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A Model about the Interaction of the Monetary Policy in an Advanced and an Emerging Economy

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Resumen: Se desarrolla un modelo tipo DSGE para dos economías abiertas (avanzada y emergente). Una distinción crítica entre ambas economías es la existencia de fricciones financieras: en la economía avanzada las imperfecciones afectan el mercado de crédito y en la emergente el tipo de cambio está sujeto a fricciones. También existe una distinción relacionada a la política monetaria que implementen los Bancos Centrales: el de la economía desarrollada dirige su política a monitorear la condición de su sistema financiero, mientras que el de la emergente concentra sus esfuerzos en regular la evolución de los pagos externos y el tipo de cambio.

Abstract: A Dynamic Stochastic General Equilibrium model is developed for two open economies (advanced and emerging). A critical distinction between the economies rests on the location of financial frictions: imperfections affect the domestic credit market in the advanced economy, and the foreign exchange market is subject to frictions in the emerging one. There is also a distinction related to the monetary policy implemented by each Central Bank: the developed economy directs its policy to monitor the condition of its own financial sector, while the emerging economy focus its efforts on regulating the evolution of external payments and the exchange rate.

Key Words: *DSGE; Open Economy; Monetary Policy; Neokeynesian Model; Central Banking; Stabilization*

JEL Clasification: E520; E580

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1 Introduction

The international financial crisis that took place in 2008 has revealed several flaws in the dominant monetary policy paradigm that prevailed in the first decade of the 21st Century. Despite the fact that the Inflation Targeting monetary regime constituted one of the most outstanding developments of the last four decades, it had to turn into a Flexible Inflation Targeting monetary regime, presumably in order to achieve greater coordination of international economic policy while consolidating price stability.

Not only academic leaders,¹ but also government officials and central bankers, have been expressing that prices and activity stabilization are not enough for preserving the financial system from fragility. Stabilizing inflation is also a too narrow objective for a Central Bank during crisis time, an aspect that was clearly illustrated by the last episode when the world-leading Central Banks were forced to implement unconventional monetary policies in order to avoid a collapse, while Central Banks in emerging economies found themselves in the need to regulate strong capital flows and intervene in the foreign exchange market as a means to mitigate the adverse effects of external shocks. In this sense, traditional measures to control financial risk have proven to be unsuccessful, and those effects of monetary policies that go beyond the boundaries of the countries that implement them have been far from negligible. Additionally, the intervention of emerging economies in the foreign exchange market has provoked a significant accumulation of international reserves in their vaults in the spirit, perhaps, of a self-insurance strategy, with the consequences for the global economy of this relatively novel behavior not being yet fully understood.²

Is in the context previously described that some questions flow naturally, namely: Will there be modifications in the practice of monetary policy both, in developed and emerging economies, after the crisis? Which should be the short run capital flow behavior? Will the internalization of spillovers reach more stability in emerging countries? And will the problems of international coordination of monetary policy raise the attention they deserve in the new context? While there have been several works dealing with the behavior of Central Banks in developed countries, most cases were always aimed at determining the actions of monetary policy only in these countries. Thus, a tentative answer is that, under certain conditions, the most convenient arrangement is a kind of “labor division” in which Central Banks in developed economies will direct their policies to monitor the conditions of their own financial sector, while Central Banks in emerging economies will focus their efforts on regulating the evolution of external payments and the exchange rate. Thus, the actions mentioned, are complementary to the conventional objectives of stabilizing prices and output, which implicitly leads to the antediluvian (but very often overlooked) issue about the number of instruments and objectives.³

There are also models that describe the influences that financial sector exerts on Macroeconomics. Thus, for example, a very comprehensive model for Argentina is that developed by Escudé (2008) where the Central Bank can influence the macroeconomic results through changes in the interest rate or interventions in the exchange rate market with the aim of

¹See, for instance, Eichengreen et al. (2011).

²See, for example, Bernanke (2005) as a reference to the Global Saving Glut hypothesis.

³We are referring to the rule formulated by Tinbergen (1952). As regards to the application of this principle to the case of monetary policy in emerging economies, see Escudé (2013).

moderating the exchange rate volatility. Aguirre and Blanco (2013) also elaborate a model for Argentina, tending to describe the monetary policy taking into account the credit market and the interest rate spreads while considering also the sterilized intervention in the exchange rate market.

Generally speaking, all proposed models are either for developed economies or for a small and open economy. In this paper, a model for both types of economies is proposed trying to shed light on the important role of their interactions and also a salient feature of the model is that intermediation is subject to imperfect competition and financial frictions, creating an additional transmission channel for monetary policy.

The remainder of the paper is organized as follows: the methodology and the model implemented is presented in the next section, taking into account the sectors involved (households, firms, banks and Central Banks and Governments for both economies) and equilibrium conditions. In section 3 some concluding remarks and possible lines for future research are presented.

2 The Model

In order to address the aforementioned issues, a model in the Dynamic Stochastic General Equilibrium (DSGE) tradition is invoked.⁴ A brief outline of it follows.

Two open economies are considered, an advanced and an emerging one. Each economy consists of the following sectors, namely: (i) Households, (ii) Firms, (iii) Banks, and the Public Sector (including both fiscal and monetary authority). The labour supply and the demands for domestic currency, deposits (denominated in domestic currency) and foreign currency (in the case of the emerging economy) are modelled with the aid of an extended Money-in-the-Utility (MIU) framework.⁵ Also, the usual Calvo pricing protocol is used to produce a Phillips Curve for domestic inflation.⁶ The banking sector raises funds by creating deposits and lends them to firms feeding the productive process. A salient feature of the model is that intermediation is subject to imperfect competition and financial frictions, which creates an additional transmission channel for monetary policy.⁷ Finally, the monetary authority sets domestic interest rates according to an “expanded” Taylor rule and uses a complementary instrument, an option made available due to the existence of alternative financial markets and frictions.⁸

There are essentially five categories of objects -goods, services and assets- involved in trade in each economy. These are: (i) goods, both for consumption and investment, (ii) productive services of capital and labour, (iii) deposits (issued by the banking sector) and bonds (issued by the government at the advanced economy), which act as a means of saving, (iv) loans and other forms of credit, such as interbank lending and discount window facilities, and (v) currency.

⁴Christiano et al (2004) and Smets and Wouters (2002) are representative of this approach.

⁵See Walsh (2010), Ch. 2.

⁶See Calvo (1983).

⁷See Hafstead & Smith (2012).

⁸See Taylor (1993).

Markets for goods, currency and bonds are open to international trade. The same is not true for productive services of capital, labour, deposits, loans and interbank credit. As anticipated, foreign currency, i.e. currency issued by the Central Bank at the advanced economy, will be demanded by households at the emerging economy.⁹ This conforms the rudiment of a foreign exchange market at the emerging economy. Moreover, a critical distinction between the economies under study rests on the location of financial frictions: while imperfections will affect the domestic credit market in the advanced economy, it is the foreign exchange market that will be subject to frictions in the emerging one.

The succeeding subsections contain a detailed discussion of the sectors. At this stage, however, it is useful to introduce some notation that will be adopted all along the paper:

- A starred symbol refers to the advanced economy while the absence of a star indicates that it represents an object related to the emerging economy.
- When treating the good and currency markets -both of which are open to international trade-, a superscript (**a** for ‘advanced economy’ or **e** for ‘emerging economy’) is added to denote its origin, that is, the economy at which the good or currency has been produced or issued, respectively. For example, M_t^a denotes the amount of foreign currency (which was issued at the advanced economy) held at the emerging economy and M_t^{a*} denotes the amount of domestic currency at the advanced economy.
- When referring to prices, a superscript (**c** for ‘consumption’ and **k** for ‘investment’) indicates the category of good to which it belongs.
- When referring to interest rates, a superscript (**d** for ‘deposits’, **b** for ‘loans to firms’, **ib** for ‘interbank credit’, **g** for ‘government bonds’ and **dwf** for ‘discount window facilities’) indicates the category of asset to which it belongs.

2.1 Households

Households at the two economies are identical except for a single -and crucial- feature, namely, that those at the emerging economy hoard foreign currency. The device employed to induce such a hoarding demand is a slight modification on the MIU scheme. The optimization problem of the household at the emerging economy is analysed next. The optimization problem of the household at the advanced economy is standard and might be obtained by ignoring the demand for foreign currency in the exposition that follows.

