Cointegration and rolling window cointegration analysis of a selected group of stock market indices

Sergio Martín Buzzi, Silvia María Ojeda

Ponencia presentada en 1º Congreso Argentino de Estadística (CAE I). XLIII Coloquio de Estadística. XX Reunión Científica del GAB realizado en 2015 en la Universidad Nacional de Tres de Febrero. Buenos Aires, Argentina

COINTEGRATION AND ROLLING WINDOW COINTEGRATION ANALYSIS OF A SELECTED GROUP OF STOCK MARKET INDICES

SERGIO MARTÍN BUZZI[1], SILVIA MARÍA OJEDA[2]

[1]*Departamento de Estadística y Matemática, Facultad de Ciencias Económicas, Universidad Nacional de Córdoba.*
[2]*Facultad de Matemática, Astronomía y Física, Universidad Nacional de Córdoba.*

sergio.buzzi@eco.unc.edu.ar

SUMMARY

In order to explore the degree of integration of international stock markets we select a group formed by the most important indices considering their market capitalization and geographical distribution. After testing for unit roots using the Augmented Dickey Fuller (ADF) and the Kwiatkowski Phillips Schmidt Shin (KPSS) tests, a full sample cointegration analysis is done provided that all the indices are found to be I(1). The existence of cointegration relationships can be interpreted economically as the existence of markets integration. Further, a rolling window cointegration testing procedure is implemented in the programming language R, in order to characterize the dynamic of the degree of integration of stock markets. This analysis provides valuable information, given that an increase in the degree of cointegration can be interpreted as a signal of the presence of contagion among markets.

Keywords: ***cointegration, rolling window cointegration, time series, stock markets***.

Introduction

In this paper we pose the question of whether there is co-movement on international stock markets. With this objective in mind, we test for cointegration a selected group of indices. The selection of the markets is based on their relative importance, measured by market capitalization. The selected indices and their corresponding countries are: MERVAL (Argentina); BVSP (Brazil); GSPC (United States of America); NDX (United States of America); FTSE (United Kingdom); FCHI (France); GDAXI (Germany); HSI (China); SSEC (China); SHENA (China); and N225 (Japan). The mentioned group accounts for more of 60% of total World market capitalization. Provided that we are interested in the MERVAL index of Argentina, we have included it and, given the influence of Brazil in our economy, the Bovespa Index is also considered.

There are reasons for believe that stock markets share common trends given the transmission of macroeconomic crisis and fluctuations, and the existence of multinational corporations. On the other hand, economies also have domestic determinants of economic activity which impact on their stock markets indices, and additionally the indices have different compositions. It should be noted that there are arguments both for and against the existence of cointegration between the various indices. If two indices are composed of similar assets, arbitration is likely to generate both indexes are cointegrated. Otherwise, the relationship could fade unless there are sufficiently strong underlying macroeconomic factors. Therefore, it is expected a priori to find evidence that some markets are interrelated long-term (while others do not). In order to learn about the degree of international markets integration, we test for cointegration using the full sample period.

A similar analysis con be made for crisis periods, because if cointegration is stronger in crisis periods it constitutes an evidence of contagion.

In the literature are mixed results on the existence of cointegration between the indices of the various stock markets, according to Chen (2012) this may be due to the changing nature over time of the interrelationships between the stock markets. That is to say, it is very likely that the cointegration relationships are not stable over time, the same occurring with the magnitudes of the same.

In order to explore whether or not the degree of stocks markets international integration is varying over time we carry out rolling window cointegration analysis. Chen (2012) points out that since Granger (1986) "studies on stock market integration tend to make reference to market efficiency hypothesis based on the finding of cointegration relationship", however it is important to note that international stock markets cointegration is not generally equivalent to market inefficiency.

Crowder (1996), identifies four sources for the existence of cointegration. First, markets are inefficient and traders are indeed wasting valuable information. Second, markets are efficient but there exist some omitted factors, such as a risk premia or regime switches, that manifest themselves as cointegration. Third, markets are inefficient but agents are ignoring the information from the ECM because it cannot engender significant profits. Fourth, finding of cointegration is due to questionable statistical properties of the tests. This discussion refrain us from using evidence on cointegration as evidence for market efficiency. Consequently, if there is cointegration it is interpreted as evidence of predictability or co-movement.

Development

We make the analysis using the adjusted close values of the selected indices in a daily base for the period from 1998-12-15 to 2014-05-09. In order to avoid the distortion produced by currency depreciation we express the series in a common currency, specifically in American dollars. Further, each index is standardized dividing its values by its initial value and multiplying by 100. Finally, natural logarithm is taken. Following, we proceed to test for unit roots using two standard procedures, the Augmented Dickey Fuller (ADF) unit root test (Dickey and Fuller 1981), and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) stationarity test (Kwiatkowski *et al*. 1992). The ADF test posses the null of unit root by estimating the following three auxiliary regressions:

$$\Delta y_t = \beta_1 + \beta_2 t + \pi y_{t-1} + \sum_{j=1}^{k} \gamma_j \Delta y_{t-j} + u_{1t}$$

$$\Delta y_t = \beta_1 + \pi y_{t-1} + \sum_{j=1}^{k} \gamma_j \Delta y_{t-j} + u_{2t}$$

$$\Delta y_t = \pi y_{t-1} + \sum_{j=1}^{k} \gamma_j \Delta y_{t-j} + u_{3t}$$

The choice of the auxiliary regression model is important to avoid loss of power. The number of lags on the difference is determined in each case by the Akaike Information Criterion (AIC). We start with the more general model testing if the trend coefficient is zero. If not, the second auxiliary regression is used to testing the drift term significance. If the drift term is not significantly different from zero, the third model is selected. Once determined the right model, this auxiliary regression is used for testing the parameter of y_{t-1}.

