in terms of

				authorship. A theorem produced in the context of a SRA is a major result, the mathematical expression of it.
Ex post budget feasibility requires that the mechanism does not need an outside subsidy to run.1 *(1. One needs to consider a budget feasibility constraint when agents can make endogenous transfers. Without such a requirement, ex post stability and incentive compatibility may be consolidated by subsidizing the system. Of course, no such condition is needed for markets without transfers).*	EC	Proof: Providing explanations/definitions	Eng:Mon (Eng:Exp:Enter Grad:For:Focus:Sharp Eng:Exp:Enter Eng:Cont:Proc:Conc Eng:Cont:Dis:Neg Grad:For:Focus:Sharp Eng:Exp:Enter)	We take the whole clause as an entity evaluated by the researchers. In this case, it is an explanation offered by the authors after announcing one result. The evaluated entity is the context in which results are produced. The authors clearly engage with the proposition introduced by adding a footnote. We conceive footnotes as expansions of the propositions in the main body of the text, and thus, as intrinsically evaluative. We could say that footnotes are a sort of macro evaluative instances (which contain several other instances). Footnotes are a highly used resource, and thus they are considered as fundamental in this analysis.
Although desirable, **ex post stability may not** be necessary for the success of some matching markets, especially when the outcome of the mechanism can be enforced.	EC		Eng:Contr Att:Val Eng:Contr Att:App:Val Eng:Cont Eng:Exp	Evaluation here is directed towards a variable or factor considered in the solution proposed. The authors acknowledge this factor as not valuable or necessary for producing a result. The use of the verb "enforce" in the passive is especially

				curious, as it implies a result produced "unnaturally" by agency of someone who is not mentioned, thus presenting it as a "distanced", hypothetical situation.
For example, **in the National Resident Matching Program, which assigns medical school graduates to residency programs, the matching outcome is binding**	PR		Eng:cont	Providing examples has been considered as a mark of engagement, as it reveals the authors' involvement with the topic, their intentions to make the topic clear to the readers
For similar markets in which agents cannot form blocking pairs after learning the outcome of the mechanism, we introduce two alternative **stability notions**.	CR	Offer: Offering a solution (Announcing result)	Eng:Mon Att:App	
The first is ex ante stability.	CR		Eng: Mon	
Even though **agents** can still unilaterally opt out of the mechanism ex post, **they** can only form blocking pairs at the ex ante stage before they learn their values.	CR	Offer: Providing explanation	Eng:Cont Eng:Exp:Enter Grad:Focus:Sharpen	This is an instance of what we call *internal evaluation*, i.e. evaluative items do not imply the authors' opinion or position in relation to the proposition presented, but rather their choice to show themselves as aware of limitations in the procedure/solution
In other words, **a matching mechanism is ex ante stable if agents get nonnegative utilities for all outcomes and there exists no firm-worker pair who could match with each other at a particular wage and both get higher**	EC		Eng:Cont	

expected utilities at the ex ante stage.				
We show that **incentive compatibility and ex ante stability** can be satisfied in conjunction with efficiency and ex ante budget balance, under the assumption that either firms and workers are ex ante symmetric or that firms and workers are equal in number (Theorem 3).	CR	Offer: Announcing result	Eng:Contr Eng:Exp	
Efficiency requires that we implement the matching that maximizes the sum of match utilities, and ex ante budget balance requires that the mechanism does not run an expected deficit or create an expected surplus.	CR	Offer:Providing explanation	Eng: Mon	
The second stability notion is interim stability.		Offer: Announcing result		
We still allow agents to opt out unilaterally ex post.	Rr	Offer: providing explanation	Eng:Mon	Authors take responsibility for what they do to obtain a result
However, **agents** can now form blocking pairs at the interim stage when **they** already know their values.	CR		Eng:Cont: Eng:Exp	Authors acknowledge variables to offer readers a detailed, informed explanation
The interim no-blocking notion presents unique **challenges**.	CR		Att:Appr:Val	Researchers acknowledge difficulties
First, agents have asymmetric information at the interim stage.				In this part, the authors explain the steps of a process. They reveal what would happen given certain circumstances. The evaluative instances that may be observed in the clauses refer to
Therefore, if a firm and a worker agree to form a blocking pair, this provides information about the firm's valuation to the				

worker and, similarly, information about the worker's valuation to the firm.				entities within the explanation of the thought process, but they are not connected to the authors' reaction towards the participants or elements in that explanation. At a discourse level, this explanation which does not contain overt evaluation from the authors, may be acting as a macro instance of evaluation through which the authors lay the groundwork for the following clause, which is certainly evaluative, as it includes expressions conveying interpersonal meanings
With this new information, they **presumably** update their estimates of expected utilities from participating in the mechanism.			ENG: Exp	
Therefore, it may no longer be beneficial for them **to block the mechanism**.2 (*2. Similar difficulties arise in auctions with collusion. See, for example, Laffont and Martimort (1997, 2000) and Pavlov (2008).*)	CR		Eng:Exp:Enter Att:App:Val (Eng:Exp:Enter)	This last clause marks a transition between the explanation of the challenge represented by the notion they introduced and the details they provide next as to the contribution they make. They acknowledge a situation that may potentially be an obstacle and reinforce it with the comment in the footnote. Thus, the use of a footnote is specially telling in this part, as reveals the presence of authors and their involvement with their propositions.