Each period t , the endogenous state vector for the representative household problem is its asset portfolio which consists of domestic currency M_{t-1}^e , deposits D_{t-1} and foreign currency M_{t-1}^a carried over from the preceding period. The household’s sources of funds are (i) labour income, (ii) interests earned on deposits i_{t-1}^d , (iii) the liquidation of its asset portfolio and (iv) profits transferred by the Central Bank, firms and banks Π_t . Those funds, net of taxes T_t , will be divided into consumption C_t , domestic currency M_t^e , deposits D_t and foreign currency M_t^a to be carried over to the succeeding period. In so doing, the agent faces the budget constraint,

$$P_t^c C_t + M_t^e + D_t + s_t M_t^a = W_t L_t + (1 + i_{t-1}^d) D_{t-1} + M_{t-1}^e + s_t M_{t-1}^a + \Pi_t - T_t \quad (1.1)$$

⁹This action is implemented very frequently in emerging economies as a means of hedging against inflation.

for all t with $(M_{-1}^e, D_{-1}, M_{-1}^a)$ given. Processes for transferred profits, taxes, prices, interest rates and exchange rates $\{\Pi_t, T_t, P_t^c, i_{t-1}^d, s_t\}_{t=0}^\infty$ are regarded as exogenous by the representative agent.¹⁰ In addition, he/she ranks processes for leisure, consumption, real balances and the real value of hoarded foreign currency according to

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\phi_c}}{1-\phi_c} - \frac{L_t^{1+\phi_l}}{1+\phi_l} + \frac{1}{1-\phi_{m^e}} \left(\frac{M_t^e}{P_t^c} \right)^{1-\phi_{m^e}} + \frac{1}{1-\phi_{m^a}} \left(\frac{s_t M_t^a}{P_t^c} \right)^{1-\phi_{m^a}} \right] \right\} \quad (1.2)$$

In order to solve the Sequential Problem (1.1)-(1.2) the optimal policy function for the agent's decision variables must satisfy four Euler equations,

$$C_t^{-\phi_c} = \beta(1 + i_t^d) \mathbb{E}_t \left[\left(\frac{P_t^c}{P_{t+1}^c} \right) C_{t+1}^{-\phi_c} \right] \quad (1.3)$$

$$\left(\frac{M_t^e}{P_t^c} \right)^{-\phi_{m^e}} = \left(\frac{i_t^d}{1 + i_t^d} \right) C_t^{-\phi_c} \quad (1.4)$$

$$\left(\frac{s_t M_t^a}{P_t^c} \right)^{-\phi_{m^a}} = \left\{ 1 - \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\phi_c} \left(\frac{P_t^c}{P_{t+1}^c} \right) \left(\frac{s_{t+1}}{s_t} \right) \right] \right\} C_t^{-\phi_c} \quad (1.5)$$

$$\frac{C_t^{-\phi_c}}{P_t^c} = \frac{L_t^{\phi_l}}{W_t} \quad (1.6)$$

together with the budget constraint (1.1). It must be noted that the solution to the problem of the household sector at the advanced economy excludes the equation (1.5).

The economic meaning of each euler equation shows the trade off that each household face in t . Equation (1.3) displays an intertemporal trade-off: the household can deposit each unit of currency in the bank and withdraw the money (capital plus interest) next period to buy consumption goods or spend this unit of money in current consumption. According to (1.3), the optimal decision should be one that equal the marginal utility of the current consumption to the expected value of the marginal utility of future consumption discounted by the real interest rate for deposits.

Equation (1.4) summarizes the intratemporal trade-off between two 'goods' that bear utility, domestic currency and consumption. It states that the household must demand the amount of money that equal the marginal rate of substitution between money and consumption to the opportunity cost of money $\left(\frac{i_t^d}{1+i_t^d} \right)$. In other words, each period the household might reduce one unit of money spent in consumption and deposit it in the bank to obtain a yield of i^d . The real value of it is $\frac{i^d}{1+i^d}$ and the present value of the real value (discounted by the real interest rate for deposits, r^d) is $\frac{i^d}{(1+i^d)(1+r^d)}$ which is equal to $\frac{i^d}{1+i^d}$.

Equation (1.5) shows the intratemporal trade-off between foreign currency and consumption and, consequently, the interpretation is similar to (1.4). Note that (1.5) incorporates the expected devaluation since it is understood as a loss in the value of the local currency. Thus, the higher the expected devaluation, the higher the demand for foreign currency.

Finally, equation (1.6) shows the intratemporal trade off between leisure and consumption. This Euler equation states that the marginal utility of leisure and consumption discounted by their respective cost should be equal.

¹⁰All along the paper, the nominal exchange rate s_t is taken to be the price of the emerging economy currency measured in units of the advanced economy currency.

Consumer Price Index: domestic vs imported consumption

Full integration of goods market will be assumed. In addition, household's consumption will be modelled as a Dixit-Stiglitz composite,¹¹

$$C_t = (C_t^a)^\gamma (C_t^e)^{1-\gamma} \quad (1.7)$$

where C_t^a represents an index of household's purchases of goods produced in the advanced economy (that is, imported consumption by the emerging economy) and C_t^e is an index of purchases of goods produced in the emerging economy (that is, domestic consumption). It is also assumed a unit elasticity of substitution between the two categories of goods and $0 < \gamma < 1$.

Households optimize (1.7) subject to a given level of consumption expenditure,

$$P_t^c C_t = P_t^{ca} C_t^a + P_t^{ce} C_t^e$$

where P_t^c is the price index of consumption goods, P_t^{ca} is the price index of consumption goods produced in the developed economy and P_t^{ce} the price index of consumption goods produced in the emerging economy. All of them are indexes of prices charged in the emerging economy and, consequently, they are expressed in terms of its currency. Identical definitions apply to the advanced economy.

The first order necessary conditions for this optimization subutility problem implies that households in the emerging economy allocate expenditure across domestic and foreign goods according to

$$P_t^{ca} C_t^a = \gamma P_t^c C_t \quad (1.8)$$

$$P_t^{ce} C_t^e = (1 - \gamma) P_t^c C_t \quad (1.9)$$

which, after some algebra, yields

$$P_t^c = \frac{(P_t^{ce})^{1-\gamma} (P_t^{ca})^\gamma}{(1 - \gamma)^{1-\gamma} (\gamma)^\gamma} \quad (1.10)$$

As noted earlier, similar expressions might be found for the advanced economy.

Consumer Price Index: individual goods

Households must also decide how to allocate their consumption expenditures for domestic goods C_t^e and imported goods C_t^a among individual categories of goods produced domestically and abroad. A *continuum* of goods, indexed by the unit interval $[0, 1]$, is assumed to exist at each economy.

Domestic consumption is represented by a Dixit-Stiglitz index, which is given by

$$C_t^e = \left(\int_0^1 C_t^e(i)^{1-\frac{1}{\eta_c}} di \right)^{\frac{\eta_c}{\eta_c-1}} \quad (1.11)$$

¹¹See Dixit & Stiglitz (1977).

in which $\eta_c > 1$ is the constant elasticity of substitution between varieties of the consumption goods and $C_t^e(i)$ is the quantity of the i -th good (produced at the emerging economy) consumed by the household in period t .¹²

The solution of maximizing the Dixit-Stiglitz subutility index C_t^e for a given level of expenditure in domestic goods $\int_0^1 P_t^e(i) C_t^e(i) di$, provide a demand equation for domestic good i such that

$$C_t^e(i) = \left(\frac{P_t^{ce}}{P_t^e(i)} \right)^{\eta_c} C_t^e \quad (1.12)$$

for all $i \in [0, 1]$, where $P_t^{ce} \equiv \left[\int_0^1 P_t^e(i)^{1-\eta_c} di \right]^{\frac{1}{1-\eta_c}}$.

Alike, imported consumption is represented by a Dixit-Stiglitz index similar to (1.11). In addition, we assume that the elasticity of substitution between varieties of imported consumption goods is equal to that for domestic consumption goods. As a consequence, the demand for imported good h is

$$C_t^a(h) = \left(\frac{P_t^{ca}}{P_t^a(h)} \right)^{\eta_c} C_t^a \quad (1.13)$$

for all $h \in [0, 1]$, where $P_t^{ca} \equiv \left[\int_0^1 P_t^a(h)^{1-\eta_c} dh \right]^{\frac{1}{1-\eta_c}}$.

The same structure holds for the advanced economy.