Provided that the ADF test could incorrectly fail to reject the null of unit root, we also employ the KPSS test which posses the null of stationarity.
This test is based in the following model:

$$y_t = \beta' D_t + \mu_t$$

$$\mu_t = \mu_{t-1} + \varepsilon_t$$

where $\varepsilon_t \sim WN\left(0, \sigma_\varepsilon^2\right)$, and μ_t is a pure random walk with innovation variance σ_ε^2. The null hypothesis is $H_0 : \sigma_\varepsilon^2 = 0$, which implies that μ_t is a constant, whereas the alternative is $H_1 : \sigma_\varepsilon^2 > 0$.

Once determined the integration order of the series, we test for cointegration using the Johansen-Juselius approach, which is based in the following model:

$$\Delta y_t = \Gamma_1 \Delta y_{t-1} + ... + \Gamma_1 \Delta y_{t-p+1} + \Pi y_{t-1} + \mu + \Phi D_t + \varepsilon_t$$

and the construction of both the maximum eigenvalue $\lambda_{\max}$, and the trace λ_{trace} statistics:

$$\lambda_{\max} = -T \ln\left(1 - \hat{\lambda}_{r+1}\right)$$

$$\lambda_{trace} = -T \sum_{i=r+1}^{n} \left(1 - \hat{\lambda}_i\right)$$

The first statistic is used for testing the existence of *r* vs *r+1* cointegration relationships, whereas the second one is used for testing if there are at most *r* cointegration vectors.

Results and Conclussions

This section summarizes briefly the main results obtained from both unit root and cointegration tests.

Table 1

Augmented Dickey Fuller unit root tests

Index	model	statistic	critical
MERV	none	0.35	-1.95
BVSP	none	0.43	-1.95
GSPC	none	0.75	-1.95
NDX	none	0.82	-1.95
FTSE	trend	-2.31	-3.41
FCHI	trend	-2.34	-3.41
GDAXI	trend	-2.38	-3.41
HSI	trend	-2.64	-3.41
SSEC	none	1.43	-1.95
SHENA	none	1.75	-1.95
N225	none	0.27	-1.95

Table 1 shows the results of the ADF test. Column 2 displays the auxiliary regression specifications selected, column 3 shows the observed values of the t statistic, and column 4 the corresponding critical values.
Provided that in all the cases the statistic value is greater than the critical one (5% significance), the null of unit root is not rejected.
Table 2, summarizes the results for KPSS stationarity tests. The critical values for a 5% significance level for mu and tau are 0.46, and 0.15 respectively. The stationary test is a one-sided right-tailed test so that one rejects the null of stationarity at the 100 · a% level if the KPSS test statistic is greater than the 100(1 - a)% quantile from the appropriate asymptotic distribution.

Table 2
KPSS stationarity tests

Index	mu	tau
MERV	8.59	1.29
BVSP	22.00	3.42
GSPC	8.24	2.90
NDX	9.48	4.14
FTSE	3.48	1.29
FCHI	3.40	1.86
GDAXI	19.51	1.61
HSI	20.08	1.46
SSEC	16.51	1.66
SHENA	20.56	2.44
N225	3.50	1.78

Given that the 5% critical values for mu and tau are 0.46 and 0.15 respectively, in all the cases the null of stationarity is rejected. Consequently, both tests shows that all the series are I(1). Further, we proceed to testing for cointegration.
Table 3 and **Table 4**, show both the maximum eigenvalue and the trace statistics, and their corresponding critical values. The maximum eigenvalue statistic suggest that there are two cointegration relationships. On the other hand, the trace statistic find one cointegration relationship at the usual 5% significance level.

