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Invariant solutions to the conformal Killing–Yano equation on Lie groups

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ABSTRACT

We search for invariant solutions of the conformal Killing–Yano equation on Lie groups equipped with left invariant Riemannian metrics, focusing on 2-forms. We show that when the Lie group is compact equipped with a bi-invariant metric or 2-step nilpotent, the only invariant solutions occur on the 3-dimensional sphere or on a Heisenberg group. We classify the 3-dimensional Lie groups with left invariant metrics carrying invariant conformal Killing–Yano 2-forms.

1. Introduction

The concept of conformal Killing–Yano p-forms (also known in the literature as twistor forms or conformal Killing forms) on Riemannian manifolds was introduced by Tachibana in [1] for $p = 2$ and later by Kashiwada in [2] for general p. Applications of these forms to theoretical physics were found related to quadratic first integrals of geodesic equations, symmetries of field equations, conserved quantities and separation of variables, among others (see, for instance, [3–7]). More recently, since the work of Moroianu, Semmelmann [8,9], a renewed interest in the subject arose among differential geometers (see, for instance, [10–16]).

Next we give the basic definitions and recall some well known properties of conformal Killing–Yano p-forms.

A p-form ω on an n-dimensional Riemannian manifold (M, g) is called *conformal Killing–Yano* (CKY for short) if it satisfies the following equation:

$$\nabla_X \omega = \frac{1}{p+1}\,\iota_X d\omega - \frac{1}{n-p+1}\,X^* \wedge d^*\omega, \quad X \in \mathfrak{X}(M) \tag{1}$$

where ∇ is the Levi-Civita connection, X^* is the 1-form dual to X and $d^* = (-1)^{n(p+1)+1} * d *$ is the co-differential. If, moreover, ω is co-closed, that is $d^*\omega = 0$, then it is called *Killing–Yano* (see [17]).

For $p = 1$, a 1-form ω is conformal Killing–Yano if and only if its dual vector field U is conformal, that is, $\mathcal{L}_U g = \varphi g$ for some $\varphi \in C^\infty(M)$. A 1-form ω is Killing–Yano if and only if its dual vector field is Killing.

We list next some properties:

- If ω is a CKY p-form on M, then $*\omega$ is a CKY $(n-p)$-form, where $*$ is the Hodge-star operator. In particular, $*$ interchanges closed and co-closed CKY forms (see [18,9]).

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