Nominal exchange rate

Since goods markets are open to international trade without barriers it is natural to assume that the law of one price holds.¹³ Hence, the following conditions must be satisfied by prices:

$$P_t^{ce} = s_t P_t^{ce*} \quad (1.14)$$

$$P_t^{ca} = s_t P_t^{ca*} \quad (1.15)$$

Taking into consideration (1.10) and assuming that $\gamma = \gamma^*$, it is possible to express the nominal exchange rate s_t as

$$s_t = \frac{P_t^c}{P_t^{c*}} = \left(\frac{P_t^{ce}}{P_t^{ce*}} \right)^{1-\gamma} \left(\frac{P_t^{ca}}{P_t^{ca*}} \right)^{\gamma} \quad (1.16)$$

2.2 Firms

The specification of the firm sector is essentially the same for both the emerging and the advanced economy, except for the fact that firms at the advanced economy are subject to

¹²In order to follow the exposition line of this subsection, the analysis here is devoted to the household sector living at the emerging economy. The role of domestic vs imported goods would need to be inverted if we aimed at analysing the advanced economy.

¹³See, for instance, Clarida, Gali and Gertler (2002). The law of one price states that if some good is sold in two markets, then its price must be equal across those markets. Here, it is applied to every individual good produced in the two economies.

credit risk. The exposition that follows treats the problem of the firm at the advanced economy. The problem of the firms at the emerging economy might be obtained by ignoring the credit risk matter at the end of this subsection.

It is assumed the presence of three types of firms: **capital good producers**, where depreciated capital and investment goods are combined to produce new capital and sell them to entrepreneurs. **Entrepreneurs** are wholesalers who take bank credit to finance the purchase of capital goods and also demand labour services to produce the goods that are demanded by **retailers**, who costless differentiate goods and sell them in a monopolistic competitive market.

2.2.1 Goods Producers

Capital good producers buy old depreciated capital goods to entrepreneurs and investment goods to retailers in order to produce homogeneous products. Investment goods are bought in a market under monopolistic competition, while depreciated capital goods are trade in a market of perfect competition. It is also supposed that they sell their products in a competitive market at a price P^{ia*} .

Specifically, and to be clear with the temporal notation, after entrepreneurs use capital in the productive process in t , capital producers purchase all the depreciated physical capital at a price ϱ_t^* . Since the transformation ratio between old and new capital good is one to one (one unit of depreciated capital good is transformed into one unit of new one) the price of them should be the same. Capital good producers also demand investment goods in t and combine them with a Dixit-Stiglitz composite¹⁴ in order to produce capital goods through the following production function:

$$K_t^*(j) = (1 - \delta^*)K_{t-1}^*(j) + I_t^*(j) \quad (2.1)$$

It follows that the j -th capital good producer decides how much to invest in order to optimize

$$\mathbb{E}_t \left[\sum_{n=0}^{\infty} \Xi_t^{(n)*} \Pi_{t+n}^{\text{cgp}*}(j) \right] \quad (2.2)$$

where $\Xi_t^{(n)*}$ is the n -period ahead nominal stochastic discount factor,

$$\Xi_t^{(n)*} = (\beta^*)^n \mathbb{E}_t \left[\frac{P_t^{\text{c}*}}{P_{t+n}^{\text{c}*}} \left(\frac{C_{t+n}^*}{C_t^*} \right)^{-\phi_c^*} \right] \quad (2.3)$$

and the profit of the j -th firm $\Pi_t^{\text{cgp}*}(j)$ is

$$\Pi_t^{\text{cgp}*}(j) = \varrho_t^*(j) [K_{t-1}^*(j)(1 - \delta^*) + I_t^*(j)] - P^{k*} I_t^*(j) - \varrho_t^*(j) K_t^*(j) \quad (2.4)$$

The optimal condition states that the producer sets $P_t^{\text{k}*} = \varrho_t^*(j)$ which imply that $\Pi_t^{\text{cgp}*}(j) = 0$ for all t, j . Since the market for capital goods is perfectly competitive and because one unit of both, depreciated physical capital and investment good, are transformed in a unit of new good, the price that they charge for a unit of new capital good is the price they pay for

¹⁴It is explained in depth in the next section.

a unit of investment good, as it had to be.

Note that the fact that all capital producers are identical means that all of them choose the same $\varrho_t^*(j)$ and allow us to aggregate (2.1). So that, the two equations to be considered in the final equation system are $P_t^{k*} = \varrho_t^*$ and $K_t^* = (1 - \delta^*)K_{t-1}^* + I_t^*$.

Aggregation of Capital: domestic vs imported investment

A scheme similar to that employed for consumption goods is used to aggregate capital goods and the assumption of open trade of capital goods for investment is retained. Firm j investment $I_t^*(j)$ is modelled through a Dixit-Stiglitz index as

$$I_t^*(j) = (I_t^{a*}(j))^{\gamma^*} (I_t^{e*}(j))^{1-\gamma^*} \quad (2.5)$$

where $I_t^{a*}(j)$ and $I_t^{e*}(j)$ denote the index of firm j purchases of domestic and imported capital goods, respectively. As always, the elasticity of substitution between imported and domestic capital goods is unity and $0 < \gamma^* < 1$. Note that γ^* is assumed equal to the analogous parameter for consumption goods.

The rest of the derivation follows the same criterion as for households. Firm j optimize (2.5) subject to a given level of expenditure in capital goods,

$$P_t^{k*} I_t^*(j) = P_t^{ka*} I_t^{a*}(j) + P_t^{ke*} I_t^{e*}(j) \quad (2.6)$$

where P_t^{k*} , P_t^{ka*} and P_t^{ke*} are the price index of investment goods, domestically-produced capital goods and imported capital goods, respectively.

The solution to the intratemporal optimization problem implies

$$P_t^{ka*} I_t^{a*}(j) = \gamma^* P_t^{k*} I_t^*(j) \quad (2.7)$$

$$P_t^{ke*} I_t^{e*}(j) = (1 - \gamma^*) P_t^{k*} I_t^*(j) \quad (2.8)$$

which, in turn, leads to

$$P_t^{k*} = \frac{(P_t^{ke*})^{1-\gamma^*} (P_t^{ka*})^{\gamma^*}}{(1 - \gamma^*)^{1-\gamma^*} (\gamma^*)^{\gamma^*}} \quad (2.9)$$

Aggregation of Capital: individual goods

Domestic and imported capital goods that feed investment are, on their own, composites of individual goods produced locally and abroad, respectively. Let $I_t^{a*}(j, i)$ denote the quantity of the i -th good produced in the advanced economy demanded by the j -th firm of that economy for $i, j \in [0, 1]$, then, the quantity of domestically-produced capital goods demanded by firm j relates to individual domestically-produced goods according to

$$I_t^{a*}(j) = \left[\int_{[0,1]} I_t^{a*}(j, i)^{1-\frac{1}{\eta_k^*}} di \right]^{\frac{\eta_k^*}{\eta_k^*-1}} \quad (2.10)$$

where $\eta_k^* > 1$ is the elasticity of substitution between varieties for the capital good produced in the advanced economy.

As a consequence, the individual demand curve of the j -th firm located at the advanced economy for the i -th good produced at the advanced economy is given by

$$I_t^{a*}(j, i) = \left(\frac{P_t^{ka*}}{P_t^{a*}(i)} \right)^{\eta_k^*} I_t^{a*}(j) \quad (2.11)$$

where P_t^{ka*} is the price index for capital goods produced and sold at the advanced economy.

An index of similar specification relates $I_t^{e*}(j)$, the quantity of imported capital goods demanded by firm j , with $I_t^{e*}(j, h)$, the quantity of the h -th individual imported good demanded by the j -th firm of the advanced economy. As for consumption, the elasticity of substitution is the same for domestic and imported capital goods. Moreover, the demand of domestic and imported capital goods at the emerging economy receives a symmetrical treatment.