Table 3
Johansen cointegration test's maximum eigenvalue statistic

	test	10pct	5pct	1pct
r <= 10	0.42	6.50	8.18	11.65
r <= 9	6.07	12.91	14.90	19.19
r <= 8	11.29	18.90	21.07	25.75
r <= 7	11.76	24.78	27.14	32.14
r <= 6	15.15	30.84	33.32	38.78
r <= 5	16.71	36.25	39.43	44.59
r <= 4	21.81	42.06	44.91	51.30
r <= 3	36.65	48.43	51.07	57.07
r <= 2	46.18	54.01	57.00	63.37
r <= 1	63.68	59.00	62.42	68.61
r = 0	75.41	65.07	68.27	74.36

Table 4

Johansen cointegration test's trace statistic

	test	10pct	5pct	1pct
r <= 10	2.40	7.52	9.24	12.97
r <= 9	8.48	17.85	19.96	24.60
r <= 8	19.80	32.00	34.91	41.07
r <= 7	34.73	49.65	53.12	60.16
r <= 6	50.75	71.86	76.07	84.45
r <= 5	68.20	97.18	102.10	111.00
r <= 4	90.01	126.60	131.70	143.10
r <= 3	126.70	159.50	165.60	177.20
r <= 2	173.10	196.40	202.90	215.70
r <= 1	236.90	236.50	244.20	257.70
r = 0	312.30	282.40	291.40	307.60

Lütkepohl *et al*. (2001), find that "the trace tests tend to have more distorted sizes whereas their power is in some situations superior to that of the maximum eigenvalue tests". Further, they advise using both test, and point out that based on their simulations they "have a preference for the trace tests. Following this line, we conclude that there is only one cointegration relationship.

Finally, we compute 5 years (1250 days) rolling cointegration tests and plot in **Figure 1** and **Figure 2** the maximum eigenvalue and the trace statistics deviations from its respective critical values, both for the first cointegration relationship and for the second one.

Figure 1

First cointegration relationship 5 years rolling window tests

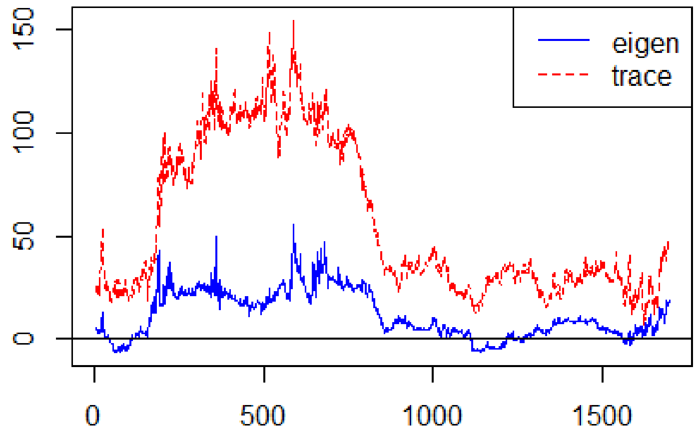

Figure 2

Second cointegration relationship 5 years rolling window tests

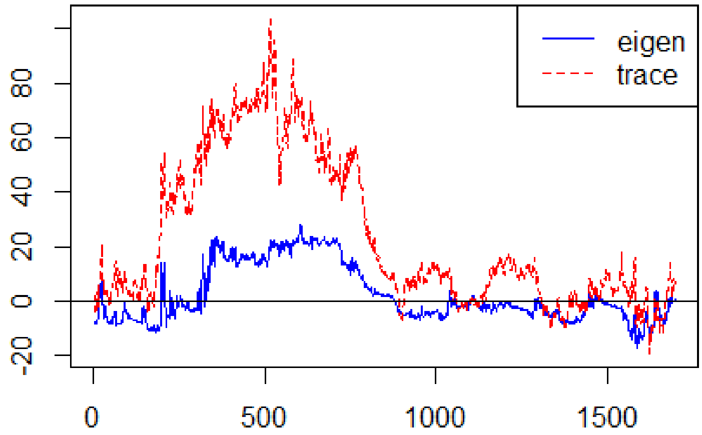

Figure 1 shows that the first cointegration relationship remains valid in almost all the period, being that an evidence of its robustness. On the contrary, **Figure 2** suggests that the second cointegration relationship may only exist for the period from February of 2007 to December of 2010.

To sum up, we find evidence of no mean reversion for all the analyzed indices, and the existence of one clear cut cointegration relationship among them. This latter fact provides evidence on the existence of stock markets integration. Also, we find that the degree of integration changes over time. In particular, exchange markets seem to be more integrated in the period from February of 2007 to December of 2010.

References

Chen, X. (2012). *Empirical investigations into stock market integration and risk monitoring of the emerging Chinese stock markets* (Doctoral dissertation, University of St Andrews).

Crowder, W. (1996). ”A Note on Cointegration and International Capital Market Efficiency: A Reply”. *Journal of International Money and Finance*, 15(4): 661-664.

Dickey, D. and Fuller, W. (1981), “Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root”. *Econometrica*, 49, 1057-1072.

Granger, C. (1986). ”Developments in the Study of Cointegrated Economic Variables”. *Oxford Bulletin of Economics and Statistics*, 48(3): 213-228.

Johansen, S. (1991). "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models". *Econometrica*, 59(6): 1551-1580.

Kwiatkowski, D.; Phillips, P.; Schmidt, P.; and Shin, Y. (1992). “Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root?”. *Journal of Econometrics*, 54: 159-178.

Lütkepohl, H., Saikkonen, P., & Trenkler, C. (2001). “Maximum eigenvalue versus trace tests for the cointegrating rank of a VAR process”. *The Econometrics Journal*, 287-310.