In our **definition of interim no blocking**, each agent has a belief that her partner's private information lies in a subset.	Current contribution	Offer:Offering a solution:Announcing result	Eng:Exp:Attr:Ack	The authors align with the idea that what they offer as a definition is different from what has been stated above. They highlight a fundamental difference in the way they have solved the problem
A pair of agents can form a blocking pair only if their beliefs are correct, in other words, when they update their beliefs, they get the same subsets back.	CR	Offer:Providing explanation	Eng:Exp	Authors reinforce statement by providing a paraphrase of the explanation, introduced by the expression "in other words", which reveals the authors 'engagement with the proposition
The second challenge is a **bargaining issue**, since agents' willingness to pay depends on their private information.	CR	Offer:Providing explanation	Att:Appr:Val Eng:Mon	The use of conjunctions is interesting, as authors usually include them to introduce an explanation of a sequence in a though process, to explain the logic of a procedure, which is the expected function of conjunctions. However, at a discourse level, they also act to show the authors' intention to make a proposition clearer. The meaning conveyed by the clause introduced by that conjunction would not be as clear without it. Its inclusion within the message makes the authors' presence and their intention to reach the reader visible. This use of conjunctions could be equated with

				Martin and Rose's notion of internal vs external conjunctions, which highlights two different uses of conjunctions: one to convey the meaning strictly attached to its cohesive function, and the other related to a more discursive usage that reveals a metaphorical or secondary function
We resolve the bargaining problem as follows: in a one firm-one worker case, if each agent's value for her potential partner can be zero, then there is a unique, incentive-compatible, individually rational, efficient, and ex post budget-feasible mechanism.	Rr	Offer:Offering a solution:Indicating steps	Eng:Cont:Proc:End/ Att:Jud:SE:Capacity	Evaluation is focused on the researchers' capacity to solve a problem, which they show by indicating the process
In this mechanism, the firm and the worker match without making any transfers.				
Therefore, **we** assume that this is the only deal they make.	Rr	Offer:Offering a solution:Expanding on steps	Eng:Exp	The alignment of the authors with the proposition is partially evidenced, as its veracity depends on the fulfillment of other conditions which go beyond the authors' scope
Even if **agents' values** are bounded away from zero, this matching becomes focal.	CR		Eng:Cont Eng:Monogloss Att: App	
Finally, **we** show that interim stability can be satisfied together with incentive compatibility, efficiency, and ex ante budget balance when there is only one agent	Rr	Offer:Offering a solution:Announcing result	Eng:Cont Eng:Exp:Enter	

on the short side of the market (Proposition 3).				
This assumption is not innocuous, but it is satisfied by some important markets such as auctions and monopolies.	CR		Att:Appr:Val Eng:Contr:Dis:Neg Eng:Exp:Att:Distance	
In Table 1, **we** provide a summary of the main results.	CR	Offer:Indicating structure	Eng:App	Researchers overtly connect with their results. They show interest in making the paper clear for their readers. Making the structure clear can also be considered a strategy for building authorship.
The first column shows whether the result is negative or positive.				
The rest of the columns refer to a property (EFF for (ex post) efficiency, IC for (ex post) incentive compatibility, IR for (ex post) individual rationality, BF for (ex post) budget feasibility, and BB for budget balance).				
Each row represents a result stating whether a mechanism with the required properties exists.				
Two strands of literature are related to the current work.	PR	Proof:Reviewing previous research (Persuading the reader of the relationship between CR and PR)	Eng:Exp	
The **first strand** considers weaker notions of incentive compatibility for matching markets without transfers to overcome the **impossibility result** of Roth (1982) that incentive-compatible and ex post stable mechanisms do not			Att:Appr Eng:Exp	The authors introduce a reference to previous works that reinforces their object of study.

exist (Roth 1989; Majumdar 2003; Ehlers and Massó 2007).				
These papers either have nonexistence results or make harsh assumptions to get existence.			Eng:Monogloss Eng:Monogloss Att:App:Val	
In contrast, **we** consider a matching market with transfers and, while retaining the incentive compatibility constraint, consider weaker stability notions.	Rr	Offer:Announcing result	Eng:Cont Eng:Exp	Previous works were introduced to mark the contrast with the current one, and thus show its relevance/validity
The second strand in the literature considers incentive-compatible core concepts for markets with transfers, following the seminal work of Wilson (1978).	PR		Att:Appr:Val Eng:Exp:Att:Ack	
Although none of the solution concepts considered in this literature are directly related to ours, **the ex ante incentive-compatible core** is worth mentioning.3 *(3. See Glycopantis and Yannelis (2005), for different solution concepts and their applications.* *Recently, Myerson (2007) provided a different approach and proved the existence of an interim core notion for games with a balanced structure.)*	PR+CR	Proof:comparing contributions/clarifying relation between PR and CR (thus indicating gap and need for investigating the proposed topic)	Eng:Cont: Att:App (Eng:Exp)	
An interim incentive compatible mechanism satisfies this property if there exists no coalition of agents that can get a higher ex ante payoff	CR	Proof:Providing explanation (of the reason why the ex ante incentive-compatible satisfies a property)	Eng: Mon	The whole clause seems to serve the purpose of reinforcing the positive opinion of researchers in relation to the

