Robustness and Weakness of the Classical Nonlinear model for circular cones at supersonic speed: New Closed-forms solutions, Crossed regions, and a new theoretical lower boundary.

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ABSTRACT

Any analytical closed-form solution can offer a deep insight into understanding the fundamentals of the associated theory, but it is not always available at a particular time. For example, in modern compressible flow literature, Anderson 2003, the author claimed that the equation of Taylor and McColl did not have closed form solution and must be solved numerically and that had been true for many years. In fact, just the numerical solution of the Taylor-Maccoll formulation, not the analytical one, has existed since 1933. In the original work, a conical body of circular cross section at zero angle of attack inside a supersonic flow and the corresponding shock wave were studied, (Taylor & Maccoll 1933

and Stone 1948 and 1952). Tables and charts presenting solutions to the Taylor-Maccoll have been available for many decades, while alternative numerical solutions to the equation are easily obtained from programmable hardware and comfortable software at the time that numerous approximate analytical representations, for this and other closely related problems, are presented as perturbations or series expansions. In particular, the Taylor-Maccoll boundary value problem, has been widely considered in the literature by numerical and perturbation methods. However, to 2011, no closed form solution to the Taylor-Maccoll equation had been presented even when closed analytical results began to appear as promissory.

Recent work by Ferreyra and Tamagno 2012 shows that this model was solved by the "conic solution", a closed-form solution. Although the handy conic solution is not similar to the existing solutions, the set of solutions provided was demonstrated to be good enough for small cone angles. To overcome the problem the aim was to demonstrate a new closed-form solution for large cone angles in good agreement with existing classical results. Later, this goal was reached, Ferreyra 2012, but it was also shown that an infinite number, of closed-form solutions, exists whatever the conical angle was. As a consequence, the validity of the traditional lower boundary in the (cone angle-shock angle-Mach number) chart is difficult to be accepted, at least under the theoretical results. Furthermore, the traditional numerical lower boundary and the new analytical one generate contradictory crossed regions that imply absurdness. Therefore, in the nearest future, some sensitive experimental results are needed to provide confidence either to the newer analytical results or to the old numerical ones, to bridge the gap between the physical behavior and theoretical results which are able to make stronger, or weaker, the original analytic model.