It can be concluded from the preceding discussion that the total investment demand for the i -th good (produced at the advanced economy), which is composed by the demand from firms of the advanced economy -indexed by j - and firms of the emerging economy -indexed by h -, is

$$\left[\int_{[0,1]} I_t^{a*}(j, i) dj \right] + \left[\int_{[0,1]} I_t^a(h, i) dh \right] = \left(\frac{P_t^{ka*}}{P_t^{a*}(i)} \right)^{\eta_k^*} I_t^{a*} + \left(\frac{P_t^{ka}}{P_t^a(i)} \right)^{\eta_k} I_t^a \quad (2.12)$$

The total investment demand plus the total consumption demand conform the demand curve faced by the i -th firm.¹⁵

Nominal exchange rate and total demand

As with consumption, the law of one price is supposed to hold. Hence, price indexes for capital are restricted by the following conditions:

$$P_t^{ka} = s_t P_t^{ka*} \quad (2.13)$$

$$P_t^{ke} = s_t P_t^{ke*} \quad (2.14)$$

Again, taking into consideration (2.12) and assuming that $\gamma = \gamma^*$, it is possible to express the nominal exchange rate s_t as

$$s_t = \frac{P_t^k}{P_t^{k*}} = \left(\frac{P_t^{ke}}{P_t^{ke*}} \right)^{1-\gamma} \left(\frac{P_t^{ka}}{P_t^{ka*}} \right)^{\gamma} \quad (2.15)$$

2.2.2 Entrepreneurs

Entrepreneurs produce homogeneous goods which are sold in a competitive market. They use labour and capital to produce them through a typical Cobb Douglas production function:¹⁶

$$Y_t^* = A_t^* (L_t^*)^{\alpha^*} (K_{t-1}^*)^{1-\alpha^*} \quad (2.16)$$

¹⁵In this expression, aggregate investment in goods produced at the advanced economy were defined as $I_t^{a*} = \int_0^1 I_t^{a*}(j) dj$ for the advanced economy and $I_t^a = \int_0^1 I_t^a(h) dh$ for the emerging economy. Analogous definitions apply to I_t^{e*} and I_t^e .

¹⁶Note that it is not necessary to specify the problem for each entrepreneur. Instead it is possible to treat it in aggregated terms because all of them are identical.

where $0 < \alpha^* < 1$ and the total factor productivity A_t^* is assumed to be equal to one.¹⁷

As anticipated, entrepreneurs use K_{t-1}^* in t and then they sell this depreciated physical capital to the capital good producers. In t , entrepreneurs also buy new capital, K_t , to capital good producers which will become productive in $t + 1$. So note that the amount of capital used in the current productive process was decided one period before, alternatively, K_t^* is chosen in t and used in the next period. The labour timing is different: entrepreneurs in t demand L_t^* unit of labour which will be used in the same period to produce Y_t^* .

In this model, entrepreneurs fund the total expenditure in capital goods with banking loans, mathematically:

$$\varrho_t^* K_t^* = B_t^* \quad (2.17)$$

This means that, alike the cost of hiring labour, acquiring capital as input has an associated additional cost which is the interest rate paid for the loan. Put it differently, there is a kind of bias to hiring labour productive services because of the way of financing each other.

The optimal demand for K_t^* is the one that equal the expected marginal return¹⁸ to the marginal cost of capital. It can be expressed in nominal terms as

$$\mathbb{E}_t \left(\frac{P_{t+1}^{ia*} \alpha^* (Y_{t+1}^*/K_t^*) + (1 - \delta^*) \varrho_{t+1}^*}{\varrho_t^*} \right) = 1 + i_t^{b*} \quad (2.18)$$

The left-hand side of (2.18) is the expected marginal nominal return of K_t^* . Specifically it is the value of the marginal productivity ($P_{t+1}^{ia*} \alpha^* Y_{t+1}^*/K_t^*$) plus the nominal income from the sale of the depreciated capital, in terms of the nominal cost of acquiring it, that is to say, the price that the entrepreneurs paid to capital good producers in t . The right-hand side is the nominal cost of borrowing to fund the capital good purchase.

On the other hand, the optimal demand for L_t^* follows the same criteria: entrepreneurs demand the amount of labour for which the marginal cost of hiring it is equal to the value of its marginal productivity:

$$W_t^* = P_t^{ia*} \left(\frac{(1 - \alpha^*) Y_t^*}{L_t^*} \right) \quad (2.19)$$

2.2.3 Retailers

There is a *continuum* of differentiated firms in the retail sector indexed by the unit interval. Each retailer buys wholesale homogeneous goods in a competitive market and sells them to households and capital good producers in a market under monopolistic competition. It is assumed that the cost of differentiation is negligible.

Since the demand faced by each retailer is formed by the world demand for consumption goods made by households and the world demand for investment goods made by capital good producers, the downward-sloping demand curve for a retailer i is:

$$Y_t^*(i) = \left(\frac{P_t^{a*}}{P_t^{a*}(i)} \right)^{\eta^*} Z_t^{a*} \quad (2.20)$$

¹⁷In the future, it will be assumed that A_t^* follows an autoregressive process in order to be able to analyse productivity shocks.

¹⁸Remember that they will be productive next period.

where Z_t^{a*} is the part of global absorption that falls on goods produced in the advanced economy¹⁹. It can be expressed as:

$$Z_t^{a*} = C_t^{a*} + C_t^a + I_t^{a*} + I_t^a + G_t^{a*} + G_t^a \quad (2.21)$$

Equation (2.20) is deduced from (1.16), (2.12) and (2.15), by including consumption demand (from households and governments²⁰) and by assuming $\eta_c = \eta_k$ and $\eta_c^* = \eta_k^* = \eta^*$.²¹

The retail sector is subject to nominal rigidities. The Calvo protocol for pricing imposes that each period t a fraction θ^* (which is chosen randomly according to an i.i.d. process) of firms cannot reset their prices. Hence, θ^* can be interpreted as the magnitude of the rigidity in the advanced economy. Each retailer who can set its price at t would do it in order to solve

$$\max_{P_t^{a*}(i)} \mathbb{E}_t \left\{ \sum_{n=0}^{\infty} (\theta^*)^n \Xi_t^{(n)*} \left[(P_t^{a*}(i) Y_{t+n}^*(i) - \text{TC}_{t+n}^*(Y_{t+n}^*(i))) \right] \right\} \quad (2.22)$$

subject to (2.20).

$P_t^{a*}(i)$ is the nominal price charged in the developed economy of the good produced by the i -th retailer and $\text{TC}_{t+n}^*(Y_{t+n}^*(i))$ is the total cost of producing $Y_{t+n}^*(i)$ units of good. In this model the total cost is simply $P^{ia*} Y^*(i)$.

The optimal value of $P_t^{a*}(i)$ is denote as $\bar{P}_t^{a*}(i)$ and is the same for all the retailers who can reset its price in t . Hence, it will be denote simply as \bar{P}_t^{a*} from now and is the one that solve the following first order necessary condition²²

$$\mathbb{E}_t \left\{ \sum_{n=0}^{\infty} (\theta^*)^n \Xi_t^{(n)*} \left[\left((1 - \eta^*) - \eta^* \text{MC}_{t+n}^* \frac{1}{\bar{P}_t^{a*}} \right) Y_{t+n}^*(i) \right] \right\} = 0 \quad (2.25)$$

where MC^* is the marginal cost.

Note the interesting point that, under no nominal rigidities ($\theta^* = 0$), the optimal setting price would be:

$$\bar{P}_t^{a*} = \frac{\eta^*}{\eta^* - 1} \text{MC}_t^* \quad (2.26)$$

which is the standard criteria used in a market under monopolistic competition: a mark-up, given by the elasticity of substitution, over the marginal cost of production.

¹⁹A similar definition applies to Z_t^e . Note that government expenditures have been added to the definition. This will be explained in detail later in the paper

²⁰The government's behaviour is analysed in section 2.7.

²¹The fact that $\eta_c = \eta_k$ and $\eta_c^* = \eta_k^*$ makes the distinction between consumption and capital price indexes vanish. This assumption is made for simplicity. It will be avoided in order to enrich future empirical analysis.

²²Equation (2.22) will be put it in different way in order to express it recursively and in terms of variables that are well defined in the steady state:

$$\begin{aligned} \Gamma_t^* &= \frac{(\bar{P}_t^{a*})^{1-\eta^*}}{(P_t^{a*})^{-\eta^*} P_{t-1}^{ia*}} Z_t^{a*} + \mathbb{E}_t (\Gamma_{t+1}^*) \\ \Delta_t^* &= \frac{\eta^*}{\eta^* - 1} \frac{P_t^{ia*}}{P_{t-1}^{ia*}} \frac{\bar{P}_t^{a*}}{P_t^{a*}} Z_t^{a*} + \mathbb{E}_t (\Delta_{t+1}^*) \\ \Gamma_t^* &= \Delta_t^* \end{aligned}$$

Finally, the dynamic of the aggregate price level is given by

$$P_t^{a*1-\eta^*} = \theta^* P_{t-1}^{a*1-\eta^*} + (1 - \theta^*)(\bar{P}_t^{a*})^{1-\eta^*} \quad (2.27)$$

Riskiness of borrowers

Each entrepreneur in the advanced economy faces an idiosyncratic shock (ζ_t^*) which is assumed i.i.d. across entrepreneurs and time with a cumulative distribution function given by

$$\mathbb{P}(\zeta_t^* \leq x) = F_t^*(x) \quad (2.28)$$

F_t^* is a log-normal distribution function with unit mean and standard deviation σ_t^{F*} . The standard deviation is allowed to change over time, representing movements in the riskiness of borrowers. A change in the riskiness of borrowers acts as a demand-side financial shock. It is further supposed that the log of σ_t^{F*} follows a first-order autoregressive process:

$$\log(\sigma_t^{F*}) = \rho_\sigma^* \log(\sigma_{t-1}^{F*}) + \xi_t^* \quad (2.29)$$

where ξ_t^* is an i.i.d. white noise process with standard deviation σ_{ξ^*} .