by an interim incentive-compatible mechanism.				mechanism. This is realised by closing the dialogic space through a monoglossic proposition and then opening it by introducing a conditional cause that functions as supporting/clarifying device in the statement.
Forges (2004) establishes the nonexistence of the ex ante incentive-compatible core for a one-to-one matching model with discrete types.	PR	Proof:Reviewing previous research	Eng:Exp	The authors bring other works and researcher to the discussion to lay the groundwork for introducing their own results
By contrast, **our main result** with ex ante stability **establishes** the existence of a (dominant strategy) incentive compatible mechanism that is immune to blocking by pairs of agents at the ex ante stage who cannot sign a contract contingent on their type realizations.	CR	Offer:Offering a solution:Announcing result	Eng:Cont: Eng: Mon	The contrast between the previous works and the current one (existence and non existence of ex ante incentive-compatible) is clearly evidenced and stated in this clause
In addition, we also require ex post individual rationality, which is stronger than the ex ante individual rationality implied by the conditions in Forges (2004).	CR		Eng:Mon Eng:Exp	Notice how researchers openly align with their results and back them up with a reference to a previous work
In a recent paper, Chakraborty, Citanna, and Ostrovsky (2010) takes a different approach than these two strands of literature.	PR	Proof:Reviewing previous research	Eng:Exp:Att:Ack Att:App:Val	It is interesting to note how authors use Engagemetn resources to bring other works and authors into the text in order to highlight differences with their own work rather than showing alignment
They study a two-sided matching problem of schools to students in which students have a common ranking of				

schools and schools have interdependent values over students.				
In this setting, they introduce a stability condition that depends on how much information the mechanism reveals and show that a stable mechanism exists if schools only observe their own matches.				
In contrast to their stability notion, **our stability notions** remain the same regardless of the mechanism considered.	CR	Offer:Offering a solution:Announcing result	Eng:Cont Eng:Exp Att:App:	
Finally, **Yenmez (2009) studies** incentive-compatible mechanisms for the allocation of discrete resources in general markets.	PR	Proof:Reviewing previous research	Eng:Exp Grad:Focus:Sharpen Eng:Exp:Att:Ack Att:App:Val	In this case, other voices are included in the text as a way to validate the following proposition, where the authors announce their results. Notice how elements of the cited author's research are positively valued (even when the terms correspond to discipline-related notions), thus laying the groundwork for the presentation of the current contribution
In particular, **he** provides necessary and sufficient **conditions for the existence of an incentive-compatible mechanism** with other **desirable properties** such as **individual rationality and ex post budget balance**.				
Our benchmark results (Theorems 1 and 2), which do not impose any stability conditions, are applications of his general results.	CR	Offer:Offering a solution:Announcing result	Eng:Cont Att:App:Val	
Whereas the **current work**'s main focus is stability, **Yenmez** (2009) does not study any such **notion**.	CR+PR	Proof:Comparing contributions	Eng: Cont Eng:Exp	
The rest of the paper is organized as follows.	CR Rr	Offer:Announcing structure		This section is especially noteworthy, since it contains several
We introduce the **formal model** and			Att:App Att:App	

establish two benchmark **results** that do not impose any stability conditions in Section II.	Rr Rr			instances of evaluative language for a stage in the Introduction normally expected to be objective and devoid of interpersonal marks. However, researchers openly assume responsibility for what they are presenting (Use of 1st person plural) and even evaluate their contribution (Appreciation resources)
In Section III, we study ex post stability.			Att:App	
We then consider two novel **stability notions**: ex ante stability in Section IV and interim stability in Section V.			Eng:Exp Att:App	
Finally, we conclude the paper in Section VI.				

Appendix 3: Interview and Observation Grid Templates

a. Interview template (personal interviews)

- Fecha:
- Sujeto entrevistado:
- Duración de la entrevista:
- Comentarios adicionales:

1. ¿Cuáles son los **objetivos** de los seminarios de su grupo de investigación? ¿Siente que coinciden con las actividades que se llevan a cabo?
2. ¿Podría mencionar si existen reglas de participación? ¿De qué manera contribuyen al éxito de los seminarios?
3. ¿Podría mencionar qué modalidades de participación existen? (trabajos completos/ideas/proyectos/posters, etc.) ¿Es el inglés el idioma esperado considerando que la mayoría de los trabajos sobre los que exponen están originalmente en esa lengua?
4. ¿De qué manera considera que la participación en los seminarios (en cualquier rol) contribuye a la mejora de su trabajo? ¿Qué le aporta esta experiencia?
5. ¿Le ayuda su participación en los seminarios a pulir sus trabajos desde el punto de vista disciplinar/lingüístico (enunciación/redacción) de ideas)? ¿Podría aportar un ejemplo concreto de cómo se transformó algún trabajo o parte del mismo a partir de esta experiencia?
6. Sobre los trabajos que se exponen en los seminarios: (1) ¿Cómo inicia la escritura de un trabajo?; (2) ¿En qué idioma?; (3) Si escribe en inglés: (a) ¿a qué fuentes recurre para guiarse? (diccionarios en línea o impresos/traductores automáticos/asesoramiento externo, etc) ¿Podría mencionarlas?; (b) ¿Desde cuándo escribe en inglés?; (c) ¿Posee alguna rutina de escritura que pueda ser considerada parte de un proceso?

b. Interview template (online form): https://forms.gle/uzVkWfBxWoBbRXUm9

c. Observation grid template

Grilla de observación

Seminarios IMASL- Grupo Teoría de Juegos

Fecha:

Presentador:

Descripción general de la situación (aula, oyentes, recursos, idioma de presentación):

Descripción general de la secuencia de presentación (orden, intervención de los oyentes, respuestas del presentador –incluye citas textuales):

Appendix 4: Consent Form Used to Access Research Articles Written at IMASL

CONSENTIMIENTO INFORMADO PARA LA AUTORIZACIÓN DE ARTÍCULOS DE INVESTIGACIÓN

San Luis, 4 de octubre de 2019.

