Nonlinear Dynamics for the prediction of Statistical Distributions: An Operational Research perspective

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Abstract A semantic connection between nonlinear or linear dynamical systems and statistical Gaussian distributions is developed as a theory. This connection is applied to predict changes in the Gaussian distribution when models of a complex system are available for making decisions in any operational research framework A typical real case is considered in the analysis and the application of the theory is explained. As a theoretical extension, modified confidence intervals and extended hypothesis testing related to inferential statistics are briefly discussed to consider the global impact of this developed tool.

Introduction

Dynamic nature of things can be visualized in a wide variety of human or non human interactions, populations, evolution processes, diseases and systems and so on. Dynamics is usually formalized throughout (deterministic or stochastic) nonlinear dynamical models which involve "a dimensional world (real world)" where time and a kind of frequency (or time and velocity) appear to be coupled to the spatial variable which together with its variations have closed relationships associated to forces. In this real world dimension exists and dynamic requires a dimensional variation. The fact is to know what sort of variations we need to study and how we should do it (See [1-2]). Nonlinear dynamic equations and statistical theory are common tools in the study of dynamic systems. In particular, statistical branches have been developing more or less separately from this dynamic scenario, but probably much more than less. It is naturally accepted that nobody knows who is going to be born tomorrow. But many people globally know a representative amount of births for tomorrow by following statistical criteria at the time that these expectations change from place to place, and from one period of time to another one. This changing behavior, which is related to space and time, probably means the existence of an implicit deterministic dynamic law that is useful to predict approximately almost all global variations. Furthermore, by assuming that knowledge (or information) for every system in the world has a tendency, a

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repetitive behavior and noise with or without sharp predictable events (or non predictable events), it appears to be wise to predict (not only to understand) the change of tendency from historical well known scientific facts (or true facts) without introducing either new restrictions or early excluded principles, but by making a different reading of the available classical models to help to generate (or to trigger) better decisions. A better decision for the future should not mean the best in a local scene or just the optimized one. Probably, it deals with a “quiet world to eyes”, like a permanent photograph. What is more, this permanent photograph should be understood as a picture from the beginning to the end of times in order to connect past, present and future. This new or imaginary world model should contain a not always perceptible but intense dynamical reality inside. In this context, a dynamic equilibrium is reached with a probably minimum of classical dynamic foundations such us energy storage capability, a dissipation mechanism, inertial properties (or their conservation rules). The ability to identify internal and external forces and the boundary of the iterative homeostatic dynamical system under study is also needed. Such system is living inside this semantically temporal-less different dynamical world. In this imaginary, parallel and quiet world to eyes, everything is assumed to have a dynamical behavior throughout a non sensorial perceptible but global independent variable. On the other hand, things into the real world are different. This real moving world is the one in which evolution of the dynamic system takes place and the one where it is possible to measure time. Now, it is postulated that a closed correlation between these two worlds exists. So, in order to connect these worlds, the key is to note that it does not matter if a body is shaking in the real, temporal intuitive, continuously moving, and sensorial dependent world, because eternal truth will be the same in the parallel quiet imaginary world. In principle, formalized rules or equations should consider the same kind of terms in both speculative words (real and imaginary) because in both worlds the universal physical laws are valid. This concept is also supported by the imaginary permanent photograph, the only one real word and two subjective ways of choosing independent variable to estimate variations as we are going to see below. However, parameters can become variables from one speculative world to the other, among other features. Then, solutions are not invariants from one world to another. In fact, they usually change providing non-temporal laws in the quiet word which can be recognized as derived rules from recognized scientific axioms of the day to day physics (at real life scale). In this paper it is proved that some classical statistical distributions are derived from this “quiet world”. These classical distributions can be modified properly by nonlinear (or linear) analysis to improve decisions, due to the fact that they are the solutions of nonlinear differential equations stated after the implementation of permanent physical laws that in many cases involve handy and adaptable parameters. In consequence, since physical rules are shared by these two worlds, the purposes of this work are two. First, to provide a formal point of view, but simple tentative connection between these two worlds, and, second, to be able to use dynamics rules as deterministic external agents allowed to modify the shape of the involved statistical distributions in order to fit decisions (and things) into their external (most real) physical support. Finally, since a wide variety of potential applications (not numerable neither here nor anywhere) begging to appear after this theoretical bridge, the most elemental and statistical concepts (or procedures) related to almost everything (health care, economy, politics and social sciences, etc.) such us confidence intervals, hypothesis testing and diagnosis assessment are considered and connected to the given theory, (For example, see [3-6]).