The variable $\bar{\zeta}_t^*$ is defined as a cutoff value: entrepreneurs who receive any value lower than the cutoff are unable to repay their loan in full. Since entrepreneurs give the loan back to banks at the beginning of the following period, they will be able to repay the loan only if the net income in the current period after the occurrence of the shock is higher than capital plus interest. Hence, $\bar{\zeta}_t^*$ is defined as the value of ζ_t^* that satisfies

$$\bar{\zeta}_t^* \left(P_t^{ia*} Y_t^* - [W_t L_t + \varrho_{t-1}^* K_{t-1}^* - \varrho_t^* (1 - \delta^*) K_{t-1}^*] \right) = (1 + i_{t-1}^{b*}) B_t^* \quad (2.30)$$

Thus, the aggregate value of the defaulted loans in nominal terms is,

$$\Omega_t^* = \left(\int_0^{\bar{\zeta}_t^*} \zeta_t^* F(\zeta_t^*) di \right) (1 + i_{t-1}^{b*}) B_t^* \quad (2.31)$$

It is assumed that an agency collects the assets of the defaulted entrepreneurs and distributes them to banks in proportion to their loan market share.

2.3 Banks: Advanced Economy

The financial sector which intermediates between households and firms in the advanced economy is characterized as a monopolistically competitive banking sector, at least with respect to deposits and loans. To be specific, banks fund loans to firms with households' deposits, but they may also take credit in a perfectly competitive interbank market and/or access a discount window facilities (DWF_t) provided by the Central Bank. All financial contracts are assumed to be of a one-period length, with deposits, interbank loans and discount window facilities being "no risk" contracts.

There is a *continuum* of banks indexed by the unit interval $[0, 1]$. Aggregate saving deposits and bank loans are structured according to standard Dixit-Stiglitz indexes,

$$D_t^* = \left(\int_0^1 D_t^*(i) \frac{\eta_d^* - 1}{\eta_d^*} di \right)^{\frac{\eta_d^*}{\eta_d^* - 1}} \quad (3.1)$$

$$B_t^* = \left(\int_0^1 B_t^*(i) \frac{\eta_b^* - 1}{\eta_b^*} di \right)^{\frac{\eta_b^*}{\eta_b^* - 1}} \quad (3.2)$$

The deposit interest rate index $(1 + i_t^{d*})^{-1}$ and the borrowing interest rate index $(1 + i_t^{b*})$ are given by:²³

$$\frac{1}{1 + i_t^{d*}} = \left(\int_0^1 \left(\frac{1}{1 + i_t^{d*}(i)} \right)^{1 - \eta_d^*} di \right)^{\frac{1}{1 - \eta_d^*}} \quad (3.3)$$

$$1 + i_t^{b*} = \left(\int_0^1 (1 + i_t^{b*}(i))^{1 - \eta_b^*} di \right)^{\frac{1}{1 - \eta_b^*}} \quad (3.4)$$

where $\eta_d^* > 1$ and $\eta_b^* > 1$ are the elasticities of substitution in the deposit and loan markets, respectively.

Each commercial bank $i \in [0, 1]$ faces the demand curves for deposits and loans,

$$D_t^*(i) = \left(\frac{1 + i_t^{d*}(i)}{1 + i_t^{d*}} \right)^{\eta_d^*} D_t^* \quad (3.5)$$

$$B_t^*(i) = \left(\frac{1 + i_t^{b*}(i)}{1 + i_t^{b*}} \right)^{\eta_b^*} B_t^* \quad (3.6)$$

In order to maximize their expected profits, banks set nominal interest rates on deposits and loans ($i_t^{d*}(i)$ and $i_t^{b*}(i)$) and the quantity of interbank borrowing and discount window facilities ($L_t^*(i)$ and $DWF_t^*(i)$). They take the aggregate amount of loans and deposits and the interest rate index for loans, deposits and discount window facilities as given. The profit function is defined as the income net of total expenditures. If income and expenditures are recorded using the cash method, income in $t + 1$ is the principal plus interest of non-defaulted loans offered in t , the defaulted assets that were collected by an specific agency and distributed into each bank in proportion to their loan market share,²⁴ capital plus interest of government bonds bought in t ($B_t^{g*}(i)$),²⁵ households' deposits of $t + 1$, money borrowed from other banks and from the Central Bank in $t + 1$. On the other hand, total expenditure is defined as the sum of the principal plus interests of deposits, interbank lending and discount windows facilities taken in t , loans to firms in $t + 1$ and purchase of government bonds in

²³The price in the deposit function is inverted because the quantity demanded of deposits made by households will increase when the bank raises its interest rate.

²⁴Let $1 + i_t^{bENNR*}(i) = \frac{\eta_b^* - 1}{\eta_b^*} [1 - F_t^*(\bar{c}_t^*)] [1 + i_t^{b*}(i)] + \frac{\Omega_t^*}{B_t^*}$ be defined as the Expected Net Nominal Return (ENNR) to bank i for each dollar of loans originated.

²⁵The government sector is explained in a succeeding section.

$t + 1$. Hence, banks' optimization problem is

$$\begin{aligned} \max_{\substack{i_t^{d*}(i), i_t^{b*}(i), L_{t+1}^*(i) \\ DWF_{t+1}^*(i), B_{t+1}^{g*}(i)}}} \Pi_{t+1}^{B*} = & [1 - F_t^*(\bar{\zeta}_t^*)] [1 + i_t^{b*}(i)] B_t^*(i) + \frac{B_t^*(i)}{B_t^*} \Omega_t^* + [1 + i_t^{g*}] B_t^{g*}(i) \\ & + D_{t+1}^*(i) + L_{t+1}^*(i) + DWF_{t+1}^*(i) - B_{t+1}^*(i) - B_{t+1}^{g*}(i) \\ & - [1 + i_t^{d*}(i)] D_t^*(i) - (1 + i_t^{b*}) L_t^*(i) - (1 + i_t^{d^{wf*}}) DWF_t^*(i) \end{aligned} \quad (3.7)$$

subject to (3.5), (3.6), (5.3) and the fact that the money they borrow from households, other banks and the Central Bank is allocated in profitable loans to firms or in government bonds,

$$D_t^*(i) + L_t^*(i) + DWF_t^*(i) = B_t^*(i) + B_t^{g*}(i) \quad (3.8)$$

The optimal policy function for the bank's decision variables must satisfy the following equations,

$$1 + i_t^{d*}(i) = \frac{\eta_d^*}{1 + \eta_d^*} (1 + i_t^{g*}) - \frac{\eta_d^*}{1 + \eta_d^*} \left[\left(1 - \frac{D_t^*}{B_t^{g*}} \right) (1 + i_t^{g*}) \left(\frac{DWF_t^*(i)}{B_t^* + B_t^{g*}} \right) \right] \quad (3.9)$$

$$\frac{\eta_b^* - 1}{\eta_b^*} [1 - F_t^*(\bar{\zeta}_t^*)] [1 + i_t^{b*}(i)] + \frac{\Omega_t^*}{B_t^*} = 1 + i_t^{ib*} \quad (3.10)$$

$$1 + i_t^{bENNR^*}(i) = 1 + i_t^{ib*}$$

$$B_t^{g*}(i) = \left[\frac{\eta_d^* (1 + i_t^{g*})}{1 + \eta_d^* (1 + i_t^{d*})} \right]^{\eta_d^*} - \frac{(1 + i_t^{ib*})}{(1 + i_t^{g*})} \left[\frac{D_t^*}{B_t^* + B_t^{g*}} \right]^{-1} \quad (3.11)$$