Por la presente, autorizo/no autorizo a la docente-investigadora Graciela Lucero Arrúa, integrante de GAECI (Gabinete de Asistencia a la Escritura Científica), a utilizar los artículos de investigación que oportunamente le fueron enviados para el análisis lingüístico que requiere su tesis de maestría, dirigida por la Dra. Mariana Pascual. Entiendo que: (a) dicho trabajo busca describir patrones de uso del inglés en mi disciplina de estudio/investigación, y (b) que la utilización de los datos que requiere dicha tesis no compromete la confidencialidad de las fuentes, ni implica la indebida divulgación de resultados u otros datos que puedan atentar contra la privacidad de los autores.

Me notifico, además, de que seré informado acerca de los resultados y conclusiones más relevantes una vez finalizada esta investigación.

Firma:

Aclaración:

Appendix 5: Interview Transcripts

1. ¿Cuáles son los objetivos de los seminarios de su grupo de investigación? ¿Siente que coinciden con las actividades que se llevan a cabo?

Researcher 1: El objetivo principal del Seminario es compartir con el grupo de investigación los trabajos, o partes de trabajos, que elaboramos así como también las dificultades que se nos presentan en los mismos. En cada sesión el expositor recibe opiniones, sugerencias, críticas, etc que le ayudan a mejorar su producción y los demás participantes amplían su formación al escucharlo. En caso de no tener material propio nuevo para exponer se selecciona algún trabajo de interés de cualquier otro autor. A veces recibimos expositores invitados. Las actividades que se llevan a cabo coinciden con los objetivos.

Researcher 2: El Seminario nació como un espacio para compartir información dentro del grupo de investigación, formarse y avanzar en sus trabajos o formación. Sí, las actividades de discusión de teoría o artículos de otros autores son claves para cumplir con los objetivos del grupo.

Researcher 3: Bien, yo creo que al seminario lo empezamos en el año 2014, y más o menos la idea era copiar lo que se hace en los departamentos de economía de afuera, que básicamente es mantener…es como que…no…para tener un horario fijo en el cual uno sabe que tiene que trabajar con investigación porque implica obligación pero por un bien común), si? Entonces, básicamente la idea era, de los seminarios, que siempre alguien presente algún trabajo en progreso, o algún tema interesante, algo relacionado a nuestros temas, a lo que estamos trabajando, y a veces, cuando hay gente disponible y plata, traer alguien de afuera que nos cuente qué está haciendo e nuestra área de investigación, no? Entonces, tiene esos dos objetivos, digamos: mantener como aceitado el grupo y además que nos visite gente, si? Esa sería la idea, y yo creo que lo estábamos haciendo, ahora, bueno, ahora estamos un poco con menos gente, estamos como muy ocupados, se está cayendo un poquito, pero la idea es que no decaiga.

Researcher 4: Sí, creo que los objetivos son más o menos…profundizar en los temas de investigación, compartir los temas, entonces ahí pueden salir ideas nuevas, o algunas correcciones, o algunos puntos de vista que uno no tuvo en cuenta, siempre en los temas de investigación que uno está trabajando.

Researcher 5: El objetivo principal del Seminario es compartir con el grupo de investigación los trabajos, o partes de trabajos, que elaboramos así como también las dificultades que se nos presentan en los mismos. En cada sesión el expositor recibe opiniones, sugerencias, críticas, etc que le ayudan a mejorar su producción y los demás participantes amplían su formación al escucharlo. En caso de no tener material propio nuevo para exponer se selecciona algún trabajo de interés de cualquier otro autor. A veces recibimos expositores invitados. Las actividades que se llevan a cabo coinciden con los objetivos.

Researcher 6: El principal objetivo del seminario del grupo de investigación es que los miembros estén al tanto de los problemas de investigación que están resolviendo los compañeros. Además los

seminarios, muchas veces sirven para presentar problemas que aún no han podido ser resueltos en su totalidad. En este caso, escuchar sugerencias y opiniones para encarar su resolución. En otras ocasiones el seminario sirve para comentar papers nuevos de la literatura.

Researcher 7: Los objetivos de dichos seminarios son que los participantes del grupo de investigación expongan (ya sea de manera formal o informal) sobre trabajos científicos en los cuales estén trabajando, de tal forma que todos los integrantes del grupo de investigación conozcan los temas abordados por los demás miembros. Como así también colaborar con ideas, comentarios, etc. que puedan enriquecer dicho trabajo presentado.

2. ¿Podría mencionar si existen reglas de participación? ¿De qué manera contribuyen al éxito de los seminarios?

