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Weighted estimates for integral operators on local BMO type spaces

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We prove the weighted boundedness for a family of integral operators T_α on Lebesgue spaces and local BMO type spaces. To this end we show that T_α can be controlled by the Calderón operator and a local maximal operator. This approach allows us to characterize the power weighted boundedness on Lebesgue spaces.

1 Introduction

In this paper we will study the weighted boundedness for a family of integral operators on different spaces of functions. To be more specific, for $n \geq 1$ and $0 < \alpha < n$, we define T_α by

$$T_\alpha f(x) = \int_{\mathbb{R}^n} \frac{f(y)}{|x-y|^\alpha |x+y|^{n-\alpha}}\, dy.$$

These operators were introduced in [6]. In that article the authors proved the boundedness on $L^2(\mathbb{R})$ in order to get the main results concerning a family of maximal operators on the three dimensional Heisenberg group. In [3] it was proved that this family of operators is of type (p, p) for $1 < p < \infty$ and of weak type $(1, 1)$ on $\mathbb{R}^n$.

Weighted boundedness for a more general family of operators is studied in [7]. The authors used the unweighted results already known to prove the weighted boundedness of type (p, p) for $1 < p < \infty$ and of weak type $(1, 1)$ for a wide family of weights in A_p. They also proved an appropriate weighted estimate from a subset of $L^\infty(\omega^{-1})$ into $BMO(\omega)$. Additionally, the boundedness of this type of operators on the Hardy spaces H^p is studied in [8].

In this article we prove the weighted boundedness on Lebesgue spaces for the operators T_α defined above, and we do this in a different way that in [7]. We bound T_α with the Calderón operator and a local maximal operator, and then we use the known results for these. This method allows us to obtain at the same time the weighted and unweighted boundedness. This approach leads us to characterize the boundedness of T_α for power weights.

Finally, based on the articles [1] and [4], we prove the boundedness of T_α from a subset of a local $BMO(\omega)$ type space into $BMO(\omega)$. This result improves the one in [7].

In Section 2 we recall some definitions and preliminary results that will be needed in this paper. In Section 3 we state our results and we give the necessary definitions. Finally, in Section 4 we prove our results.

2 Preliminaries

It is well known that a weight is a locally integrable and non-negative function, and the Muckenhoupt class A_p, $1 \leq p < \infty$, is defined as the class of weights ω such that for all balls B

$$\left(\frac{1}{|B|}\int_B \omega\right)\left(\frac{1}{|B|}\int_B \omega^{-\frac{1}{p-1}}\right)^{p-1} < C, \quad \text{if } 1 < p < \infty, \tag{2.1}$$

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