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# *Pure type systems with explicit substitutions*

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**Abstract**

We introduce a new formulation of pure type systems (PTSs) with explicit substitution and de Bruijn indices and formally prove some of its meta-theory. Using techniques based on Normalisation by Evaluation, we prove that untyped conversion can be typed for predicative PTSs. Although this equivalence was settled by Siles and Herbelin for the conventional presentation of PTSs, we strongly conjecture that our proof method can also be applied to PTSs with $\eta$.

## 1 Introduction

In this article, we introduce a new formulation of pure type systems (PTSs) with explicit substitutions and de Bruijn indices. We consider two formulations: a *semantical* variant with typed equality and a *syntactical* version with untyped conversion. For both of them, we formally develop some of its meta-theory including the substitution lemma.

Whereas the first formulation is convenient for theoretical considerations (Streicher, 1989; Miquel & Werner, 2002), the latter is often used in implementations like Coq (2004) and Matita (Asperti *et al.*, 2011). The kernels of these systems are implemented with terms using de Bruijn indices instead of named variables (Barras, 1998) and substitutions are handled explicitly rather than as meta-level operations.

The equivalence between these two formulations bridges the gap between the systems usually considered in theoretical considerations and the ones actually implemented in proof assistants. This result has been established by Siles and Herbelin (2012) for the conventional presentation of PTSs using syntactic methods.

Our main contribution is the proof of that equivalence for a class of PTSs which we call predicative, by semantical means, using techniques based on Normalisation by Evaluation (NbE). We have a strong confidence that this method can be used to prove the key result for other systems. In particular, we are starting to work out the details for PTSs with $\eta$ and plan to report them in a forthcoming paper.

### *1.1 Related work*

As far as we know, our formulation of PTSs is new, although the variations that we add were already considered separately. The three aspects in which we depart from usual PTSs are: explicit substitutions, de Bruijn indices, and abstractions without