Researcher 1: La regla que más se sigue es la de fijar al principio del cuatrimestre un cronograma de expositores. Entonces cada participante preve con tiempo la selección de un trabajo propio (preferentemente) o ajeno para exponer. Prácticamente no se observan reglas de participación durante las sesiones, resultan espontáneas.

Researcher 2: Ninguna, sólo exponer algo que sea pertinente para el grupo.

Researcher 3: En principio, yo creo que la regla más…que está más o menos implícita, pero la idea es que toda la gente que participa, en algún momento del año tiene que dar una charla, no? Presentar algún trabajo. Esa es más o menos la idea y (interrupción de comida jajaja) Bueno, te sigo contando, a veces la regla..es…no hay…a veces is alguien tiene algo que es un poco largo podríamos tardar varios encuentros para que termine su trabajo, así que…pero no hay como un…que son 40 minutos y hay que presentar. Este…y en general, se pueden presentar no sólo trabajos propios, sino que si alguien lee algo que le parece interesante relacionado a lo que hacemos también se puede presentar, así que no hay mucha regla. El ppt no es necesario. Yo alguna vez he agrarrado fibrón y pizarrón, pero siempre es así tipo clase. Pero no es que hacemos una puesta en común de algo; siempre es uno y el resto escuchando y debatiendo sobre eso, sobre lo que está presentando el orador.

Researcher 4:

Si…no…muchas reglas no hay. Sí un poco tratamos de respetar el tiempo por una cuestión de orden nada más, pero después no. Creo que no hay muchas reglas…cada uno va presentando, ya sea trabajos que hizo o cosas que está leyendo, o cosas que quiere hacer. Y después las preguntas van en cualquier omoento, las intervenciones. A veces es más el tiempo de intervenciones que de presentación, pero…

En el caso de trabajos que se están leyendo sirven para reslatar un interés en un área específica. Por ahí surgen preguntas que refuerzan ese interés o no, entonces creo que también sirve para eso

Researcher 5: La regla que más se sigue es la de fijar al principio del cuatrimestre un cronograma de expositores. Entonces cada participante preve con tiempo la selección de un trabajo propio (preferentemente) o ajeno para exponer. Prácticamente no se observan reglas de participación durante las sesiones, resultan espontáneas.

Researcher 6: Casi no hay reglas de participación, sólo el día y horario. En general se utiliza alguna presentación pero puede usarse sólo el pizarrón. Se puede interrumpir si hay dudas, no hace falta esperar al final. Creo que en este caso informal, este hecho es importante, permite quitar la duda y continuar comprendiendo la presentación. Muchas veces, cuando uno debe esperar al final para preguntar se pierde el hilo de la presentación.

Researcher 7: Básicamente las reglas se basan en que todos los miembros del grupo de investigación participen de forma activa en el seminario, es decir, comprometiéndose en exponer y como así también asistir a los seminarios como oyente. De esta forma, se aseguraría el éxito del seminario.

3. ¿Podría mencionar qué modalidades de participación existen? (trabajos completos/ideas/proyectos/posters, etc.) ¿Es el inglés el idioma esperado considerando que la mayoría de los trabajos sobre los que exponen están originalmente en esa lengua?

Researcher 1: El tipo de trabajos se describió en las respuestas anteriores. En principio no se esperan exposiciones en inglés. Pero surgió la idea de exponer en inglés, a modo de práctica, ya que los integrantes suelen participar en eventos científicos internacionales.

Researcher 2: Presentación de trabajos propios o ajenos. Sin son propios, puede estar en curso. También se presentan ideas para potenciales trabajos que se presentan al grupo para que opinen sobre la viabilidad de su concreción o para aportar a su desarrollo. Cando se presentan trabajos ajenos, la idea es aclarar conceptos y disparar ideas para investigar, que se discuten entre todos. También se exponen trabajos de tesis, como una especie de taller para ensayar la defensa.

Researcher 3: Siempre se expone en español, aunqe las ppt estén en inglés. No es obligatorio que las ppt estén en inglés, pero es conveniente en general porque a los trabajos sí o sí hay que publicarlos en inglés, entonces si sacamos algo de lo que estamos escrbiendo, va a estar en inglés, y a veces vamos a congresos internacionales y tenemos que presnetar una charla, un poster en inglés, asi que la idea es…y bueno, lo que sí nos gustaría potenciar un poco es hablar en inglés.

Researcher 4: No es condición necesaria el uso de ppt, lo que pasa es que …eso me parece que no hace a la formalidad, es una cuestión más de comodidad porque por ahí como aparece mucha notación, y eso, si uno lo quiere hacer en el pizarrón…ese es el punto, porque de hecho es más informal, que si lo hicieramos en el pizarrón, igual. Ya uno está acostumbrado, no le cuesta nada, es una cuestión de comodidad.

El inglés no es obligatorio, de hecho lo intentamos poner en práctica el año pasado, pero no. El motivo por el que las ppt están en inglés es porque uno escribe así, ya para presentar, y lo que lee también está inglés.

Researcher 5: El tipo de trabajos se describió en las respuestas anteriores. En principio no se esperan exposiciones en inglés. Pero surgió la idea de exponer en inglés, a modo de práctica, ya que los integrantes suelen participar en eventos científicos internacionales.

Researcher 6: Se puede presentar un paper terminado enviado a publicar o uno ya publicado. En este caso el idioma usado es ingles. La mayoría de las veces la presentación es en ingles. Cuando lo que se expone es una idea que aún esta verde o que recién se está comenzando a desarrollar puede que este en castellano.