In order to explain the economic meaning of (3.9),²⁶ it is necessary to advance that the interest rate of discount windows facilities for bank $i \in [0, 1]$ is a function of its risk: the higher its liabilities ($D^*(i)$) the higher the access cost to $DWF(i)$, and the higher its stock of risk-free assets ($B^{g*}(i)$) the fewer the funding cost of $DWF(i)$. Thus, for each unit money of deposits, bank i has two costs. One of them is direct: the cost paid to households ($i^{d*}(i)$) (first term of the left-hide side of the equation in the corresponding footnote); the another one is an indirect cost: the marginal increase in $i_t^{d^{wf*}}(i)$ due to a rise in the risk of bank i perceived by the Central Bank, ceteris paribus (second term of the left-hide side of the equation in the footnote). Assuming that the bank allocate that unit of money in government bonds, then the marginal benefit of that unit of money are two. Again, one of them is direct, capital plus the interest rate paid by the public bond (first term of the right-hide side of the footnote) and the another one is indirect: the marginal decrease in $i_t^{d^{wf*}}(i)$ due to a reduction in the risk of bank i after buying the risk-free asset. Equation (3.9) states that the optimal nominal rate on saving deposits is the one that equal the total marginal cost (note that the direct cost is adjusted by a mark-up given by the elasticity of substitution of deposits since the bank has a monopolistic power) to the total marginal

²⁶The extended expression of it is

$$\frac{\eta_d^*}{1 + \eta_d^*} (1 + i_t^{d*}(i)) + (1 + i_t^{g*}) \left(\frac{DWF_t^*(i)}{B_t^* + B_t^{g*}} \right) = (1 + i_t^{g*}) + (1 + i_t^{g*}) \left(\frac{D_t^*}{B_t^* + B_t^{g*}} \right) \left(\frac{1}{B_t^{g*}} \right) DWF_t^*(i)$$

profit.

The optimal nominal rate for loans is such that the expected net nominal return for each unit of money lent is equal to the marginal opportunity cost of money, as it is shown in (3.10). Recall that the expected net nominal return for each unit of money lent is the interest rate charged to those loans which were repaid $(1 + i_t^{d*}(i))$ adjusted by a “mark-up” $(\frac{\eta_b^* - 1}{\eta_b^*})$ and weighted by $1 - F_t^*(\bar{c}_t^*)$ (the proportion of total loans that were paid it back) plus the value of the defaulted loans that were distributed between banks according to their loan market share.

Finally, (3.11) states that the optimal demand for government bonds is the result of two forces. On one hand, since a government bond is a profitable investment, the higher its yield (in terms of the marginal cost of money), the higher the demand for it (first term in the right-hand side of (3.11)). On the other hand, since government bonds are understood as a risk-free asset, the higher the banking sector risk, the higher the amount of government bond that the bank i should have to access to a given discount window facility rate (last term in the right-hand side of (3.11)).²⁷

It is possible to obtain the deposit interest rate index from (3.9),(3.3) and from the fact that $\int_0^1 DW F_t^*(i) di = DW F_t^*$,

$$1 + i_t^{d*} = \frac{\eta_d^*}{1 + \eta_d^*} (1 + i_t^{g*}) - \frac{\eta_d^*}{1 + \eta_d^*} \left[\left(1 - \frac{D_t^*}{B^* B_t^{g*}} \right) (1 + i_t^{g*}) \left(\frac{DWF_t^*}{B_t^* + B^* B_t^{g*}} \right) \right] \quad (3.12)$$

the loan interest rate index from (3.10) and (3.4),

$$1 + i_t^{bENNR^*} = 1 + i_t^{ib*} \quad (3.13)$$

and the total amount of government bonds demanded by commercial banks from (3.11) and the fact that $\int_0^1 B^* B_t^{g*}(i) di = B^* B_t^{g*}$,

$$B^* B_t^{g*} = \left[\frac{\eta_d^* (1 + i_t^{g*})}{1 + \eta_d^* (1 + i_t^{d*})} \right]^{\eta_d^*} - \frac{(1 + i_t^{ib*})}{(1 + i_t^{g*})} \left[\frac{D_t^*}{B_t^* + B^* B_t^{g*}} \right]^{-1} \quad (3.14)$$

2.4 Banks: Emerging Economy

Banking sector at the emerging economy is specified in a simpler manner. As their pairs in the advanced economy, banks in the emerging one raise funds by creating deposits and lend those to firms, which require funds to finance investment.

There is, again, a *continuum* of banks indexed by the unit interval $[0, 1]$. But they differ from banks in the advanced economy in that the market for deposits is perfectly competitive. Yet, the market for loans to firms is subject to imperfect competition. Aggregation in the loan market is described with the aid of a Dixit-Stiglitz index,

$$B_t = \left(\int_0^1 B_t(i)^{\frac{\eta_b - 1}{\eta_b}} di \right)^{\frac{\eta_b}{\eta_b - 1}} \quad (4.1)$$

²⁷See equation (5.3) and the associated explanation.

And the interest rate index is analogously deduced,

$$1 + i_t^b = \left(\int_0^1 (1 + i_t^b(i))^{1-\eta_b} di \right)^{\frac{1}{1-\eta_b}} \quad (4.2)$$

where $\eta_b > 1$ is the elasticity of substitution in the loan market.

It follows from the latter expression that the demand curve for loan funds faced by bank $i \in [0, 1]$ is given by

$$B_t(i) = \left(\frac{1 + i_t^b}{1 + i_t^b(i)} \right)^{\eta_b} B_t \quad (4.3)$$

Additionally, banks in the emerging economy may accumulate government bonds issued both in the advanced (${}^B B_t^{g*}$) and in the emerging economy (${}^B B_t^g$).

The problem of an individual bank consists of choosing the amount of deposits it creates ($D_{t+1}(i)$), the interest rate it charges on new loans ($i_t^b(i)$), and the amount of government bonds (${}^B B_{t+1}^{g*}(i)$) and (${}^B B_{t+1}^g(i)$) it will accumulate and carry on to the next period. The objective function is defined in a similar way as it was done for banks in the advanced economy, that is, taking into account the cash method. Thus, the problem of the bank in the emerging economy reduces to

$$\begin{aligned} \max_{\substack{D_{t+1}(i), i_t^b(i) \\ {}^B B_{t+1}^{g*}(i), {}^B B_{t+1}^g(i)}} \quad & \Pi_{t+1}^B = [1 + i_t^b(i)]B_t(i) + s_t(1 + i_t^{g*}){}^B B_t^{g*}(i) + (1 + i_t^g){}^B B_t^g(i) + D_{t+1}(i) \\ & - B_{t+1}(i) - s_t{}^B B_{t+1}^{g*}(i) - {}^B B_{t+1}^g(i) - [1 + i_t^d(i)]D_t(i) \end{aligned} \quad (4.4)$$

subject to (4.3) and to its balance sheet restriction, that is, the condition that total deposits must equal the sum of loans to firms and government bonds,

$$D_t(i) = B_t(i) + s_t{}^B B_{t+1}^{g*}(i) + {}^B B_{t+1}^g(i) \quad (4.5)$$

The optimal policy function for the bank must satisfy the following equations,

$$1 + i_t^b = \frac{\eta_b}{\eta_b - 1} (1 + i_t^d) \quad (4.6)$$

$$i_t^d = i_t^g \quad (4.7)$$

$$\mathbb{E}_t \left[\Xi_t^{(1)} \left((1 + i_t^g) - (1 + i_t^{g*}) \frac{s_{t+1}}{s_t} \right) \right] = 0 \quad (4.8)$$

since the optimal decision is the same for all banks the index was omitted in the last equations. Interpretation is almost direct. Equation (4.6) shows that, because of monopolistic competition, banks charge an interest rate on loans higher than the deposit interest rate, the marginal cost, with the mark-up being determined by the elasticity of substitution in the loan market as usual. Equation (4.7) is an arbitrage condition while (4.8) contains the uncovered interest parity on government bond rates.

2.5 Monetary Policy: Advanced Economy

The Central Bank sets the reference interest rate according to a Taylor Rule extended by a term through which it responds to conditions in the financial system. Mathematically, the interest rate obeys the following rule:

$$1 + i_t^{\text{ib}^*} = v^* \left(\frac{P_t^{\text{c}^*}}{P_{t-1}^{\text{c}^*}} \right)^{\phi_\pi^*} \left(\frac{Y_t^*}{Y_t^{\text{n}^*}} \right)^{\phi_y^*} \left(\frac{i_t^{\text{b}^*}}{i_t^{\text{d}^*}} \right)^{-\phi_w^*} \quad (5.1)$$

where v^* , ϕ_π^* , ϕ_y^* and ϕ_w^* are non-negative coefficients and $Y_t^{\text{n}^*}$ represents the natural GDP of the developed economy. Thus the monetary authority raises the interest rate if inflation increases or the GDP growth rate is over its natural value²⁸ and reduces the interest rate if the spread of interest rates in the banking sector increases.