A broad perspective in OR research

The developed point of view could produce consequences in the Operational Research, OR, field. As a first sight, descriptive statistics and inferential statistics provide two ways for direct consequences. From descriptive statistics, the μ parameter is obtained and it is one of the terms in the relationship

$$\alpha = y - \mu \tag{16}$$

This relation connects inferential statistics (many times useful in OR context) to the "Twin World Theory", (or just TWT), (see [7]). The μ parameter indicates the center, or mean, of the distribution. Now, α is working as a kind of error, just because y is measuring something. According to TWT, standard deviations obey universal rules (physical, economical,..). Let Z be the variable of a standard normal distribution with parameters μ and σ, and let σ^* be the standard deviation obtained by applying TWT. Thus, the relationships with the rest of variables and parameters are

$$Z = \frac{y-\mu}{\sigma} = \frac{\alpha}{\sigma} = \frac{\alpha}{\sigma^*} = \frac{y^* - \mu}{\sigma^+} \tag{17}$$

In applying TWT, when you read the measure y, you should also have expectations of reading a more representative y^*. In other words, these equalities make us aware of the sense that TWT can be satisfied at the time that classical terms do not. The reciprocal situation is true (your expectations belong to TWT). This observation should alert somebody before making a decision.

A sample average will approximately follow a normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$, where n is the size of the sample. If we follow a procedure to approximately capture the value of μ, we will finally know with probability $1-\delta$ that μ will fall inside the confidence interval

$$\overline{X} - z_{1-\frac{\delta}{2}} \frac{\sigma}{\sqrt{n}} < \mu < \overline{X} + z_{1-\frac{\delta}{2}} \frac{\sigma}{\sqrt{n}} \tag{18}$$

In this expression, $z_{1-\frac{\delta}{2}}$ is the point z associated with a standard normal curve such that for random variable Z, the probability of $Z < z_{1-\frac{\delta}{2}}$ is equal to $1-\frac{\delta}{2}$. Therefore, it is possible to see

$$z_{1-\frac{\delta}{2}} \frac{\sigma}{\sqrt{n}} \neq z_{1-\frac{\delta}{2}} \frac{\sigma^*}{\sqrt{n}} \tag{19}$$

Frequently, the confidence interval is stated by choosing δ for a given σ. However, it is possible to change the confidence by fixing δ and considering σ^*, or keeping σ as it is, and changing the confidence through δ. For instance, these procedures imply practical considerations during hypothesis testing. For a given n, once the confidence is adopted by selecting δ (so the critical region is stated), the acceptance region in terms of physical variables can be rejected with σ and accepted with σ^* (or the opposite according to the nature of the problem). Finally, in diagnosis assessment with prevalence P, the confidence interval for prevalence can be bigger (or smaller) than the classical confidence interval, when the dynamic analysis is considered. The same fact can occur with, sensitivity, specificity, predictive positive values, predictive negative values. So, it is worth to suppose that these consideration can sensitively affects the decision process related to health care, economy, politic and social sciences. Then, major changes should be implemented in the decision process.

Conclusion

A supply of dynamic equilibrium for randomness should be given as an external support (or as a deterministic order) based on universal but basic physical laws, at daily real world life scale. The standard deviation versus dissipation curve was connected in context to be the backup to decisions which consider available statistical and dynamical information from the model. The popular belief that the order and randomness coexist together at the scale of common life was adopted to support the model. Confidence intervals, hypothesis testing, diagnosis assessment as complementary tools for the developed theory were briefly discussed under the idea of being powered in order to be applied in health care, economy, politics and other social sciences. So, in the future, practical applications with straight and emergent real connotations have to be done in humanitarian sciences.

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REFERENCES

1. R. T. Ferreyra, E. Fogliato, M. A. Ferreyra, S. García, Oscilaciones de Relajación en la Dinámica No Lineal de la Depredación entre Sistemas, Revista de MECANICA COMPUTACIONAL Vol XXVII, 2008, págs. 2411-2417, ISSN 1666-6070.

2. R.T. Ferreyra, C.A. Prato, Análisis No Lineal de Respuesta en Tiempo y Frecuencia de una Placa Plana con Grandes Rotaciones por Elementos Finitos y Escalas Múltiples en Órdenes Superiores de Aproximación, Revista de MECANICA COMPUTACIONAL, vol. XXVI, 2007, págs. 2471-2478.
3. http://escuela.med.puc.cl/publ/Boletin/obesidad/ObesidadManejo.html
4. http://www.nhlbi.nih.gov/health/public/heart/obesity/wecan/healthy-weight-basics/balance.htm
5. http://www.labstein.com/informacion-para-medicos/calculo-de-requerimientos.html
6. http://es.wikipedia.org/wiki/Ecuaci%C3%B3n_de_Harris-Benedict
7. R.T. Ferreyra, M. A. Ferreyra, Dynamical Laws for Statistical Distributions: Application to Complex System Analysis, Journal of Mechanical Engineering and Automation, 2013, vol. 3, Number 2.

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