Researcher 7: En los seminarios llevados a cabo en nuestro grupo de investigación, como ya hice mención en la primera pregunta, son de carácter formal e informal. Esto quiere decir, que podemos presentar trabajos propios completos ya publicados. O bien ideas que se estén trabajando para los cuales es importante la participación de los demás miembros del grupo de investigación. En este ultimo caso, pueden llegar a producirse debates muy útiles para el expositor. También, pueden presentarse trabajos no propios que nos hayan parecido interesantes, ya que esto hace enriquecernos y estar actualizados en nuestra área de investigación. Sabemos que la mejor opción es llevar a cabo los seminarios en el idioma inglés, sin embargo, esto no siempre sucede ya que para algunos miembros del grupo de investigación es un trabajo extra prepararlo en inglés.

4. ¿De qué manera considera que la participación en los seminarios (en cualquier rol) contribuye a la mejora de su trabajo? ¿Qué le aporta esta experiencia?

Researcher 1: El expositor recibe opiniones, sugerencias, críticas, etc que le ayudan a mejorar su producción y los demás participantes amplían su formación al escucharlo.

Researcher 2: Nos obliga a producir trabajos, explicaciones claras, lo más clara posibles teniendo en cuenta que si bien el eje del grupo es una misma teoría, no todos inestigan en la misma dirección. Hay matemáticos y economistas.

Researcher 3: Son varias cosas, no? A nivel así bien, bien personal de la persona que está presentando, bueno, obviamente todas las críticas buenas y malas sirven, sirve eso más que nada para empezar a pulir ya para tener un working paper para empezar a circular, y además sirve para los que están de oyentes, sirven para, bueno, eso para mantenerse entrenado, todo el tiempo hablando sobre los temas de investigación sirve para que no te duermas o no te, bueno, y adenás te sirve porque hay mucho feedback entre la gente que está ahi, surgen ideas para alguna colaboración entre la misma gente del instituto, que ha pasado eso, ya aparte, bueno, para conocer más al grupo, para integrar más al grupo; se integra más, me parece si cada uno sabe lo que está haciendo el resto, porque aparte te sirve si tenés alguna duda especial,

sabés que tal persona está haciendo eso y le podés consultar, y dudas que no sé, eso sirve mucho, uno tiene una duda super especifica y se puede pasar una tarde entera buscando papers y a lo mejor vas, tocas una puerta y en cinco minutos te sacan la duda.

Researcher 4: Creo que discutir. Siempre es bueno un poco discutir el proyecto que uno tiene, el problema que está pensando, sobre todo cuando intervienen la gente que tiene más experiencia, porque la literatura hoy en día es tan amplia, que uno no puede acceder, en cambio, por ahí a medida que va pasando el tiepo, la gente va adquiriendo experiencia, conoce la literatura y da una opinión más calificada, y también vernos y conversar con los otros.

Researcher 5: El expositor recibe opiniones, sugerencias, críticas, etc que le ayudan a mejorar su producción y los demás participantes amplían su formación al escucharlo.

Researcher 6: Cómo expositor, cuando el trabajo es propio, creo que es seminario contribuye enormemente, aún más si está la presencia de Alejandro Neme. Muchas veces las preguntas que te hacen en el seminario son las mismas que te preguntan los referes en los paper o la audiencia en algún congreso. El seminario en este caso es una etapa previa en dónde uno puede medir su trabajo, qué partes son claras y cuáles quizás reorganizar para facilitar su comprensión. Cuando uno es parte de la audiencia, el seminario ayuda al mantenerse al tanto de otros temas.

Researcher 7: La participacion en el seminario contribuye de forma positiva en cualquier rol.

5. ¿Le ayuda su participación en los seminarios a pulir sus trabajos desde el punto de vista disciplinar/lingüístico (enunciación/redacción) de ideas)? ¿Podría aportar un ejemplo concreto de cómo se transformó algún trabajo o parte del mismo a partir de esta experiencia?

Researcher 1: La participación en seminarios también ayuda a pulir los trabajos desde el punto de vista disciplinar/lingüística. En mi caso, por ejemplo he descubierto errores de tipeo (Apreciación -) en medio de la presentación, cuyo texto usualmente es copiado y pegado del paper original. Algunos errores de tipeo son significantes (cuando se producen en fórmulas, subíndices, etc) porque cambian el sentido del objeto matemático.

Researcher 2: Me ayuda más desde el punto disciplinar. Hay instancias en las que la intervención de los colegas es clave para redireccionar argumentos/refirzarlos.

Researcher 4: Ehh siii, ayudar seguro que ayuda, depende de la situación de los trabajos, por ahí ayuda un montón o cambia un montón el proyecto a como lo tenía pensado, por ahí lo modifica menos, por ahí lo modifica muy poco, depende el estadío en el que esté. En cuanto a la naturaleza de las intervenciones, hay distintos tipos de participantes en el seminario. Algunos se fijan más en las cuestiones de forma, de cómo está escrito el ppt, o verbal como un todo, cómo está presentado todo, qué destacar, qué poner primero, no tanto en particular, sino como una presentación general, qué es lo central del paper, qué es accidental,

qué vincula con otro trabajo y qué no. Eso sí a veces se ha conversado. Respecto a la presetación escrita hay algunos que participan más detalladamente en eso y creo que lo hacen bien y después hay todo un marco que es de la discusión del problema en sí, que no tiene que ver tanto con la presentación, que ahí creo es donde hay más participación. También eso depende de cada uno, pero creo que es ahí donde a gente espera más respuestas, donde más tiene interés.