Since the monetary authority may be in a liquidity trap where the interest rate is near to zero and the economy still does not shows signals of recovery, the Central Bank is also able to use discount window facilities as an alternative policy instrument. Hence the Central Bank's balance sheet is constituted by government bonds and discount windows lent to commercial banks (asset side) and money in circulation (liability side).

Table 1: **Central Bank's Balance Sheet. Advance Economy**

Assets	Liabilities
Govt Bonds ($^{\text{CB}^*}B_t^{\text{g}^*}$)	Money (M_t^*)
DW Facilities (DWF_t^*)	

The purpose of a discount window is to provide short term cash to any fundamentally sound bank. However, it charges a penalty interest rate $i^{\text{dwf}^*}(i)$ for discount window facilities to bank i . This interest rate pay a premium over a risk-free rate ($1 + i^{\text{g}^*}$) given by the risk of the banking sector and the risk of the bank (i). The banking sector risk is defined by the total liabilities related to the total assets of the sector as a whole, while the risk associated with bank i is given by its liabilities net of its free-risk assets (both, as fractions of the aggregate values of the sector). Specifically, $i^{\text{dwf}^*}(i)$ is defined as:

$$1 + i_t^{\text{dwf}^*}(i) = (1 + i_t^{\text{g}^*}) \left(\frac{D_t^*}{B_t^* + {}^{\text{B}^*}B_t^{\text{g}^*}} \right) \left(\frac{D_t^*(i)}{D_t^*} - \frac{{}^{\text{B}^*}B_t^{\text{g}^*}(i)}{{}^{\text{B}^*}B_t^{\text{g}^*}} \right) \quad (5.3)$$

Each period, the monetary authority obtains benefits that are transferred to households. These profits come from the interest earned for the stock of national government hold and for the discount windows facilities borrowed to commercial banks. Mathematically:

$$\Pi_t^{\text{CB}^*} = i_{t-1}^{\text{dwf}^*} DWF_{t-1} + i_{t-1}^{\text{g}^*} {}^{\text{CB}^*}B_{t-1}^{\text{g}^*} \quad (5.4)$$

2.6 Monetary Policy: Emerging Economy

Monetary policy in the emerging economy consists of a Taylor Rule for deposits interest rate in a formulation similar to that employed for the advanced economy but with the difference

²⁸This value is defined as the level of GDP in steady state in the absence of price rigidities ($\theta^* = 0$).

that the Central Bank in this economy does not respond to changes in the interest rate spread (which is constant, indeed),

$$1 + i_t^d = v \left(\frac{P_t^c}{P_{t-1}^c} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^n} \right)^{\phi_y} \quad (6.1)$$

where v , ϕ_π and ϕ_y are non-negative coefficients.

The Central Bank may incur in sterilized interventions into the foreign exchange market by varying its stock of foreign bonds, which could be motivated by the aim of compensating fluctuations in the exchange rate.²⁹ The Central Bank balance sheet shows government bonds, both foreign and domestic, in the asset side, and high-powered money on the liability side.

Table 2: **Central Bank's Balance Sheet. Emerging Economy**

Assets	Liabilities
AE Govt Bonds (${}^{\text{CB}}B_t^{g*}$)	Money (M_t^*)
EE Govt Bonds (${}^{\text{CB}}B_t^g$)	

Even though space for an exchange rate policy is extremely narrow it is tempting to specify a rule for international reserves -i.e. the Central Bank's stock of foreign bonds-, which could convey the spirit of an intervention aimed at moderating exchange rate fluctuations,

$${}^{\text{CB}}B_t^{g*} = \kappa {}^{\text{CB}}B_{t-1}^{g*} + \nu \frac{s_{t-1}}{s_t} + \epsilon_t^{s*} \quad (6.2)$$

where $\nu > 0$ and ϵ_t^{s*} is Gaussian white noise with standard deviation σ^{s*} . This rule dictates that the Central Bank will increase its stock of foreign bonds whenever the nominal exchange rate is falling and viceversa, so to smooth fluctuations in the value of domestic currency.

Again, there is a quasifiscal result generated on interests earned on bonds and currency depreciation,

$$\Pi_t^{\text{CB}} = [s_t(1 + i_{t-1}^{g*}) - s_{t-1}] {}^{\text{CB}}B_{t-1}^{g*} + i_{t-1}^g {}^{\text{CB}}B_{t-1}^g \quad (6.3)$$

2.6.1 Dynamic of price indexes

It is by no means unusual to find in the literature that the citizens' welfare of a country is measured in terms of consumption prices. Furthermore, since P_t^{k*} and P_t^k do not affect directly either the utility function or the budget constraints of citizens of each country, it may be appropriate to turn thoughts to P^{c*} and P^c . This is why although the dynamic of capital goods inflation will be presented here, only the dynamics of the aggregate price level for consumption goods will be deduced mathematically. The analysis is developed for the emerging economy. A similar argument applies to the advanced economy.

According to the Calvo framework, it is assumed that a set $S(t) \subset [0, 1]$ of firms are unable to change their prices in period t . Considering the definition of the aggregate price

²⁹However, in this model the capability of the Central Bank of affecting the exchange rate is severely limited by the law of one price.

level for consumption goods P_t^{ce} (for domestic goods) and P_t^{ca} (for imported goods from the emerging economy's perspective), both denominated in the currency of the emerging economy, and the fact that all firms which change their prices in t choose the same price \bar{P}_t^{ce} in the emerging economy and \bar{P}_t^{ca*} in the advanced one (the latter being, this time, measured in units of the advanced economy currency), the following expressions are obtained:

$$P_t^{ce} = \left[\int_{S(t)} P_{t-1}^{ce}(i)^{1-\eta_c} di + (1-\theta)(\bar{P}_t^{ce})^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} \quad (6.4)$$

$$P_t^{ca*} = \left[\int_{S(t)} P_{t-1}^{ca*}(i)^{1-\eta_c^*} di + (1-\theta^*)(\bar{P}_t^{ca*})^{1-\eta_c^*} \right]^{\frac{1}{1-\eta_c^*}} \quad (6.5)$$

Gross inflation rates for consumption goods produced in the advanced economy (in foreign currency) $\Pi_t^{ca*} = P_t^{ca*}/P_{t-1}^{ca*}$ and for consumption goods produced in the emerging economy (in local currency) $\Pi_t^{ce} = P_t^{ce}/P_{t-1}^{ce}$ can be expressed as:

$$(\Pi_t^{ca*})^{1-\eta_c^*} = \theta^* + (1-\theta^*) \left(\frac{\bar{P}_t^{ca*}}{P_{t-1}^{ca*}} \right)^{1-\eta_c^*} \quad (6.6)$$

$$(\Pi_t^{ce})^{1-\eta_c} = \theta + (1-\theta) \left(\frac{\bar{P}_t^{ce}}{P_{t-1}^{ce}} \right)^{1-\eta_c} \quad (6.7)$$

These expressions clearly show that θ^* and θ could be interpreted as measures of price stickiness in each economy since each firm may reset its price only with probability $1-\theta^*$ and $1-\theta$ in any given period independently of the time elapsed since the last adjustment.

Finally, according to (1.10), (6.6) and (6.7), consumer price inflation in the emerging economy $\Pi_t^c = P_t^c/P_{t-1}^c$ obeys

$$\frac{P_t^c}{P_{t-1}^c} = \left(\frac{P_t^{ce}}{P_{t-1}^{ce}} \right)^{1-\gamma} \left(\frac{P_t^{ca}}{P_{t-1}^{ca}} \right)^\gamma \quad (6.8)$$

$$\Pi_t^c = \left(\frac{s_t}{s_{t-1}} \right)^\gamma (\Pi_t^{ce})^{1-\gamma} (\Pi_t^{ca*})^\gamma \quad (6.9)$$

The last equation shows that consumer price inflation at the emerging economy is the result of changes in the value of the domestic currency, of 'imported' inflation (Π_t^{ca*}) and domestically produced inflation (Π_t^{ca}).