Researcher 5: La participación en seminarios también ayuda a pulir los trabajos desde el punto de vista disciplinar/lingüística. En mi caso, por ejemplo he descubierto errores de tipeo en medio de la presentación, cuyo texto usualmente es copiado y pegado del paper original. Algunos errores de tipeo son significantes (cuando se producen en fórmulas, subíndices, etc) porque cambian el sentido del objeto matemático.

Researcher 6: Si, ayuda mucho. Muchas veces después de un seminario cambié por ejemplo de orden de la presentación. Aquí hace falta un ejemplo para comprender la definición, o este ejemplo debería estar antes de esta definición. Falta en la introducción dar más motivación al problema que queremos resolver. A veces la notación no es la adecuada, tratamos de simplificarla, etc.

Researcher 7: Claramente la participación en los seminarios ayuda a pulir los trabajos. Principalmente en la mejora de redacción de los resultados, como así también en la manera de contar dicho trabajo, es un ejercicio muy buena. Particularmente, recuerdo cuando un compañero me sugirió modificar notaciones de un trabajo que estaba presentando, las cuales fueron muy útiles ya que agilizaba la lectura del trabajo.

6. Sobre los trabajos que se exponen en los seminarios: (1) ¿Cómo inicia la escritura de un trabajo? (2) ¿En qué idioma? (3) Si escribe en inglés: (a) ¿a qué fuentes recurre para guiarse? (diccionarios en línea o impresos/traductores automáticos/asesoramiento externo, etc) ¿Podría mencionarlas? (b) ¿Desde cuándo escribe en inglés? (c) ¿Posee alguna rutina de escritura que pueda ser considerada parte de un proceso?

Researcher 1: Actualmente inicio la escritura de un trabajo directamente en inglés. Recurro permanentemente al diccionario de ejemplos linguee y al programa ant conc. Luego pido asesoramiento a expertos en la escritura para una revisión final. A partir de 2012/13 escribo en inglés. No tengo rutina de escritura.

Researcher 2: Actualmente inicio la escritura de un trabajo directamente en inglés. Recurro permanentemente En mi caso particular, tengo conocimientos de inglés de pequeño porque viví en el exterior, pero cuando volvimos lo dejé hasta la etapa de la universidad, en la qe tuve que leer mucho. Eso fue clave para tener modelos. Respecto de la ayuda, escribo con la ayuda de colegas y de asesores de GAECI. Antes hacía todos los cálculos y luego escribía el texto. Hoy hago todo de manera integrada

Researcher 3: Primero, acá hay algo distinto, a lo mejor, con respecto a la escritura, que como nosotros hacemos matemática, muchas veces, lo que tenemos es alguna conjetura (hipótesis) sobre si algo es cierto o no. Si es cierto, bueno, hay que armar la demostración. O sea, que primero viene toda la parte matemática. Una vez que uno tiene todo el esqueleto matemático de los resultados que puede llegar a tener. Recién ahí empezamos a escribir en palabras. Entonces recién ahí, uno escribe después de que ya tiene toda la estructura matemática con las pruebas armadas. Entonces es como que uno ya tiene todo un camino, pero bueno, el tema es, para generar los resultados matemáticos, las conjeturas que uno tiene a veces son falsas, entonces tiene que empezar a cualificar un poco las propiedades e ir viendo, ver por qué falla. Entonces, a lo mejor, uno quería presentar un cierto teorema, ve que no es cierto ero tener un buen contraejemplo también es algo que se puede publicar (o sea, probar que ese teorema que pensó está mal?), entonces es como que depende…uno no sabe de entrada lo que va a …que es cierto y qué no, entonces dependiendo de los resultados matemáticos que uno obtiene…después viene a parte de la escritura…pero uno de entrada no está bien seguro de lo que qué es lo que va a pasar ni qué puede llegar a obtener, no?, pero bueno, lo que se obtiene primero es todos los resultados matemáticos y en base a eso se escribe armando, bueno, el review, la literatura, los resultados que obtuvo, por qué son interesantes, y a lo mejor uno presenta una propiedad que es nueva en la literatura y tiene que defender por qué es buena, para qué sirve, qué aplicación económica en general tiene, porque nuestros trabajos son de teoría económica en general, pero bueno más o menos sería esa la forma.

Únicamente con el esqueleto matemático no se podría presentar un trabajo, en nuestro caso, que son de teoría económica, uno tiene que poner una buena motivación económica para todos los teoremas que presenta, un contexto y decir por qué son relevantes…porque un teorema, uno puede armar un teorema de cualquier cosa, ahora que sea interesante o no, tiene que ver con la motivación económica de fondo y si tiene o no importancia, y eso es muy difícil de hacer. Muchas veces nosotros tenemos muchos teoremas, un paper que tiene todos los teoremas, pero rebota en una revista porque dicen que no tiene relevancia. Entonces darse cuenta de qué es importante y qué no es la parte complicada para la mayoría de nosotros que no hay mucho entrenamiento en economía.

En cuanto a la escritura, al principio, los primeros papers eran en castellano y luego trataba de traducirlos, ahora hago ya todo en inglés porque el otro proceso es muy costoso y armar la idea en inglés o en español es más o menos lo mismo en términos de tiempo, porque también el inglés que usamos no es…digamos tan…florido, es co o más técnico y no hay que ponerle…mucho, no?

La mayoría de nuestros trabajos son hipotéticos, en el sentido de teóricos, uno presume una teoría, algo que puede ser razonable, pero después no es que hacemos algo empírico, alguna cosa como para ver si funciona. De eso se ocupan los economistas aplicados. Nosotros no. Sólo trabajamos en el plano de lo teórico. No estamos capacitados, no tenemos los métodos y no nos interesa.

Cómo se convence al lector/editor de lo que se propone en el trabajo sirve? Me parece que tiene que ver más con saber contextualizar lo que hacés con las referencias, la literatura que está relacionada y mostrar ahí que es importante, establecer conexiones y mostrar que están presentando algo nuevo, una cuestión que dejó abierta algún otro paper, o estás generalizando algún resultado que ….por ejemplo, muchas veces se trabaja de apoquito, entonces se presenta un resultado, la prueba para el caso adonde hay solo dos agentes de un resultado x y vos lo generalizas a N agentes utilizando una prpiedad, lo que tenes que demostrar es que esa propiedad no es tan fuerte o no demanda tanto, entonces si conveces desde ahí, parece que ya es razonable, porque bueno, estás generalizando de dos a cualquier cantidad de agentes, por ejemplo, no? A ese nivel es la argumentación.

Usa diccionarios, colaboracion con colegas, libro de William Thompsom, que además fue mi tutor. No tengo rutina de escritura, salvo lo de los resultados matemáticos: teorema, prueba, teorema, prueba. Una vez que lo tenga, escribo el resto.

Researcher 4: Yo creo que lo primero que hay es el problema que se quiere tratar, que puede ser más o menos teórico dependiendo del marco donde uno se encuentre. Una vez que uno considera que…o encuentra un problema interesante. Por lo menos en el area que nosotros manejamos, hay que ver si uno es capaz de encontrar algun resultado, de descubrir algun resultado en la direccion que uno pensaba, si tiene una conjetura a ver si encuentra algún resultado.

Después si uno da con algun resultado, que esa es una etapa, que en la matemática creo es la central, vienen las demostraciones, después de las demostraciones viene el hecho de contar cuál era el proyecto, que sería explicar la introducción, contextualizar, así funciona en general mi manera de investigar.

Respecto propiamente del inglés, uno cuando hace las pruebas, y escribe los resultados básicos, en general eso no le ofrece a uno resistencia, eso como es muy técnico, encontrar los conectores, se repite. Después la otra etapa sí es la difícil, la de contar el proyecto esa sí es la más difícil desde el punto de vista de la escritura. Encontrar las palabras adecuadas, los errores de ortografía, entonces lo que uno hace un poco es…bueno, va escribiendo, va consultando otra bibliografía, después hace varias revisiones. Y bueno, el tema de escribir la introducción también es un tema difícil. Pare escribir uso con referencia otros trabajos y diccionarios, ayuda de otros colegas.

Yo creo que, si el problema en sí es interesante, que a veces puede estar más vinculado a lo real y a veces puede ser más teórico, después todo lo demás es más fácil. Decir que es un problema estudiado, que otra gente se ha interesado, vincularlo con la literatura existente. A veces pasa que uno arranca con algo que parece interesante, pero después se encuentra con que no lo es tanto. Pero también hay lugar para eso, y bueno, pasa que uno resolvió un problema interesante, pero terminó en otro lugar. Parte de una conjetura (hipótesis) o a veces no partimos de una conjetura, no tenemos. La parte de esta de vincular con otro resultado es ingrediente para que el paper tenga éxito. Mucho de lo aprendido en inglés lo hice de grande,

en la etapa de produccion cientifica. En realidad, gran parte de la aceptacion o rechazo de un trabajo depende de la revista a donde se lo mande. En general son de economia, y por hi no importa tanto el problema sino qué tan bien motivado/fundamentado esté. Ese contar de la historia es lo complicado.

Researcher 5: Actualmente inicio la escritura de un trabajo directamente en inglés. Recurro permanentemente al diccionario de ejemplos linguee y al programa ant conc. Luego pido asesoramiento a expertos en la escritura para una revisión final.

Researcher 6: En general uno hace cuentas al principio en papeles. Bosquejos de demostraciones. Cuando escribo en papel en castellano. Cuando ya empiezo a usar la compu que la cuenta ya salió, en ingles. 3) Traductores online al comienzo. Graciela al final. :) Escribo en ingles desde que comencé mi doctorado, mi director primero me hizo escribir los resultados en ingles, formato paper. Luego comencé a escribir en castellano la tesis. Rutinas creo que no.

Researcher 7: La escritura del trabajo comienza introduciendo el modelo matemático que vamos a utilizar y definiciones previas las cuales serán utilizadas para presentar los resultados principales del trabajo. Cabe aclarar que los trabajos científicos previamente presentan una introducción, en mi caso siempre es lo ultimo que termino de escribir porque a medida que avanzo en el trabajo voy completando la introducción.