Similarly, the inflation of capital goods in the emerging economy is

$$\Pi_t^k = \left(\frac{s_t}{s_{t-1}} \right)^\gamma (\Pi_t^{ke})^{1-\gamma} (\Pi_t^{ka*})^\gamma \quad (6.10)$$

2.7 Governments

In each economy, the government sector is assumed to play a passive role. That is to say that expenditures and bonds supply are determined by autorregressive processes while the budget

constraint determines the amount of lump-sum taxes (or transfers) that must be collected from (or given to) the household sector. Thus, for the advanced economy,

$$G_t^* = \rho^{G^*} G_{t-1}^* + \epsilon_t^{G^*} \quad (7.1)$$

$$B_t^{g^*} = \rho^{gb^*} B_{t-1}^{g^*} + \epsilon_t^{gb^*} \quad (7.2)$$

$$(1 + i_{t-1}^{g^*}) B_{t-1}^{g^*} + G_t^* = T_t^* + B_t^{g^*} \quad (7.3)$$

with $0 < \rho^{G^*}, \rho^{gb^*} < 1$ and both $\epsilon_t^{G^*}$ and $\epsilon_t^{gb^*}$ are gaussian white noise with standard deviations σ^{G^*} and σ^{gb^*} , respectively.

Similar assumptions apply to the emerging economy. Also, once the expenditure G_t^* is determined by the autorregressive process (7.1), the composition of expenditure, that is, its division into domestic and imported goods and those, in turn, into individual goods, follows the same specification as for consumption goods (with identical elasticities of substitution assumed). This feature was already included in the analysis of the firms problem. It is worth highlighting that government demand for goods reduces to a consumption role, i.e. it does not invest or accumulate capital.

2.8 Equilibrium Conditions

2.8.1 Market clearing conditions and balance of payments

1. Produced goods equilibrium in the advanced economy

$$C_t^{a^*} + C_t^a + I_t^{a^*} + I_t^a + G_t^{a^*} + G_t^a = Y_t^* \quad (8.1)$$

The left-hand side of the equation represents the consumption (private and public) of domestically produced goods (from the perspective of the advanced economy), plus the demand of capital goods (both, domestic and external). While the right-hand side is the aggregate production level.

2. Produced goods equilibrium in the emerging economy.

$$C_t^{e^*} + C_t^e + I_t^{e^*} + I_t^e + G_t^{e^*} + G_t^e = Y_t \quad (8.2)$$

The same considerations made above for the advanced economy are applied to the emerging one.

3. The level of expenditure in the advanced economy results equal to the income level when the production is sold.³⁰

$$P_t^{c^*}(C_t^{a^*} + C_t^e + G_t^{a^*} + G_t^e) + P_t^{k^*}(I_t^{a^*} + I_t^e) = P_t^* Y_t^* \quad (8.3)$$

³⁰Where $P_t^* = \left[(\alpha_t^{ca^*} + \alpha_t^{ca} + \alpha_t^{ga^*} + \alpha_t^{ga}) P_t^{ca^*} + (\alpha_t^{ia^*} + \alpha_t^{ia}) P_t^{ka^*} \right]$ and $\alpha_t^{ca^*} = \frac{C_t^{a^*}}{Y_t^*}$, $\alpha_t^{ca} = \frac{s_t C_t^a}{Y_t^*}$, $\alpha_t^{ga^*} = \frac{G_t^{a^*}}{Y_t^*}$, $\alpha_t^{ga} = \frac{s_t G_t^a}{Y_t^*}$, $\alpha_t^{ia^*} = \frac{I_t^{a^*}}{Y_t^*}$ and $\alpha_t^{ia} = \frac{s_t I_t^a}{Y_t^*}$.

The left-hand side of the equation includes the expenditure in the consumption of domestic and foreign goods incurred by households in the advanced economy, plus the expenditure in capital goods, also domestic and imported. The right-hand side is the nominal income which emerges from the aggregate production of the firms in the advanced economy.

4. The level of expenditure of the emerging economy results equal to the income level when the production is sold

$$P_t^c(C_t^a + C_t^e + G_t^a + G_t^e) + P_t^k(I_t^a + I_t^e) = P_t^e Y_t \quad (8.4)$$

The same considerations made above for both sides of the equation are similar.

5. Financial equilibrium in the advanced economy

$$D_t^* + DWF_t^* = B_t^* + {}^B B_t^{g*} \quad (8.5)$$

The left-hand side of the equation is the sum of the deposits and the discounts provided by the Central Bank, and the right-hand side includes credits to the non-financial private sector plus government bonds.

6. Financial equilibrium in the emerging economy

$$D_t = B_t + s_t B_t^{g*} + {}^B B_t^g \quad (8.6)$$

The considerations made above for both sides of the equation are similar, with the exception that in this case the discount window facilities do not exist, and government bonds from the advanced economy are included taking into account the exchange rate.

7. Exchange rate market equilibrium in the advanced economy

$$M_t^* = M_t^{a*} + M_t^a \quad (8.7)$$

The left-hand side of the equation represents the foreign currency supply, while the right-hand side includes the sum of the demand (domestic and external) of advanced economy currency.

8. Bonds market equilibrium in the advanced economy

$$B_t^{g*} = {}^B B_t^{g*} + {}^B B_t^{g*} + {}^{CB} B_t^{g*} + {}^{CB} B_t^{g*} \quad (8.8)$$

This equation set that the supply of bonds is equal to the sum of bonds demanded by commercial banks (in both economies) plus the demand of bonds by Central Banks (also in both economies).

9. Bonds market equilibrium in the emerging economy

$$B_t^g = {}^B B_t^g + {}^{CB} B_t^g \quad (8.9)$$

Since any agent from the advanced economy demands bonds from the emerging economy, the equilibrium condition sets that the supply of government bonds of the emerging economy equals the demand that commercial banks of its economy and the central bank made.

10. Balance of Payments result. As usual this equation reflects the sum of the result of current account plus capital and financial account:

$$NX_t + RENTS_t = s_t({}^B B_t^{g*} + {}^{CB} B_t^{g*} + M_t^a) - s_t({}^B B_{t-1}^{g*} + {}^{CB} B_{t-1}^{g*} + M_{t-1}^a) \quad (8.10)$$

in which, net exports NX_t are defined as

$$NX_t = Y_t - (C_t + I_t + G_t)$$

and “rental income” $RENTS_t$ is defined as

$$RENTS_t = s_t i_{t-1}^{g*} ({}^B B_{t-1}^{g*} + {}^{CB} B_{t-1}^{g*}) + (s_t - s_{t-1}) ({}^B B_{t-1}^{g*} + {}^{CB} B_{t-1}^{g*} + M_t^a)$$

3 Conclusion

The common wisdom about the policy framework followed by Central Banks revolves around price stability and GDP stability. What this traditional wisdom did not consider was financial stability and the cross-border effects of the monetary policy. The last financial crisis shed light on the importance of a Central Bank that responds to those matters and made many researchers to recognize that the conventional approach needs to be kept under analysis and trying to find new answers. Since this paper is part of a broader project devoted to answer many practical and theoretical questions in this regard, the authors are aware that many aspects are not completely developed.

Thus the proposed theoretical model sets the necessary basis on which the whole project will be built. In this sense, the authors want to contribute to the literature in two matters: first, the differentiation between an advanced economy and an emerging one; second, the incorporation of unconventional monetary policy in the advanced economy which generates spillovers effects on the emerging country that forces its Central Bank to take decisions that would not be taken in the absence of those effects. To cut a long story short, the main features of the emerging economy rest on the existence of a hoarding demand for foreign currency and the fact that its Central Bank intervenes in the exchange rate market with the aim of moderating the exchange rate volatility; while the main peculiarity of the advanced economy is the existence not only of a Central Bank that provides liquidity aid to any fundamentally sound bank but also the existence of risky entrepreneurs.

There is a lengthy path ahead. There are two directions in which more work will be done in the project, namely: (i) enriching the monetary policy instruments that Central Banks

are able to use (ii) analysing the comparison between a world with and without monetary policy cooperation.

The aim of incorporating alternative tools to be used alongside the monetary instruments developed in the body of this paper is to pursue a better incentive structure for key institutions which could deeply hurt the real economy and make it more volatile. For instance, financial institutions that were considered “Too Big to Fail” did not have the incentive to take appropriate levels of risk since the Central Bank eventually would work as a Lender of last Resort. However, incorporating microprudential tools could mitigate these incentives that do not lead to optimal solutions and, consequently, could reduce financial instability.

On the other hand, (ii) will allow the authors to illuminate whether there is a gain to be obtained out of international coordination of monetary policy for certain economies or not. There are many doubts on this regard: while Liu and Pappa (2005) are in favour of an international monetary integration due to it raises the profit level between economies which are not symmetric, Coenen et al (2008) found that given the openness degree of the United States and the European Community, the gains from monetary policy coordination are small.

And in order to find new answers, following that old phrase, the suggestion should be “keep calm and carry on”.

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